Problem Set 2 Solutions

1. 2-25

**Problem 2–25.** Find $v_x$ and $i_x$ in Figure P2–25.

The current $i_x$ points in the opposite direction as the 500-$\mu$A current source, so $i_x = -500 \, \mu$A. Using Ohm’s law, we have $v_x = Ri_x = (68 \times 10^3)(-500 \times 10^{-6}) = -34 \, \text{V}$.

2. 2-37

**Problem 2–37.** Find the equivalent resistance $R_{\text{EQ}}$ in Figure P2–37.

Working from the right to the left, combine the 10-k$\Omega$ resistor in parallel with the 15-k$\Omega$ resistor to get an equivalent resistance of 6 k$\Omega$. That resistance is in series with the 33-k$\Omega$ resistor, which yields an equivalent resistance of 39 k$\Omega$. Finally, combine the 39-k$\Omega$ resistance in parallel with the 56-k$\Omega$ resistor to get $R_{\text{EQ}} = 22.99 \, \text{k}\Omega$.

3. EK2-2

![Diagram of circuit](EK2-2.png)

KVL Loop A: \( v_6 - v_4 - v_2 = 0 \)

KVL Loop B: \( v_2 + v_3 - v_1 = 0 \)

KVL Loop C: \( -v_3 + v_4 + v_5 = 0 \)

With the given numbers,

\[ 15 - v_4 - 20 = 0 \Rightarrow v_4 = -5 \, \text{V} \]

\[ 20 + v_3 - v_1 = 0 \]

\[ -v_3 + v_4 + 10 = 0 \Rightarrow v_3 = 5 \, \text{V} \]

\[ v_1 = 25 \, \text{V} \]
4. EK2-3

Combine $R_2$ and $R_3$ in parallel:

Then $v_1 = (25\Omega) i_1 = (25\Omega)(0.002A) = 50mV$ [i.e. $i_1 = 2mA$]

$\therefore v_1 = v_2 = (75\Omega) i_1 = (75\Omega)(0.002A) = 150mV$

5. EK2-4

\[ P_{R1} = v_1 i_1 = (0.05V)(0.002A) = 0.1mW \]
\[ P_{R2} = v_2 i_2 = (0.15V)(0.0015A) = 0.225mW \]
\[ P_{R3} = v_3 i_3 = (0.15V)(0.0005A) = 0.075mW \]
\[ P_{source} = (v_1 + v_2) i_1 = (0.2V)(0.002A) = 0.4mW \]

Check: $0.1mW + 0.225mW + 0.075mW = 0.4mW$
6. 2-45

**Problem 2-45.** Using no more than four 1-kΩ resistors, show how the following equivalent resistors can be constructed: 2 kΩ, 500 Ω, 1.5 kΩ, 333 Ω, 250 Ω, and 400 Ω.

The following table presents the solutions.

<table>
<thead>
<tr>
<th>$R_{\text{EQ}}$ (Ω)</th>
<th>Combination of 1-kΩ Resistors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>Two resistors in series: $R + R$</td>
</tr>
<tr>
<td>500</td>
<td>Two resistors in parallel: $R \parallel R$</td>
</tr>
<tr>
<td>1500</td>
<td>One resistor in series with a parallel combination of two resistors: $R + (R \parallel R)$</td>
</tr>
<tr>
<td>333</td>
<td>Three resistors in parallel: $R \parallel R \parallel R$</td>
</tr>
<tr>
<td>250</td>
<td>Four resistors in parallel: $R \parallel R \parallel R \parallel R$</td>
</tr>
<tr>
<td>400</td>
<td>Two resistors in series in parallel with two resistors in parallel: $(R + R) \parallel R \parallel R$</td>
</tr>
</tbody>
</table>

7. 2-47

**Problem 2-47.** Find the equivalent practical voltage source at terminals A and B in Figure P2-47.

A current source in series with a resistor is equivalent to just the current source, so we can remove the 5-Ω resistor without affecting the performance of the circuit between terminals A and B. That leaves a 5-A current source in parallel with a 10-Ω resistor. The current source and parallel resistor can be converted into a voltage source in series with the same resistor. The value for the voltage source follows Ohm’s Law, so $v_S = i_S R = (5)(10) = 50$ V. Figure P2-47 shows the resulting circuit.

![Figure P2-47](image)

8. 2-55

**Problem 2-55.** Use voltage division in Figure P2-55 to obtain an expression for $v_L$ in terms of $R$, $R_L$, and $v_S$.

The two right resistors are in parallel and the voltage $v_L$ appears across that combination. Combine the parallel resistors and then use voltage division to develop the expression for $v_L$.

$$R_{\text{EQ}} = R \parallel R_L = \frac{RR_L}{R + R_L}$$

$$v_L = \left[ \frac{R_{\text{EQ}}}{R + R_{\text{EQ}}} \right] v_S = \left[ \frac{RR_L}{R + R_L} \right] v_S = \left[ \frac{RR_L}{\left( R^2 + RR_L + RR_L \right)} \right] v_S$$

$$v_L = \frac{R_L v_S}{R + 2R_L}$$
9. 2-60

**Problem 2–60.** (A) The 1-kΩ load in Figure P2–60 needs 5 V across it to operate correctly. Where should the wiper on the potentiometer be set \( R_x \) to obtain the desired output voltage?

Figure P2–60 shows an equivalent circuit with the potentiometer split into its two equivalent components. To solve the problem, find an equivalent resistance for the parallel combination of resistors and then apply voltage division to find an expression for \( R_x \). Solve for \( R_x \) and select the positive result.

\[
R_{EQ} = R_x \parallel 1000 = \frac{1000R_x}{1000 + R_x}
\]

\[
5 \text{ V} = \frac{R_{EQ}}{5000 - R_x + R_{EQ}} (24 \text{ V})
\]

\[
5 = \left[ \frac{1000R_x}{1000 + R_x} \right] \left( \frac{1000R_x}{5000 - R_x + \frac{1000R_x}{1000 + R_x}} \right) \quad (24) \quad = \quad \frac{24000R_x}{(5000 - R_x)(1000 + R_x) + 1000R_x}
\]

\[
1 = \frac{4800R_x}{5 \times 10^6 + 4000R_x - R_x^2 + 1000R_x}
\]

\[
-R_x^2 + 5000R_x + 5 \times 10^6 = 4800R_x
\]

\[
R_x^2 - 200R_x - 5 \times 10^6 = 0
\]

\[
R_x = -2138 \text{ or } 2338 \Omega
\]

\[
R_x = 2.338 \text{ k}Ω
\]
10. 2-63

Problem 2-63. (A) Figure P2-63 shows a voltage bridge circuit, that is, two voltage dividers in parallel with a source \( v_S \). One resistor \( R_X \) is variable. The goal is often to “balance” the bridge by making \( v_X = 0 \) V. Derive an expression for \( R_X \) in terms of the other resistors when the bridge is balanced.

Let the node between resistors \( R_A \) and \( R_B \) have a voltage \( v_1 \) and let the node between resistors \( R_C \) and \( R_X \) have a voltage \( v_2 \). The goal is to make \( v_1 \) equal \( v_2 \) so that \( v_X \) is zero. Use voltage division to derive expressions for \( v_1 \) and \( v_2 \), set those expressions equal, and solve for \( R_X \).

\[
\begin{align*}
v_1 &= \frac{R_B}{R_A + R_B} (v_S) \\
v_2 &= \frac{R_X}{R_C + R_X} (v_S) \\
\frac{R_B v_S}{R_A + R_B} &= \frac{R_X v_S}{R_C + R_X} \\
R_B (R_C + R_X) &= R_X (R_A + R_B) \\
R_B R_C + R_B R_X &= R_A R_X + R_B R_X \\
R_X &= \frac{R_B R_C}{R_A}
\end{align*}
\]