INTRODUCTION TO CIRCUITS AND ELECTRONICS

ECEN 2250 – Fall Semester 2015/16

Prof. Milos Popovic

Week 4 – Lecture 9
Today’s Summary

• Plan
  • Administrative
  • Finish a few circuit reduction examples
  • Systematic node-voltage analysis (Section 3.1)
### August

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<td>Lecture 9</td>
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Engineering Fellow

- Matthew Haney (matthew.haney@colorado.edu)

- Will provide additional 2h office hours Thursdays 2-4pm
- Will do a review session before each Test and Final Exam
- Will be reachable for ad-hoc assistance
- I will inform the class shortly of his office
Test #1: Friday, September 25

• Will post sections covered by end of week, but count on Chapters 1-3, and all Homework until then.

• HW#4 due this Friday. Short HW#5 due next Wednesday. Will be posted today.

• Best times for review session: Mon aft/eve, Thu mor/aft/eve
  • Brief input from class
  • Doodle poll
Example 3-44

[Diagram of an electrical circuit with a 15 V source, 5 Ω, 10 Ω, and 15 Ω resistors, and a load symbol.]
Example 3-44

(a) 15 V

(b) 15 V

(c) 15 V

(d) 10 V

(e) 545 mA

Figure 3-44
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(Ladder Example from Lecture 7)
Exercise 3-28 but find Norton eq.

We got 4V, 5 kohms Thevenin Norton: 0.8A, 5 kohms.
Ex 3-18

- Find Thevenin equivalent
- Find power to 10 kohm resistor, or to 5V source.
Ex 3-18

Figure 3-47
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Ex 3-19

- Find Norton equivalent
- Find output current $i$ when load is dissipating 5W.
Node-Voltage Method

- Section 3.1
All circuit analysis

1. Connection constraints
   - KCL (zero net current out of a node)
   - KVL (zero net voltage drop around a closed circuit)

2. Device constraints
   - i-v relationship (e.g. \( V = I*R \))

Complexity to solve:
   - \( E \) circuit elements: 2\( E \) equations with 2\( E \) unknowns (i, v for each)
All circuit analysis

• Complexity to solve:
  • E circuit elements: 2E equations with 2E unknowns (i, v for each)

• Connection constraints: KCL, KVL
  • Elements: E eqs
  • KCL: N-1 eqs
  • KVL: E-N+1 eqs
  • Total: 2E equations, 2E unknowns (1 current + 1 voltage per element)
Node-Voltage Method

• Instead of voltages across an element...

• ...define node voltages relative to a reference point.

• Voltage across any element is a difference between two node voltages.
Node-Voltage Method

- Voltage across any element is a difference between two node voltages

- Case A: \( v_1 = v_A \)
- Case B: \( v_1 = v_A - v_B \)
- “Local KVL”
Today’s Summary

- **Plan**
  - Systematic node-voltage analysis (Section 3.1)
    - Supernodes
  - Matrix formulation
  - Examples

- **Next week:**
  - AC (sinusoidal) signals
  - Capacitors and inductors

- **Next Friday: Test #1 (of 3 tests + 1 final exam)**
  - Up to Lecture 12 (end of Monday)
### Where we are

**August**

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### Lecture and Reading Schedule

<table>
<thead>
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Exercise 3-2

First find node voltages, then find $v_x$ and $v_y$.

$v_D = 0$
$v_A = 5\text{V}$
$v_C = 6\text{V}$
$v_B = +6-10 = -4\text{V}$

$v_X = v_A - v_B = 9\text{V}$
$v_Y = v_A - v_C = -1\text{V}$
Formulating Node-Voltage Equations

- No need for KVL (it’s implicit now)
- Write only KCL and element laws

**Example:** 1 ground node, 4 element currents, 2 node voltages
Formulating Node-Voltage Equations (Fig 3-5)

- **KCL**
  - $-i_0 - i_1 - i_2 = 0$
  - $i_2 - i_3 = 0$

- **Elements:**
  - $i_1 = \frac{1}{R_1} v_A$
  - $i_2 = \frac{1}{R_2} (v_A - v_B)$
  - $i_3 = \frac{1}{R_3} v_B$
  - $i_0 = i_S$
Method

N nodes, E elements circuit:
1. Pick a reference node. Label all other voltage node voltages
2. Write KCL equations for all N-1 nodes except reference node
3. Use i-v relationship of each of the E elements to write currents in terms of node voltages and constants
4. Sub element eqs for currents in step 3 into KCL eqns in step 2.

Now you have just N-1 equations with N-1 unknown node voltages (reduced down from 2E). Can write it as a matrix equation, and put into Matlab.
Formulating Node-Voltage Equations

- **KCL**
  - \(-i_0 - i_1 - i_2 = 0\)
  - \(i_2 - i_3 = 0\)
- **Elements:**
  - \(i_1 = \frac{1}{R_1} v_A\)
  - \(i_2 = \frac{1}{R_2} (v_A - v_B)\)
  - \(i_3 = \frac{1}{R_3} v_B\)
  - \(i_0 = i_S\)

- **Matrix formulation:**
  - One equation per KCL eqn
  - Unknowns?
  - Number?
  - Size of matrix?
  - Please read Sec 3-1 part on solving matrix eqns using Cramer’s rule
General approach to writing node voltage equations in matrix by inspection

1. One line = one KCL equation

2. Each KCL eqn: Currents leaving node A are either
   1. \( i = \frac{1}{R} (v_A) \) if R is connected to ground, or...
   2. \( i = \frac{1}{R} (v_A - v_B) \) if R is connected to node B

3. Why symmetric? Because if R is connected from A to B, then it’s connected from B to A.’

4. Unknowns are node voltages. Currents out through resistors into matrix, currents into node from sources into right vector.
Example

\[ \begin{bmatrix} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} \end{bmatrix} \]
Example
Example

\[
\begin{bmatrix}
\frac{1}{R_1} & -\frac{1}{R_1} \\
\frac{1}{R_1} & \frac{1}{R_1}
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix}
= 
\begin{bmatrix}
i_s
\end{bmatrix}
\]
Example

\[
\begin{bmatrix}
\frac{1}{R_1} & -\frac{1}{R_1} \\
-\frac{1}{R_1} & \frac{1}{R_1}
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix}
= \begin{bmatrix}
i_S
\end{bmatrix}
\]
Example

\[ \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1 + R_2} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} i_S \end{bmatrix} \]
Example

- Symmetric matrix… check!
  - BTW, computers are faster at solving symmetric $A x = b$ problems than arbitrary matrices (look up e.g. LAPACK online)
Example Fig 3-8
Example 3-2
Supernodes: What about when there are voltage sources?

- How many nodes N? (KCL equations?)
- Two less (1 per transformation)
Supernodes: What about when there are voltage sources? Another example.
Supernodes

Method 1

\[ v_A = \frac{v_S}{R_S} \]

Method 2

Method 3

Supernode

Figure 3-13
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Example Fig 3-15
Example Fig 3-17
SINUSOIDAL STEADY STATE, CAPACITORS AND INDUCTORS
DC (steady state) response

![Diagram showing steady state response](image)

*Figure 5-1
© John Wiley & Sons, Inc. All rights reserved.*
Types of waveforms

- Step
- Sinusoid
- Pulse train
- Square wave
- Exponential
- Damped sinusoid
- Sawtooth
- Triangular wave

Figure 5-2
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Sinusoidal signals
Sinusoidal signals

\[ v(t) = V_A \cos\left( \frac{2\pi t}{T_0} \right) \text{ V} \]

\[ T_S = 0 \]

\[ V_A \]

\[ -V_A \]

\( T_0 \)

\( T_S \)
Sinusoidal signals

\[ v(t) = V_A \cos\left(\frac{2\pi t}{T_0} - \phi\right) \text{V} \]

\[ T_S > 0 \]

Figure 5-22b
© John Wiley & Sons, Inc. All rights reserved.
Sinusoidal signals

\[ v(t) = V_A \cos \left( \frac{2\pi t}{T_0} + \phi \right) V \]

\[ T_S < 0 \]

Figure 5-22c
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*Any* function can be represented as a linear superposition (sum) of sinusoids.
Therefore if you find the circuit response for a sinusoidal waveform, you have it for any waveform!
Capacitor
Capacitor

Figure 6-1
© John Wiley & Sons, Inc. All rights reserved.
Capacitor

(a)

Figure 6-3a
© John Wiley & Sons, Inc. All rights reserved.
Capacitor

\[ i_C(t) \quad (b) \]

\[ v_C(t) \quad (V) \]

\[ t \quad (ms) \]

\[ i_C(t) \quad (mA) \]

\[ t \quad (ms) \]
Capacitor

\[ i_C(t) \quad C \quad v_C(t) \]

(a)

(b)
Inductor

\[ L \]

\[ v_L(t) \]

\[ i_L(t) \]

\[ \lambda(t) \]

\[ di_L(t) \]

\[ L \]

\[ 1 \]