Negative feedback

Feedback is widely used to accurately control an output signal, making it a) closely follow a reference input, and b) be independent of other unwanted quantities ("disturbances").

We have two examples so far:

**Robot speed control**

![Robot speed control diagram]

**Op amp circuit**

![Op amp circuit diagram]
Introduction to negative feedback and control

We have studied lots of op-amp circuits, such as the inverting amplifier.

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = -\frac{R}{(sC)} = -sRC
\]

- This transfer function depends on \( R \) and \( C \).
- It doesn't depend on anything inside the op amp.
- What is going on inside the op amp?

Bode plot of predicted transfer function:

- OK at low frequency, but high-frequency behavior is non-physical: \( \left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| \to \infty \) as \( \omega \to \infty \)
- What really happens?
To answer—

We need an op-amp model

Op amp circuits are examples of negative feedback systems.

Block diagram of a basic negative feedback system:

\[ \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{G}{1+GH} \]

\[ = \frac{1}{H} \frac{GH}{1+GH} \]

\[ = \frac{1}{H} \frac{T}{1+T} \]

with \( T = GH \)

Negative feedback can

- make the transfer function \( \frac{V_{\text{out}}}{V_{\text{in}}} \) nearly independent of the forward gain \( G(s) \) and strongly dependent on the feedback gain \( H(s) \)
- reduce the output impedance
- reduce the influence of disturbances (temperature variations, power supply voltage, etc.) on the output voltage
- increase the bandwidth

These statements are true when the loop gain is large: \( |M| \gg 1 \)

Let's model the op-amp transfer function \( G(s) \), and then compute the actual \( \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \)
A simple op-amp model

![Diagram of op-amp model]

Typical values:

- $G_o = 10^5 \Rightarrow 100\text{dB}$
- $f_1 = 10\text{ Hz}$
- $\omega_1 = 2\pi f_1$
- Unity gain frequency $= G_o f_1 = 1\text{ MHz}$

$G_{op}(s) = \frac{G_o}{1 + \frac{s}{\omega_1}}$
Insert model into circuit:

\[ V_{in}(s) \]

\[ V(s) \]

\[ G_{op}(s)(v^+-v^-) \]

with \( v^+=0 \)

\[ = -G_{op}(s)v^-(s) \]

Equations of circuit:

\[ V_{out}(s) = G_{op}(s)(v^+-v^-) \]

\[ = -G_{op}(s)v^-(s) \]

also, \( v^-(s) = V_{in}(s) \cdot \frac{R}{R+\frac{1}{SC}} + V_{out}(s) \cdot \frac{\frac{1}{SC}}{R+\frac{1}{SC}} \)

(by superposition)
Put the diagrams together:

Now manipulate into the form on page 2:

Combine series blocks:
Finally, push the minus sign through the summing node:

which is of the form

Simplify feedback loop:

Transfer function is

\[
\frac{v_{out}(s)}{v_{in}(s)} = (-1) \frac{G(s)}{1 + G(s)H(s)}
\]
\[ = (-1) \frac{1}{H(s)} \frac{GH}{1+GH} \]

\[ = (-1) \frac{1}{H(s)} \frac{T(s)}{1+T(s)} \]

where \( T(s) = G(s) H(s) = "\text{loop gain}" \)

The loop gain \( T(s) \) is a measure of how well the feedback loop works. When the loop gain is large, \( |T| \gg 1 \), then

\[ 1 + T(s) \approx T(s) \]

\[ \frac{T(s)}{1 + T(s)} \approx \frac{T(s)}{T(s)} = 1 \]

and

\[ \frac{V_{out}}{V_{in}} \approx (-1) \frac{1}{H(s)} = -sRC \]

which is the transfer function predicted by the virtual short principle.

But if the loop gain is not large in magnitude, then the \( T/(1+T) \) term causes the transfer function to deviate from ideal op amp behavior.
Suppose $R = 1 \text{k}$ and $C = 1 \mu \text{F}$
and $G_{sp}(s)$ is as given on p. 3

Let's construct $T(s)$ and $\frac{v_{out}}{v_{in}} = \frac{1}{s} \frac{T}{1+T}$

\[
T(s) = GH = \frac{R}{R+\frac{1}{sC}} G_{sp}(s) \frac{1}{sRC} = \frac{1}{(1+sRC)} G_{sp}(s)
\]

\[
= \frac{G_o}{(1+sRC)} \left( \frac{1}{1+\frac{s}{\omega_1}} \right)
\]

\[
= \frac{G_o}{(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_2})}
\]

$G_o = 10^5 \rightarrow 100 \text{dB}$

$f_1 = \frac{\omega_1}{2\pi} = 10 \text{ Hz}$

$f_2 = \frac{1}{2\pi RC} = 159 \text{ Hz}$

\[f_c = \text{"crossover frequency"} = \text{frequency where } \|T\| = 1\]