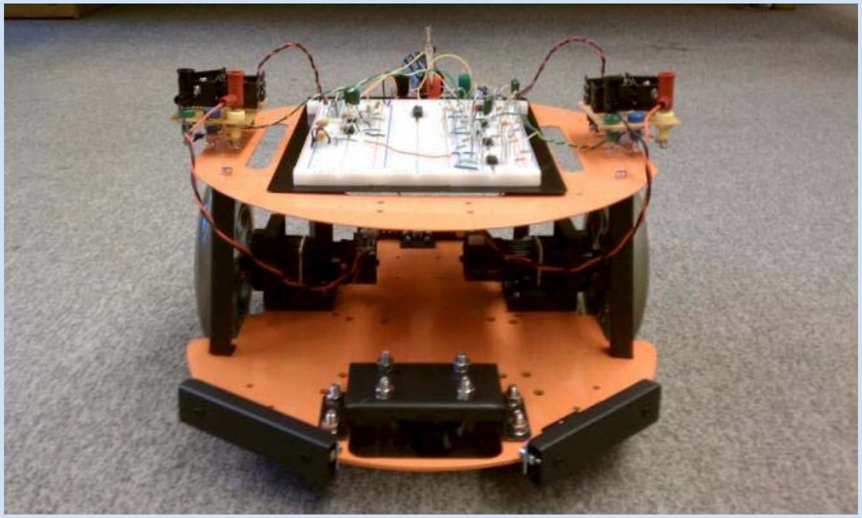
Lab 2 Part B: Supplement

How to approach a complex circuit

(or: Analysis of the tachometer circuit)

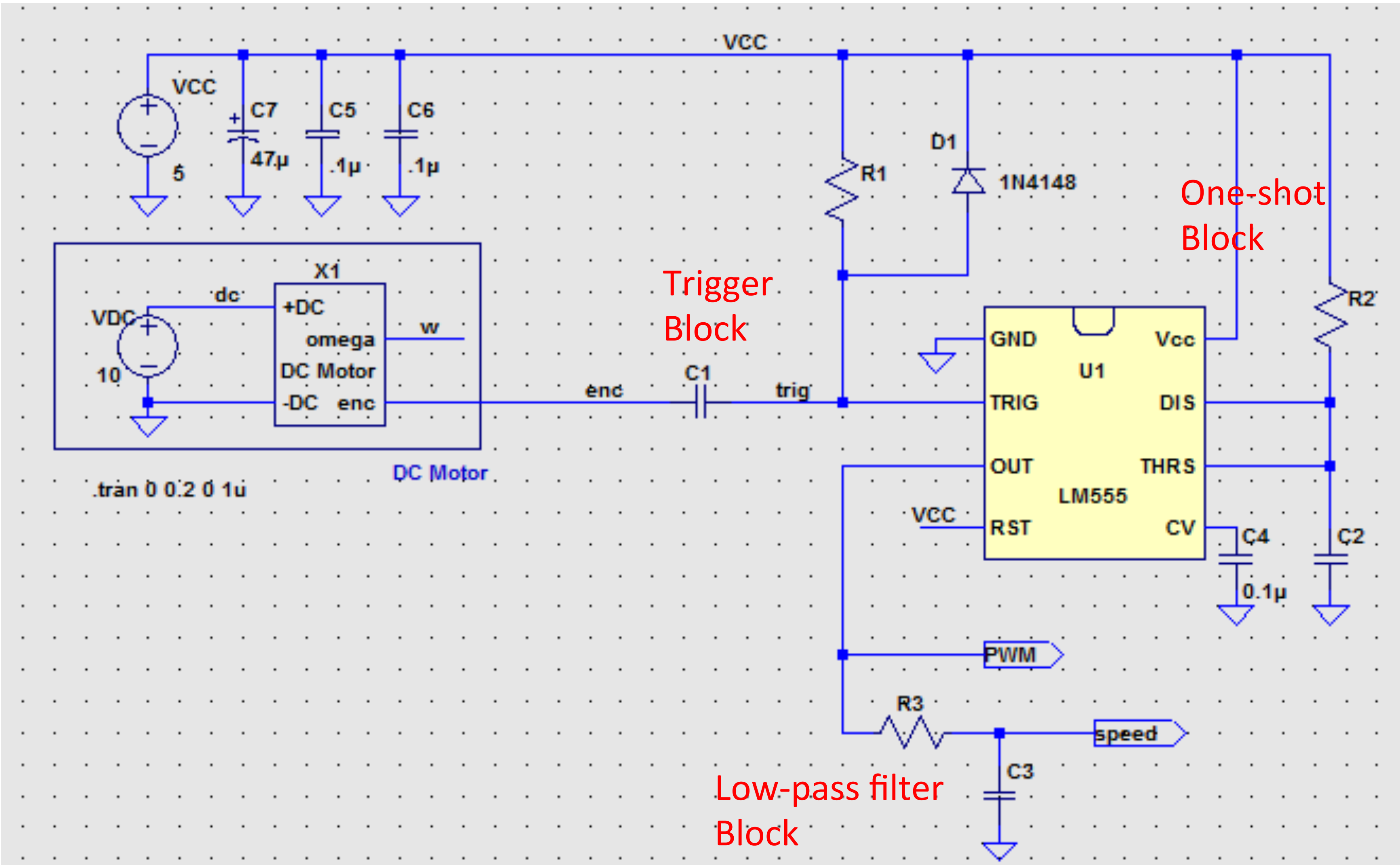


How to approach the analysis (or design, or debugging) of a complex circuit (i.e., a circuit having more than 3 components)

- **1. DON'T**: dive right in and start writing a lot of loop and node equations
  - You will make an algebraic mess and get nowhere
  - *When debugging*: don't just build the whole thing and turn it on, expecting it to work first time
- **2. DO**: Break the circuit down into smaller functional blocks that can be separately understood
  - First try to explain in words (a paragraph) how each block works
  - Replace semiconductors (IC's) with equivalent circuits
  - *When debugging*: get the first block to work before moving on to the next

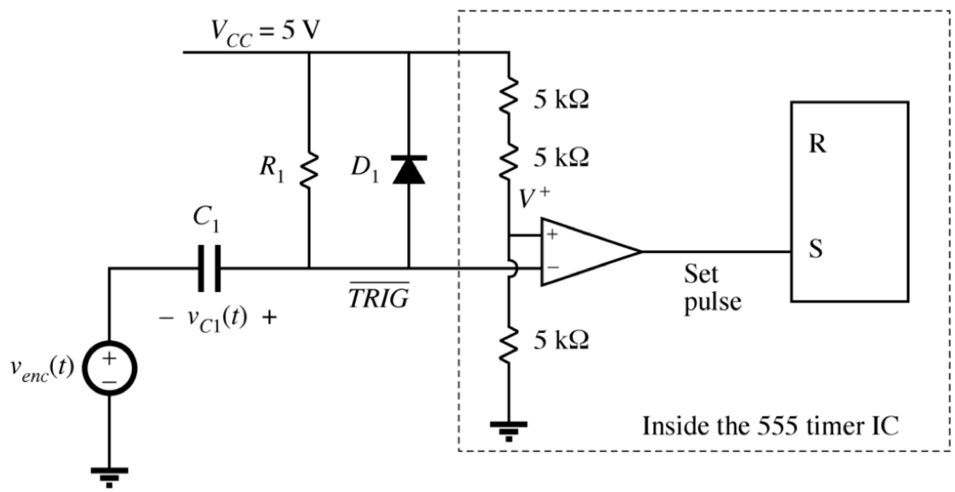
- **3. DO**: For each block, decide what you need to know, and what analysis will be tractable and feasible
  - Identify the input and output signals. The goal is to find the functional relationship between these.
  - Write (relatively small) equations that you can reasonably solve
- **4. DO**: Develop design equations from the results of #3 above
  - Solve your equations for the element values
  - Invariably, you will need more equations, and there is more than one valid answer
    - Develop additional constraints based on your understanding (from part #2) of the circuit
    - Choose impedance levels so that currents and power consumption are reasonable: mA not A, etc.

Breaking the tachometer circuit into blocks



Trigger circuit

Equivalent circuit



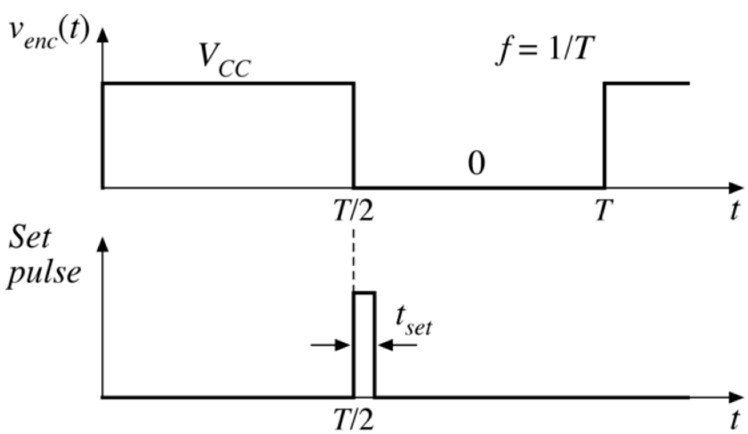
Input: $v_{enc}(t)$, square wave from encoder

Output: Set pulse going to latch

This circuit sets the latch at the falling edge of $v_{enc}(t)$

Soon after the latch is set, we want the set pulse to go away: the set pulse needs to terminate before it is time to reset the latch. But the datasheet for the 555 timer specifies a minimum pulse width of approximately 1 μsec.

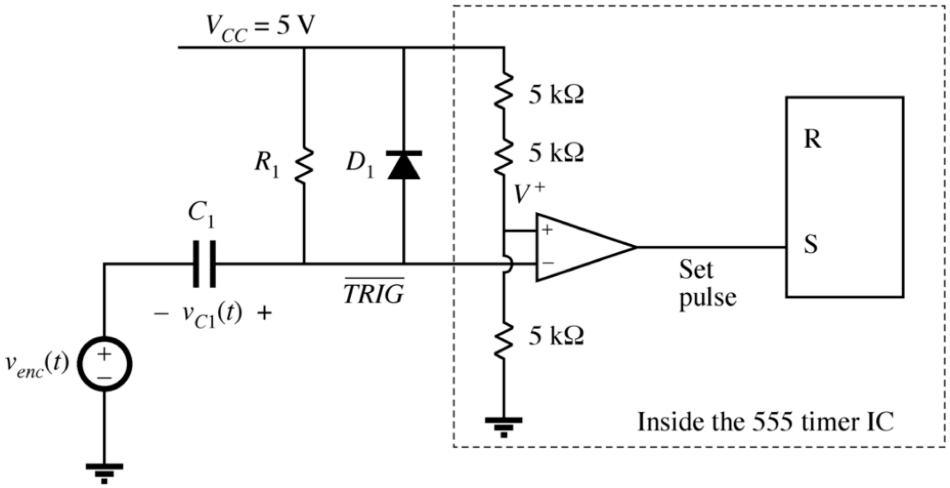
Input and output waveforms

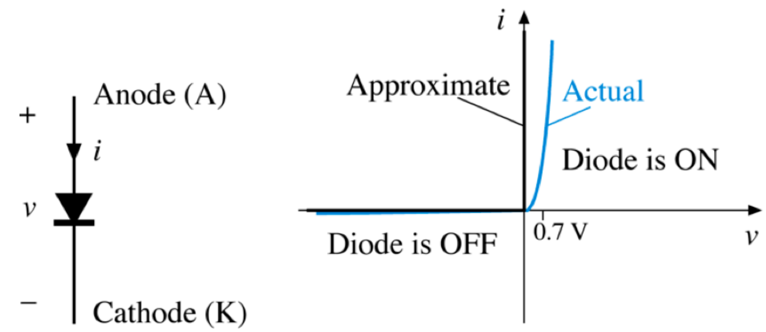


Things we want to know:

- How is f related to the ground speed of the wheels? (left as exercise for students)
- How does this circuit generate the set pulse?
- How long is t_{set}, and how should we choose R_1 and C_1?

Trigger circuit analysis: preliminaries





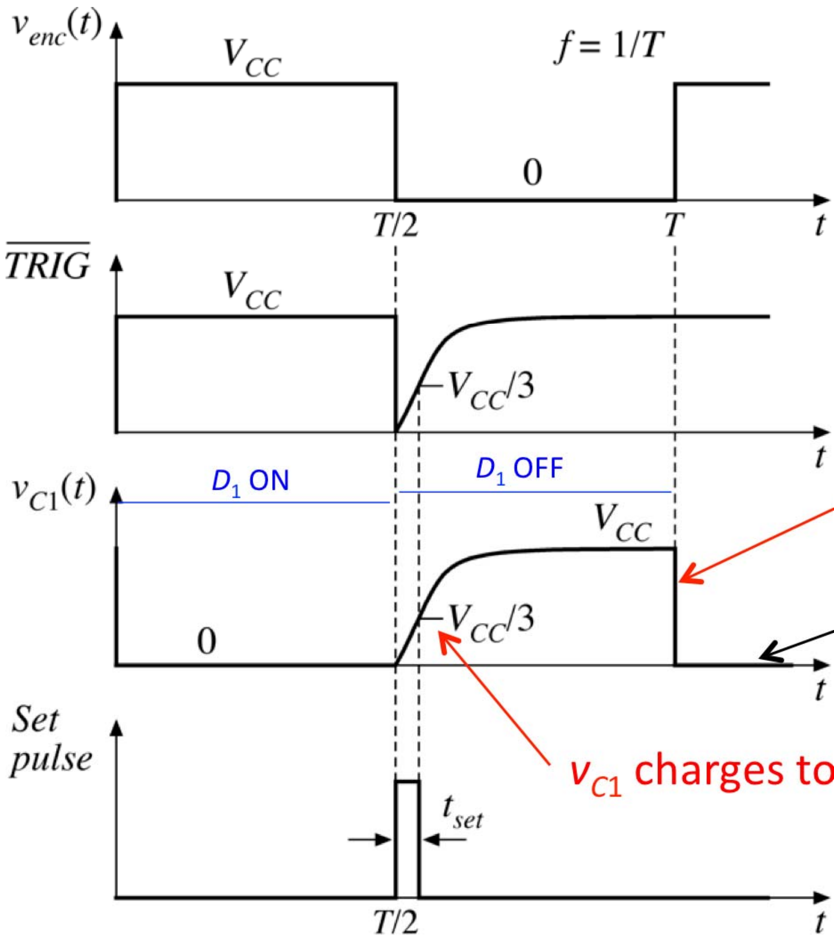
Characteristics of silicon *p*–*n* diode

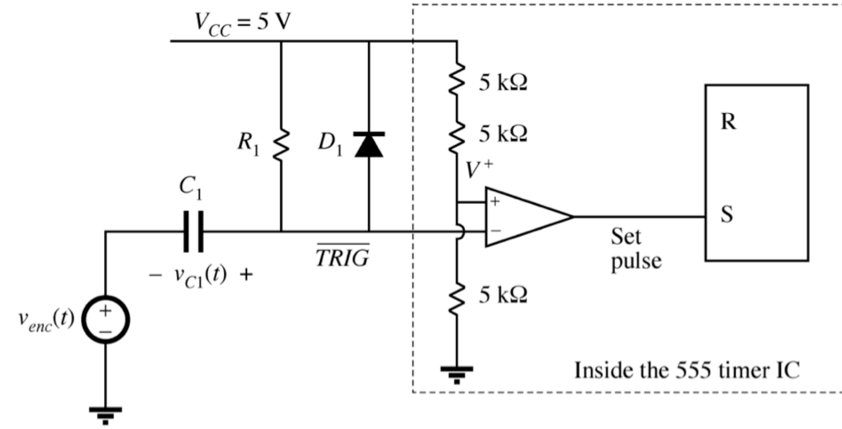
1. V^+ is determined by voltage divider: $V^+ = V_{CC}$ (5 kΩ)/(15 kΩ) = $V_{CC}/3$ = 1.67 V

2. Diode D_1 behaves as an on-off switch ("a one-way valve"). The conducting state depends on the voltage and current applied to the diode by the circuit:

- If the circuit makes the voltage at the anode terminal greater than the voltage at the cathode (or if the circuit applies positive current to the diode), then the diode turns on. This clamps the anode to a voltage slightly greater than the cathode voltage (i.e., the diode is approximately a short circuit).
- If the circuit attempts to apply reverse current (from cathode to anode) or reverse voltage, then the diode turns off. This causes the diode to become (approximately) an open circuit.

Trigger circuit: waveforms





v_{C1} discharges to 0 through D_1

D_1 does not allow $\overline{TRIG} > V_{CC}$
so $v_{C1} = 0$

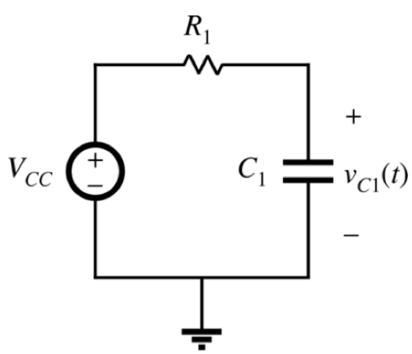
v_{C1} charges to V_{CC} through R_1

Trigger circuit: analysis to find t_{set}

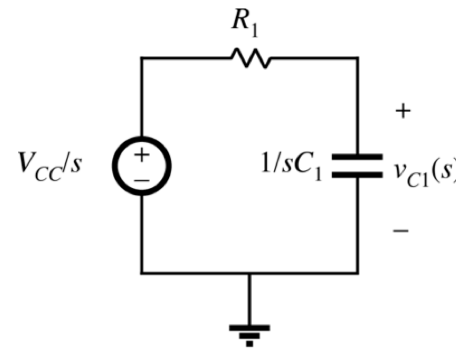
During the set pulse interval:

- Diode D_1 is OFF (open circuit)
- $v_{enc}(t) = 0$
- v_{C1} is initially zero, and C_1 charges through R_1

Therefore, for this interval the circuit reduces to:



Let's define $t = 0$ at the beginning of this interval, and take the Laplace transform of the circuit to solve for the capacitor voltage:



$$v_{C1}(s) = \frac{V_{CC}}{s}\frac{\left(1/sC_1\right)}{\left(R + 1/sC_1\right)} = V_{CC}\frac{1}{s\left(1 + sR_1C_1\right)}$$

Now use partial fraction expansion to take inverse Laplace transform (exercise for the student); the result is:

$$v_{C1}(t) = V_{CC}\left(1 - e^{-t/R_1C_1}\right)$$

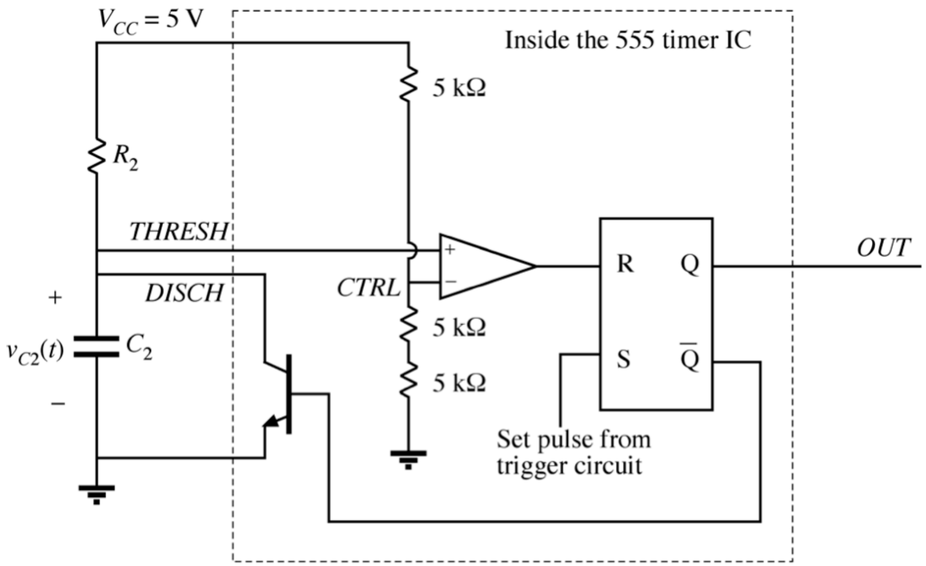
The trigger circuit comparator causes the set pulse to end when $v_{C1} = V_{CC}/3$, at time $t = t_{set}$. Hence:

$$v_{C1}(t_{set}) = V_{CC}\left(1 - e^{-t_{set}/R_1C_1}\right) = \frac{V_{CC}}{3}$$

Solve for t_{set}: $\quad t_{set} = R_1C_1 \ln\left(\frac{3}{2}\right)$

One-shot circuit

Equivalent circuit

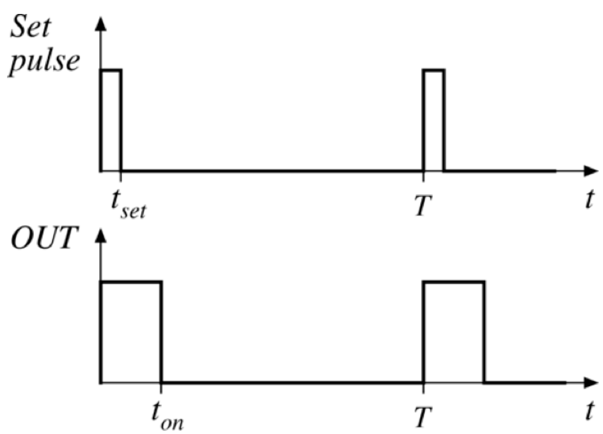


Input: Set pulse from trigger circuit

Output: "Out" signal having pulse width t_{on}

This circuit resets the latch when the capacitor voltage $v_{C2}(t)$ reaches the threshold voltage CTRL

Input and output waveforms



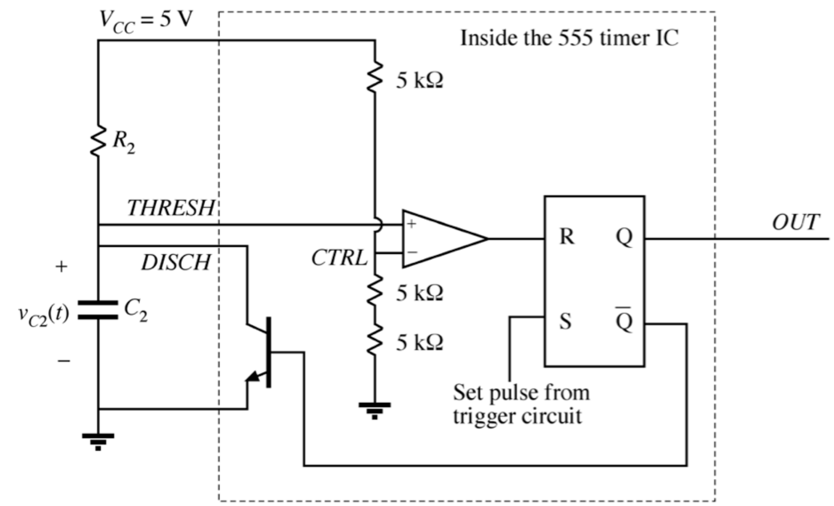
Things we want to know:

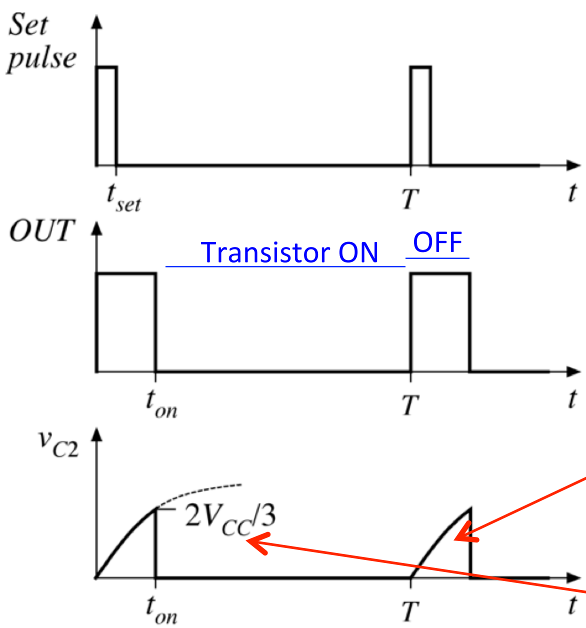
- How does this circuit generate the t_{on} pulse?
- How long is t_{on}, and how should we choose R_2 and C_2?

One-shot circuit: waveforms

The transistor operates as an on/off switch:

- When Q is high then the transistor is off, and vice-versa.
- When on, the transistor shorts out C_2 and holds $v_{C2} = 0$.
- When the transistor is off, capacitor C_2 charges through R_2.





v_{C2} charges towards V_{CC} through R_2

Comparator resets latch when v_{C2} reaches *CTRL* (= $2V_{CC}/3$)

One-shot circuit: analysis

Exercise for the student: derive an expression for t_{on}

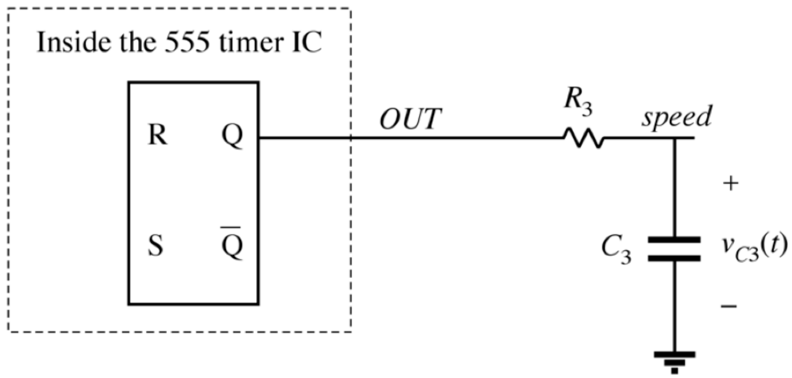
Show the steps of:

- The Laplace-transformed circuit to be solved
- Derive relevant equation and take inverse transform
- Solve for t_{on}

Then use your result to select values for R_2 and C_2.

Low-pass filter circuit

Equivalent circuit

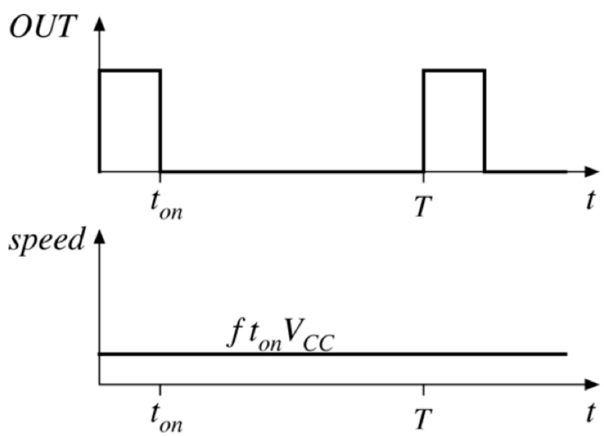


Input: Pulse-width-modulated signal *OUT*

Output: "speed" signal having a DC value proportional to *f*

This circuit is a low-pass filter that passes the DC component of *OUT* but removes the fundamental and higher frequency components

Input and output waveforms



Things we want to know:

- How does this circuit operate on the pulse-width-modulated OUT signal to produce the *speed* signal?
- How should we choose R_3 and C_3?

Low-pass filter circuit: analysis

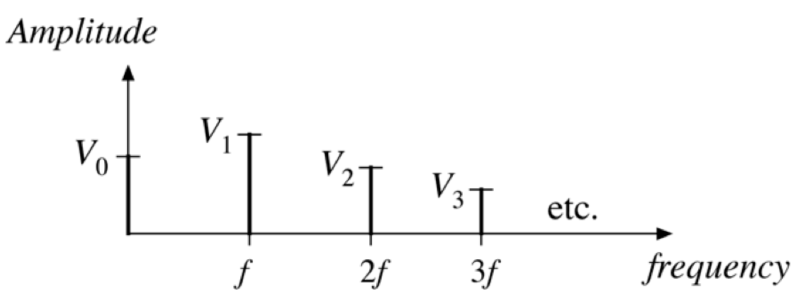
The pulse-width-modulated signal *OUT*(*t*) can be represented by Fourier analysis as a DC component V_0 plus a sum of sinusoids called *harmonics*:

$$OUT(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n2\pi ft + \theta_n)$$

The harmonics have frequencies that are integral multiples of the fundamental frequency *f*. The DC component is given by the average value:

$$V_0 = \frac{1}{T}\int_0^T OUT(t)\,dt$$

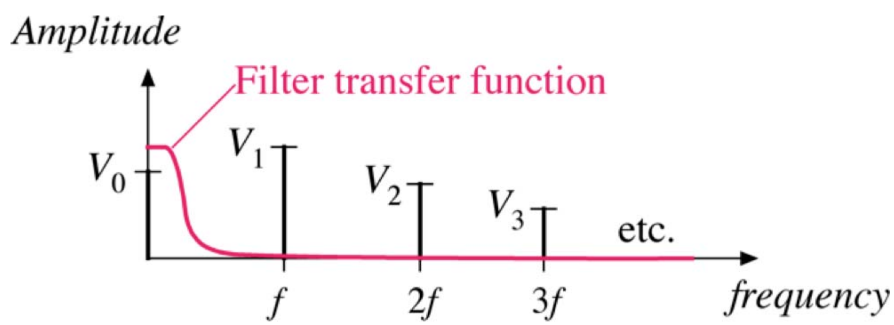
The *amplitude spectrum* is a plot of the harmonic amplitudes vs. frequency:



Filter design

The effect of the R_3–C_3 filter on each individual harmonic can be found by phasor analysis of the circuit: use phasors to solve the circuit and find how the amplitude of a sinusoid is changed by the circuit, as a function of frequency.

We want to choose R_3 and C_3 so that the filter passes the DC component and any very low-frequency variations that occur as a result of the changing speed of the robot. But we want the filter to reject the components of *OUT* at the fundamental frequency f and its harmonics. So the filter should have a *transfer function* (i.e., the ratio of its output voltage amplitude to its input voltage amplitude, vs. frequency) that looks like this:



Exercise for students: use phasor analysis to solve for the transfer function of the R_3–C_3 filter. Select appropriate values for R_3 and C_3.