ECEN 3400  ELECTROMAGNETIC FIELDS AND WAVES  11/5/2010

Hour Exam 2

One 8-1/2 \times 11 sheet of notes and a calculator allowed. Neither text nor course notes may be consulted. Show (i.e., explain) all work.

1. (20 points) Two strip conductors (denoted 1 and 2) are located above a ground conductor as shown in cross-section in Figure (a) below.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure_a.png}
\caption{(a)}
\end{figure}

A shield conductor in the form of a third vertical strip (denoted s) is placed in between the two strips as shown in Figure (b) or (c) above. In Figure (b) the shield is not connected to anything else, while in Figure (c) the shield is connected to the ground. Assume that strip (1) is at a positive potential $V$ with respect to ground, while the total charge on conductor (2) is zero. Draw a rough sketch of what the electric field lines look like for each of Figures (a), (b) and (c). In which case is the potential of strip (2) with respect to ground the smallest?

2. (20 points) Derive the expression for the capacitance of a conducting sphere of radius $a$ in infinite empty space. [Hint: Start by using Gauss’ Law to find the field.]

3. (20 points) A uniform but time-varying magnetic field $\mathbf{B}(t) = u_x B_0 \cos \omega_0 t$ is produced in a region of space. The field amplitude is $B_0 = 0.3$ T, and the frequency of the field is $f_0 = \omega_0/2\pi = 1$ MHz. A square loop of conducting wire whose side is $a = 2$ cm is placed in the field, such that its surface normal makes an angle of $60^\circ$ to the $x$-axis. The loop has a small gap, across which an induced voltage appears. Evaluate this induced voltage.
4. (20 points) A resistor consists of two rectangular solid portions of resistivities $\rho_1$ and $\rho_2$ arranged as shown below.

![Diagram of the resistor](image)

Derive an expression for the resistance of this composite resistor. If the dimensions are $w = h = 1$ cm, $a = 0.5$ cm and $b = 1.5$ cm, and the resistivities are $\rho_1 = 4$ $\Omega$m and $\rho_2 = 0.5$ $\Omega$m, give the value for this resistance.

5. (20 points) A long solenoid of radius $a$ and length $l \gg a$ (you can neglect end fringing effects) is wound tightly with $N'$ turns of wire per meter around a ferrite rod whose relative permeability is $\mu_r$. A planar loop of wire is located completely outside the solenoid as shown. Begin by using Ampère’s Law to find the magnetic field $\mathbf{B}$ of the solenoid if a current $I_1$ flows in its windings. Then use this result to find an expression for the mutual inductance of the loop and the solenoid.

![Diagram of the solenoid and loop](image)

[Note: You do not need to know anything about the area of the loop to do this problem.]