

# 3

## ***Coulomb's Law in Vector Form and Electric Field Strength***

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### 3.1 Introduction

We have seen that sources of an electrostatic field are stationary and time-constant electric charges. This is the simplest form of the general electromagnetic field: since there are no moving or time-varying charges, there is no magnetic field. Although the electrostatic field is only a special case of the electromagnetic field, it occurs frequently. It is essentially the field that drives the electric current through wires and resistors in electric circuits; the field driving tiny signals in our nerves and brain; the field in depletion layers of transistors in computer chips; or the field that ionizes air (makes it conducting) just before a lightning bolt.

In this introductory course, the electrostatic field is of specific importance. The simplicity of the physical concepts in the electrostatic field allows us to develop mathematical models in a straightforward way. Later on, in more complex fields, we will be able to solve difficult practical problems using these concepts and tools as they are or with minor modifications.

### 3.2 Coulomb's Law in Vector Form

We have seen that Coulomb's law is an experimentally established law which describes the force between two charged bodies that are small compared to the distance between them. Such charged bodies are referred to as point charges. Coulomb's law in Eq. (1.1) is an algebraic expression that needs an additional explanation in words. The force directed along the line joining the two bodies is either repulsive or attractive. It is repulsive if the two charges are of the same kind, or sign, and attractive if they are of different kind.

Using vector notation, it is not difficult to write Coulomb's law in a form that does not need such additional explanations. Let  $\mathbf{r}_{12}$  be the vector directed from charge  $Q_1$  to charge  $Q_2$  (Fig. 3.1), and  $\mathbf{u}_{r12} = \mathbf{r}_{12}/|\mathbf{r}_{12}|$  be the unit vector of  $\mathbf{r}_{12}$ . (A number of notations have been used for unit vectors, e.g.,  $\mathbf{a}_r$ ,  $\mathbf{u}_r$ , or  $\hat{\mathbf{r}}$  for the unit vector of vector  $\mathbf{r}$ . We will adopt  $\mathbf{u}_r$  to remind us that it is a *unit* vector.) We can then express Coulomb's law in Eq. (1.1) in the following form:

$$\mathbf{F}_{e12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \mathbf{u}_{r12} \quad \text{newtons (N).} \quad (3.1)$$

*(Coulomb's law in vector form)*

The unit vector  $\mathbf{u}_{r12}$ , and thus also the force  $\mathbf{F}_{e12}$ , is directed from  $Q_1$  toward  $Q_2$ , so that the additional explanation in words is not necessary anymore. Moreover, we have adopted the convention that a positive charge implies a *positive sign*, and a negative charge a *negative sign*. Complete information about the direction of the force is therefore also contained in this vector expression (recall that  $-\mathbf{r}$  means the same vector, but in the opposite direction). If the two charges are of the same sign, the vector  $\mathbf{u}_{r12}$  is as in Fig. 3.1, and if they are of different signs, we have instead  $-\mathbf{u}_{r12}$ , which means that the force is attractive. (If necessary, before proceeding further read Sections A1.1–A1.3 of Appendix 1, "Brief survey of vectors and vector calculus.")

The constant  $\epsilon_0$  is known as the *permittivity of free space* or *of a vacuum*. According to Eq. (3.1) its unit is  $\text{C}^2/(\text{m}^2 \text{N})$ . Usually a simpler unit,  $\text{F/m}$  (farads per meter), is used, as will be explained in Chapter 8. The value of  $\epsilon_0$  is

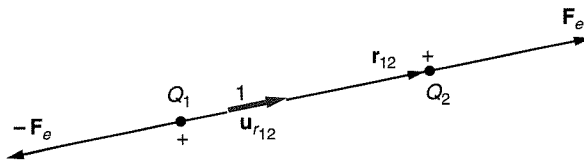


Figure 3.1 Notation in the vector form of Coulomb's law

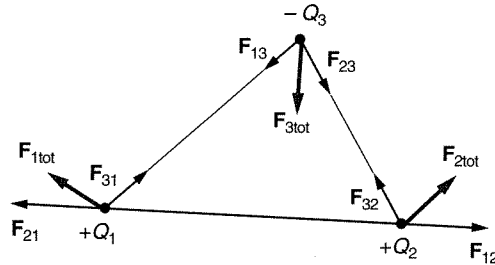


Figure 3.2 Example of vector addition of Coulomb forces

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ farads per meter (F/m)} \approx \frac{1}{36\pi \cdot 10^9} \text{ F/m.} \quad (3.2)$$

*(Permittivity of a vacuum)*

The reason for writing  $1/(4\pi\epsilon_0)$  in Coulomb's law instead of, say, simply  $k$ , is purely practical, removing the factor  $4\pi$  from many other commonly used equations. Also, in many equations the permittivity of free space then appears as it is,  $\epsilon_0$ , and not as its reciprocal.

Coulomb measured the electric force in air. We will see later that the electrical properties of air are very nearly the same as those of a vacuum, i.e., of space with no elementary particles of matter. Coulomb's law in Eq. (3.1) is therefore valid for charges that are strictly in a vacuum, but the presence of air does not change the result substantially. Therefore, the term "free space" usually implies vacuum or air.

Coulomb also measured the force on one point charge (e.g.,  $Q$ ) due to several point charges (e.g.,  $Q_1, Q_2, \dots$ ). He concluded that the total force is obtained by *vector addition* of Coulomb's forces acting on  $Q$  by charges  $Q_1, Q_2, \dots$  individually. We know that mechanical forces on a body are summed in the same way, which is known as the *principle of superposition for forces*. An example of vector addition of Coulomb forces is sketched in Fig. 3.2.

How large are charges and electric forces we encounter around us? The charges rarely exceed a few nanocoulombs ( $1 \text{ nC} = 10^{-9} \text{ C}$ ). The largest electric forces around us do not exceed about 1 N, which is the weight of a small glass of water. Thus, measured by our standards, electric forces are very small.

*Questions and problems:* Q3.1 to Q3.10, P3.1 to P3.11

### 3.3 Electric Field Strength of Known Distribution of Point Charges

Let us repeat the definition of the electric field strength vector given in Eq. (1.5) in somewhat different notation. We first define the *test charge*,  $\Delta Q$ , to be a very small

body with negligibly small charge. (Such a test charge placed in an electric field will not affect the field, so that we can measure the electric field strength at a particular point of the field as it is in the absence of the test charge.) Then at any point in the electric field, the electric field strength vector is given by

$$\mathbf{E} = \frac{\mathbf{F}_{\text{on } \Delta Q}}{\Delta Q} \quad \text{newtons per coulomb (N/C) = volts per meter (V/m).} \quad (3.3)$$

*(Definition of the electric field strength vector)*

The unit of the electric field strength is newtons per coulomb (N/C). For reasons to become clear in the next chapter, the equivalent unit, volts per meter (V/m), is used instead.

Combining this definition with the expression for the force exerted by one point charge on another point charge, Eq. (3.1), we see that the electric field vector due to a point charge  $Q$  is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \mathbf{u}_r \quad (\text{V/m}), \quad (3.4)$$

*(Electric field strength of a point charge)*

where  $\mathbf{u}_r$  is the unit vector directed *away* from charge  $Q$ . This is the electric field strength of a single point charge. It is a vector function of the distance from the point charge producing it and is directed away from a positive charge or toward a negative charge.

What if we have more than one charge producing the field? How is the electric field strength then obtained? The answer is fairly obvious: since the principle of superposition is valid for electric forces just as for mechanical forces, the equation follows directly from Eqs. (3.3) and (3.4). Assume that we have  $n$  point charges,  $Q_1, Q_2, \dots, Q_n$ . The electric field strength at a point that is at distances  $r_1, r_2, \dots, r_n$  from the charges is simply

$$\mathbf{E} = \sum_{i=1}^n \frac{1}{4\pi\epsilon_0} \frac{Q_i}{r_i^2} \mathbf{u}_{ri} \quad (\text{V/m}). \quad (3.5)$$

**Example 3.1—Superposition applied to the electric field strength.** As an example, Fig. 3.3 shows how we obtain the electric field strength resulting from three point charges,  $Q$ ,  $2Q$ , and  $3Q$ . Assume that the three charges are in air, in the plane of the drawing, and let us determine the total electric field at the point  $P$  which is at the same distance from the three charges. To obtain the total field, we first add up the field of the charges  $Q$  and  $2Q$ , and then add to this sum the field of the charge  $3Q$ , as indicated in the figure.

It is important to note that Eq. (3.5) can be used in this case because both vacuum and air, in which the charges are placed, are *linear media* (i.e., electrical properties of the medium, in this case of  $\epsilon_0$ , do not depend on the electric field strength in the medium). This is not always the

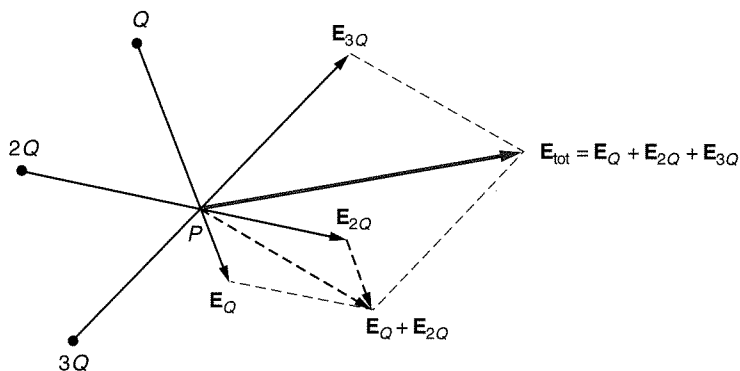


Figure 3.3 The electric field strength resulting from the three point charges  $Q$ ,  $2Q$ , and  $3Q$ , situated in the plane of the drawing, at a point that is at the same distance from each of them

case—many important and practical media are nonlinear. For nonlinear media, superposition cannot be applied. Superposition allows us to break up a complicated problem into several easier ones and then add up their solutions to get the solution to the complicated problem. We will use it often.

*Questions and problems:* Q3.11 to Q3.14, P3.12 to P3.15

### 3.4 Electric Field Strength of Volume, Surface, and Line Charge Distributions

Elemental charges (electrons and protons) that create a field are always so small that they can be considered as point charges. Therefore Eq. (3.5) can be used, in principle, to calculate the electric field of any charge distribution. However, even for a tiny amount of charge, the number of elemental charges is very large. For example,  $-1$  pC ( $-10^{-12}$  C) contains about  $10^7$  electrons. Therefore, in macroscopic electromagnetism, it is convenient to introduce the concept of *charge density*, and then to use integral calculus to evaluate the field of a charge distribution.

#### 3.4.1 VOLUME CHARGE DENSITY

Consider first a cloud of static charges. (Of course, it must be kept in place by some means; otherwise it would move as a result of the electric forces the charges exert on each other.) Let them be packed so densely that even inside a small volume  $dv$  there are many charges, amounting to a total charge  $dQ_{\text{in } dv}$ . We then define the *volume charge density*,  $\rho$ , at the point enclosed by that small volume:

$$\rho = \frac{dQ_{\text{in } dv}}{dv} \quad \text{coulombs per cubic meter (C/m}^3\text{)}. \quad (3.6)$$

*(Definition of volume charge density)*

Note that the unit of volume charge density is  $\text{C/m}^3$ .

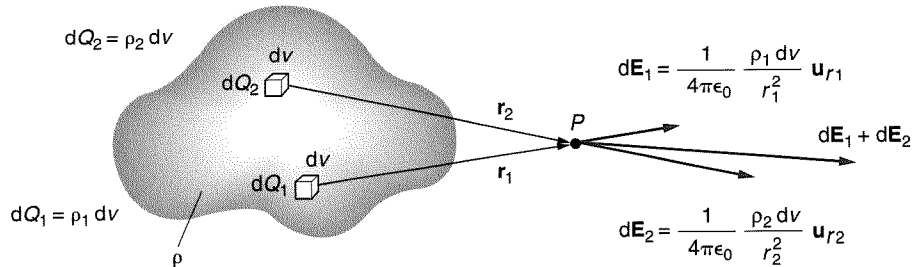


Figure 3.4 Calculating the electric field strength of a charged cloud of known charge density

According to this definition, inside a small volume  $dv$  where the charge density is  $\rho$ , there is a small charge

$$dQ = \rho dv. \quad (3.7)$$

This charge can be considered a point charge. If we have a charged cloud with known charge density  $\rho$  at all points, we can obtain the electric field strength at any point using, essentially, Eq. (3.5) but with a very large (theoretically infinitely large) number  $n$  of point charges. Such a sum is an integral (see Fig. 3.4):

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_v \frac{\rho dv}{r^2} \mathbf{u}_r \quad (\text{V/m}). \quad (3.8)$$

*(Electric field strength of volume distribution of charges)*

Note that the charge density,  $\rho$ , and the ~~position vector,  $\mathbf{r}$~~  vary from one elemental volume  $dv$  to another. **DISTANCE  $r$  FROM THE FIELD POINT  $\mathbf{Q}_1$**

Suppose we know the shape of the cloud and volume charge density at all points of the cloud. It is possible only rarely to evaluate the integral in Eq. (3.8) analytically. However, we can approximately calculate the electric field strength at any point in space by dividing the volume charge into a finite number of very small volumes  $\Delta v$ , taking the value of  $\rho$  at the center of this small volume, and then summing all the vector electric field strengths resulting from these point charges. In this case, Eq. (3.8) becomes a sum over all the little volumes. This sum is not hard to evaluate on a computer, and the result will be more accurate with a greater number of small volumes.

### 3.4.2 SURFACE CHARGE DENSITY

Strictly speaking, volume charge is the only type of charge that appears in nature. However, in some cases this charge is spread in an extremely thin layer (of thickness on the order of a few atomic radii) and can be regarded as a *surface charge*. Such is, for example, the excess charge on a conducting body. To describe the surface charge distribution, we introduce the concept of *surface charge density*,  $\sigma$ . Figure 3.5 shows a body with surface charge. Consider a small area  $dS$  of the body surface. Let the charge on that small surface patch be  $dQ_{\text{on } dS}$ . The surface charge density at a point

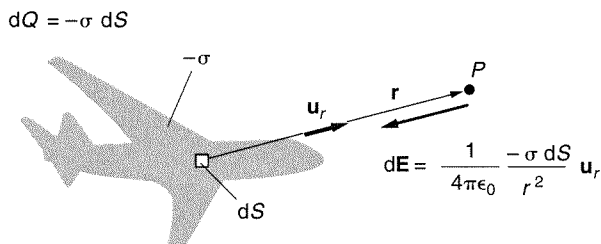


Figure 3.5 A body charged over its surface

of  $dS$  is then defined as

$$\sigma = \frac{dQ_{\text{on } dS}}{dS} \quad \text{coulombs per square meter (C/m}^2\text{)}. \quad (3.9)$$

*(Definition of surface charge density)*

(The symbol  $\rho_s$  is sometimes used instead of  $\sigma$ .) From this definition, it follows that if we know the surface charge density at a point on the body surface, the charge on a small patch of area  $dS$  enclosing this point is obtained as

$$dQ_{\text{on } dS} = \sigma dS. \quad (3.10)$$

The unit of surface charge density is  $\text{C/m}^2$ . Note that in general, the surface charge differs from one point of a surface to another.

The field of a given distribution of surface charge is obtained if in Eq. (3.8) we substitute the elemental charge,  $\rho dv$ , by  $\sigma dS$ :

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma dS}{r^2} \mathbf{u}_r \quad (\text{V/m}). \quad (3.11)$$

*(Electric field strength of a surface distribution of charges)*

### 3.4.3 LINE CHARGE DENSITY

Finally, we frequently encounter thin charged wires. Wires are usually conductors and the charge is distributed in a very thin layer on the wire surface. If the wire is thin compared to the distance of the observation point, we can consider the charge to be distributed approximately along a geometric line, for example along the wire axis. This type of charge is known as *line charge*. Its distribution along the line (i.e., along the wire the line approximates) is described by the *line charge density*,  $Q'$  (Fig. 3.6).

Let the charge on a very short segment  $dl$  of the line be  $dQ_{\text{on } dl}$ . The line charge density is defined as

$$Q' = \frac{dQ_{\text{on } dl}}{dl} \quad \text{coulombs per meter (C/m)}. \quad (3.12)$$

*(Definition of line charge density)*

(The symbol  $\rho_\ell$  is sometimes used instead of  $Q'$ .) Thus if we know the line charge density at a point along a wire, the charge on a short segment  $dl$  of the wire containing

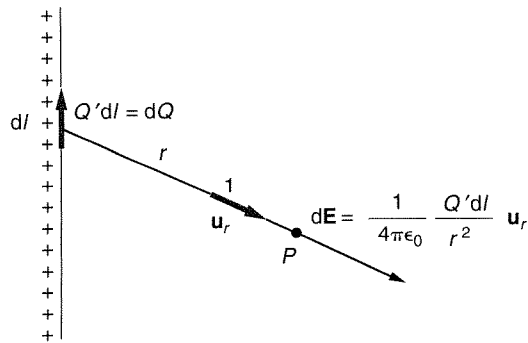


Figure 3.6 Electric field strength due to a thin charged wire

that point is simply

$$dQ_{\text{on } dl} = Q' dl. \quad (3.13)$$

The unit of line charge density is C/m. Note that  $Q'$  may differ from one point of the line to another.

The field of a given distribution of line charge along a line  $L$  is obtained as

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_L \frac{Q' dl}{r^2} \mathbf{u}_r \quad (\text{V/m}). \quad (3.14)$$

*(Electric field strength of line distribution of charges)*

The integrals in Eqs. (3.8), (3.11), and (3.14) usually cannot be evaluated analytically, but they can always be evaluated numerically, as described in connection with the volume charge distribution. We do not give any examples of analytical evaluation of these integrals because for those that can be evaluated, it is usually possible to obtain the result in a much simpler way, described in the next two chapters.

If we know the distribution of volume, surface, and line charges, i.e., if we know the volume, surface, and line charge density at all points, it is a simple matter to evaluate the electric field strength of these distributions at any point. As we shall see, however, the charge distribution is rarely known in advance. Instead, in practical problems we need to *determine* the charge distribution in order to calculate the electric field around it. Therefore, the formulas to determine the electric field strength from a known distribution of volume, surface, or line charges are mainly of academic interest. The concepts of volume, surface, and line charge are very useful, however, as we shall see later. Equations in the form of Eqs. (3.11) and (3.14) are also indispensable in determining unknown charge distributions numerically.

*Questions and problems:* Q3.15 to Q3.18, P3.16 to P3.27

### 3.5 Lines of the Electric Field Strength Vector

The electrostatic field is a vector field. A useful concept for visualizing vector fields is *field lines*. The lines of vector  $\mathbf{E}$  are defined as imaginary, generally curved lines,



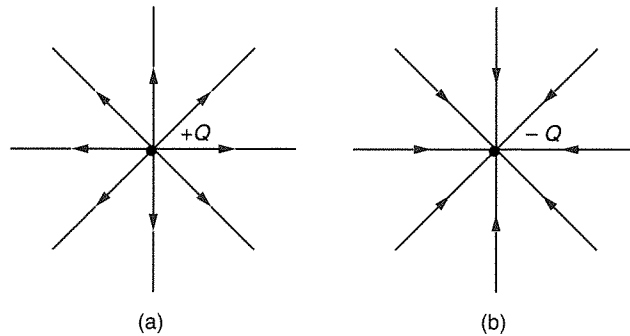


Figure 3.7 Electric field lines of (a) a positive and (b) a negative point charge

having the property that  $\mathbf{E}$  is tangential to these lines at all points. For example, lines of vector  $\mathbf{E}$  of the field of a point charge are straight lines emanating from the charge (Fig. 3.7). It is usual to add an arrow to the lines of vector  $\mathbf{E}$  indicating the direction of  $\mathbf{E}$  along the lines.

**Example 3.2—Electric field lines of a very large, uniformly charged flat plate.** As a further example of electric field lines, consider a very large, uniformly charged flat plate. Let the charge on the plate be positive. Since the plate is very large, if we consider the field close to the plate, the electric field strength vector must be normal to the plate and pointing away from it (why?). The electric field lines are as sketched in Fig. 3.8. This kind of electric field, which has in a region of space the same direction and magnitude, is called a *uniform electric field*.

For a negative plate, the lines are of the same form, only the arrowhead (indicating the direction of vector  $\mathbf{E}$ ) points toward the plate, i.e., the field is also uniform.

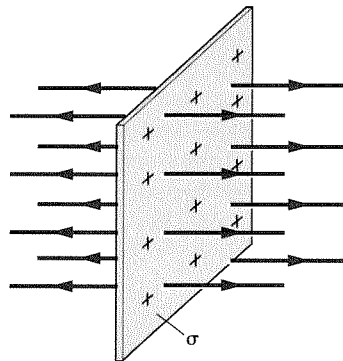


Figure 3.8 Electric field lines for a flat, large plate with uniform positive surface charge distribution

## 3.6 Chapter Summary

1. If written in vector form, Coulomb's law does not require additional explanations in words—all information is then contained in the formula. For formulating Coulomb's law in this manner, the positive and negative sign convention for charges is essential.
2. The electric field strength vector  $\mathbf{E}$  of a point charge is defined from the vector form of Coulomb's law.
3. The expression for the electric field strength of a point charge can be used for obtaining vector  $\mathbf{E}$  resulting from any distribution of point charges, or from volume, surface, and line distributions of charges. To do this, it is necessary to define volume, surface, and line charge distributions.
4. The charge distribution is usually not known in advance. Therefore, the formulas for the electric field strengths of known distributions of charges are of limited practical usefulness. However, they may be used for numerical evaluation of the vector  $\mathbf{E}$  by means of integral equations.
5. We say that in a region of space the electric field is uniform if the electric field strength at all points of the region has the same direction and magnitude.

### QUESTIONS

- Q3.1.** Discuss the statement that Eq. (3.1) indeed shows not only the magnitude but also the correct direction of the force  $\mathbf{F}_{e12}$ . Does Eq. (3.1) need an additional explanation in words? Explain.
- Q3.2.** Would the vector form of Coulomb's law (Eq. 3.1) be possible if plus and minus signs were not associated with the two types of charges? For example, suppose that they were denoted by subscripts  $A$  and  $B$  instead of plus and minus signs. Explain your answer. (This question is intended to show how important proper conventions are for simplifying the mathematical description of physical phenomena.)
- Q3.3.** Is it possible to *derive* the principle of superposition of Coulomb's forces, starting from Coulomb's law? Explain.
- Q3.4.** Prove that there can be no net electric force on an isolated charged body due to its charge only.
- Q3.5.** Similarly to the electric field, the gravitational field also acts "at a distance." But whereas we understand and accept that there is a downward force on an object we lift (e.g., a stone), with no visible reason, such an electric force with no visible reason is somewhat astonishing. Explain why this is so.
- Q3.6.** Of five equal conducting balls one is charged with a charge  $Q$ , and the other four are not charged. Find all possible charges the balls can obtain by touching one another, assuming that two balls are allowed to touch only once, and that while two balls are touching, the influence of the other three can be neglected.
- Q3.7.** If an electrified body (e.g., a plastic ruler rubbed against a wool cloth) is brought near small pieces of thin aluminum foil, you will see that the body first attracts, but after the contact repels, the small pieces. Perform this experiment and explain.

- Q3.8. Imagine that you electrified a body, e.g., by rubbing it against another body. How could you determine the sign of the charge on the body? Try to perform the experiment.
- Q3.9. You have two identical small metal balls. How can you obtain identical charges on them?
- Q3.10. Two small balls carry charges of unknown signs and magnitudes. Experiment shows that there is no electric force on a third charged ball placed at the midpoint between the first two. What can you conclude about the charges on the first two balls?
- Q3.11. An uncharged small ball is introduced into the electric field of a point charge. Is there a force on the ball? Explain.
- Q3.12. Is it correct to write the following: (1)  $Q$  ( $Q > 0$ ); (2)  $-Q$  ( $Q < 0$ ); (3)  $Q$  ( $Q < 0$ ); and (4)  $-Q$  ( $Q > 0$ )? Explain.
- Q3.13. To measure the electric field strength at a distance  $r$  from a small charge  $Q$ , a test charge  $\Delta Q$  ( $\Delta Q \ll Q$ ) in the form of a sphere of radius  $a = r/2$  is centered at that point. Discuss the correctness of the measurement.
- Q3.14. What would the form of the expression in Eq. (3.4) be if  $\mathbf{u}_r$  is *toward* the charge? What form does Eq. (3.4) take if we do not associate a sign with the charge  $Q$ ?
- Q3.15. Is  $\rho$  in Eq. (3.6) a function of coordinates, in general?
- Q3.16. Assuming  $\rho$  in Eq. (3.8) to be known, explain in detail how you would numerically evaluate the *vector* integral to obtain  $\mathbf{E}$ .
- Q3.17. Repeat question Q3.16 for a surface distribution of charges over a surface  $S$ , and for a line distribution of charges along a line  $L$ .
- Q3.18. Why are the formulas in Eqs. (3.8), (3.11), and (3.14) only of limited practical value?

## PROBLEMS

- P3.1. What would be the charge of a copper cube, 1 cm on a side, if one electron were removed from all the atoms on the cube surface? A cubic meter of copper has about  $8.4 \cdot 10^{28}$  atoms.
- P3.2. Evaluate the force that would exist between two cubes as described in problem P3.1 when they are (1)  $d = 1$  m, and (2)  $d = 1$  km apart.
- P3.3. Three small charged bodies arranged along a straight line are at distances  $a$ ,  $b$ , and  $(a + b)$  apart. Determine the conditions that the charges on the bodies have to satisfy so that the electric forces on all three are zero.
- P3.4. Assume that the earth is electrified by a charge  $2Q$ , and the moon by a charge  $Q$ . How large does  $Q$  have to be so that the repulsive electric force between the earth and the moon becomes equal to the attractive gravitational force? The masses of the earth and the moon are  $m_e = 5.983 \cdot 10^{24}$  kg and  $m_m = 7.347 \cdot 10^{22}$  kg. The gravitational constant is  $\gamma = 6.67 \cdot 10^{-11}$  N  $\cdot$  m<sup>2</sup>/kg<sup>2</sup>.
- P3.5. Evaluate the specific charge of the electron (charge over mass). Estimate the charge of the book you are reading if it had the same specific charge. What would the force between two such charged books be if they were at a distance of 10 m?
- P3.6. A given charge  $Q$  is divided between two small bodies, so that one has the charge  $Q'$ , and the other has the rest. Determine the ratio  $Q/Q'$  resulting in the greatest electric force between them, assuming the distance between them is fixed.

**BOTH CHARGES HAVE THE SAME SIGN.**

- P3.7.** Three small charged bodies of charge  $Q$  are placed at three vertices of an equilateral triangle with sides of length  $a$ . What is the direction and magnitude of the electric force on each of them if  $a = 3 \text{ cm}$  and  $Q = 1.8 \cdot 10^{-10} \text{ C}$ ?
- P3.8.** A charge  $Q$  exists at all vertices of a cube with sides of length  $a$ . Determine the direction and magnitude of the electric force on one of the charges.
- P3.9.** Two identical small, conducting balls with centers that are  $d$  apart have charges  $Q_1$  and  $Q_2$ . The balls are brought into contact and then returned to their original positions. Determine the electric force if charges  $Q_1$  and  $Q_2$  are (1) of the same sign; (2) of opposite signs.
- P3.10.** Evaluate the velocity of an electron orbiting around the nucleus of a hydrogen atom along an approximately circular orbit of radius  $a = 0.528 \cdot 10^{-10} \text{ m}$ . How many revolutions does the electron make in one second?
- P3.11.** Two small balls of mass  $m$  each have a charge  $Q$  and are suspended at a common point by separate thin, light, conducting filaments of length  $l$ . Assuming the charges are located approximately at the centers of the balls, find the angle  $\alpha$  between the filaments. Suppose that  $\alpha$  is small. (Such a system can be used as a primitive device for measuring charge, and is called an *electroscope*.)
- P3.12.** A small body with a charge  $Q = 1.8 \cdot 10^{-10} \text{ C}$  is situated at a point  $A$  in the electric field. The electric force on the body has an intensity  $F = 5.4 \cdot 10^{-4} \text{ N}$ . Evaluate the magnitude of the electric field strength vector at that point.
- P3.13.** A point charge  $Q$  ( $Q > 0$ ) is located at the point  $(0, d/2)$ , and a charge  $-Q$  at the point  $(0, -d/2)$ , of a rectangular coordinate system. Determine and plot the magnitude of the total electric field strength vector at any point in the  $xy$  plane.
- P3.14.** An electric dipole consists of two equal and opposite point charges  $Q$  and  $-Q$  that are a distance  $d$  apart, Fig. P3.14. (1) Find the electric field vector along the  $x$  axis in the figure. (2) Find the electric field vector along the  $y$  axis. (3) How does the electric field strength behave at distances  $x \gg d$  and  $y \gg d$  away from the dipole? How does this behavior compare to that of the field of a single point charge?

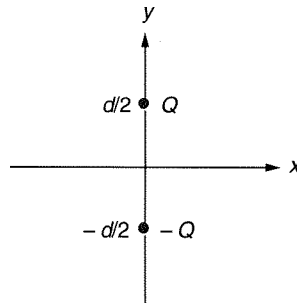


Figure P3.14 An electric dipole consists of two equal charges of opposite signs.

- P3.15.** Find the  $x$  and  $y$  components of the electric field vector at an arbitrary point in the field of the electric dipole from problem P3.14, assuming that the distance of the observation point from the dipole center is much greater than  $d$ . Plot your results.

- P3.16.** A thin, straight rod  $a = 10$  cm long is uniformly charged along its length with a total charge  $Q = 2 \cdot 10^{-10}$  C. The rod extends from the point  $(-a/2, 0)$  to the point  $(a/2, 0)$  in an  $xy$  rectangular coordinate system. Evaluate the electric field strength vector at points  $A(0, a/4)$  and  $B(3a/4, 0)$ .
- P3.17.** Solve problem P3.16 approximately, by dividing the rod into  $n$  segments. Compare the results with the exact solution for  $n = 1, 2, 3, 4, 5, 6, 10,$  and  $20$ .
- \*P3.18.** An L-shaped rod with sides  $a = 10$  cm extends from the origin of an  $xy$  rectangular system to the point  $(a, 0)$ , and from the origin to the point  $(0, a)$ . The rod is charged uniformly along its length with a total charge  $Q = 2.6 \cdot 10^{-9}$  C. Evaluate the electric field strength vector at points  $A(a, a)$  and  $B(3a/2, 0)$ .
- \*P3.19.** Solve problem P3.18 approximately, by dividing the L-shaped rod into  $2n$  segments. Compare the results with the exact solution for  $n = 2, 3, 4, 5, 6,$  and  $20$ .
- P3.20.** A thin ring of radius  $a$  is uniformly charged along its length with a total charge  $Q$ . Determine the electric field strength along the ring axis.
- P3.21.** A thin circular disk of radius  $a$  is charged uniformly over its surface with a total charge  $Q$ . Determine the electric field strength along the disk axis normal to its plane and plot your result. What do you expect the expression for the electric field to become at large distances from the disk? What do you expect the expression to become if the radius of the disk increases indefinitely, and the surface charge density is kept constant?
- P3.22.** Calculate the electric field along the axis of the disk in problem P3.21 if the charge is not distributed uniformly but increases linearly along the disk radius, and it is zero at the disk center. Plot your result and compare it to those for problem P3.21.
- P3.23.** A dielectric cube with sides of length  $a$  is charged over its volume with a charge density  $\rho(x) = \rho_0 x/a$ , where  $x$  is the normal distance from one side of the cube. Determine the charge of the cube.
- P3.24.** The volume charge density in a spherical charged cloud of radius  $a$  is  $\rho(r) = \rho_0(a-r)/a$ , where  $r$  is the distance from the cloud center, and  $\rho_0$  is a constant. Determine the charge of the cloud.
- P3.25.** Determine and plot the electric field strength at a distance  $r$  from a straight, very long, thin charged filament with a charge  $Q'$  per unit length.
- P3.26.** A wire in the form of a semicircle of radius  $a$  is charged with a total charge  $Q$ . Assuming the charge to be uniformly distributed along the wire, determine the electric field strength vector at the center of the semicircle.
- P3.27.** A hemispherical shell of radius  $a$  is charged uniformly over its surface by a total charge  $Q$ . Determine the electric field strength at the center of the sphere, one-half of which is the shell.