Additional Homework Problems for ECEN 3400

These homework problems are referred to in homework assignments using the prefix: “EK”.

EK1.1 Consider two parallel wire segments as shown in the figure for Problem P1.8 in the text. Suppose the lengths of the wire segments are again 1 mm each, but their distance apart is \( d = 5 \, \text{mm} \), as might be the case for the power cord of an electrical appliance. Suppose also that the currents flow in opposite directions rather than in the same direction as shown in the figure. If the wire segments carry 5 A of current each, find the magnitude and direction of the force between the segments.

EK1.2 A small particle of mass \( m = 10^{-16} \, \text{g} \) and carrying a charge \( Q = 10^{-12} \, \text{C} \) is moving along a circular path of radius \( r = 1 \, \text{cm} \) with velocity \( v \). The circular path is maintained due to the force exerted by the earth’s magnetic field (whose strength is \( B = 50 \, \mu \text{T} \)), directed perpendicular to the plane of the circular path.

(a) Make a sketch of this motion, showing arrows denoting the velocity of the particle at some point of the circle, the direction of the magnetic field vector and the direction of the magnetic force on the particle.

(b) Recall that the centrifugal force acting on this particle due to its circular motion is \( mv^2/r \). What must the velocity of the particle be in order to maintain this circular motion?

EK1.3 A small spherical particle of mass \( m_1 \) and charge \( Q_1 \) is anchored (fixed) at the bottom of a ramp inclined at an angle \( \theta \) to the horizontal direction as shown in the Figure. A second spherical particle of mass \( m_2 \) and charge \( Q_2 \) is located up the ramp at a distance \( r \) from the first particle as shown. Find an expression for the distance \( r \) at which the Coulomb force between the particles is exactly balanced by the gravitational force on the second particle. Assume the gravitational force is vertical, and downward. What condition must hold on the charges \( Q_1 \) and \( Q_2 \) for this balance to be possible?

EK2.1 Two parallel wires of radius \( a \) and length \( l \) are separated by a distance \( d \) between their centers as shown. With the ends of the wires open circuited, there is a capacitance given approximately by

\[
C_0 \simeq \frac{\pi \epsilon_0 l}{\ln(d/a)}
\]
between the wires, where \( \epsilon_0 = 8.854 \ldots \times 10^{-12} \text{ F/m} \) (this can be determined based on the principles described in chapter 8 of the text). If the wires are connected together at both ends, the resulting thin loop has an inductance given approximately by

\[
L_0 \simeq \frac{\mu_0 l}{\pi} \ln \frac{d}{a}
\]

where \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \) [see eqn. (15.12) of the text].

If wires are connected together at only one of the ends, let us assume that the resulting circuit at the other end appears approximately as the series combination of \( C_0 \) and \( L_0 \) (in chapter 18 we will see how to model this circuit in a more accurate way using the theory of transmission lines).

(a) Give a formula for the resonant frequency \( f_r \) of this circuit, based on these assumptions.
(b) If \( l = 1 \, \text{m}, \, a = 1 \, \text{mm} \) and \( d = 1 \, \text{cm} \), at what frequencies will the impedance of this circuit have a magnitude of \( 1 \, \text{kΩ} \)?

**EK2.2** Consider the situation of Problem **EK2.1**, except that instead of a short circuit at the end of the parallel wires, there is a 500 \( \Omega \) resistor, which can be assumed to appear in series with the inductance of the loop, while the capacitance appears in parallel with this \( RL \) combination. If \( l = 1 \, \text{m}, \, a = 1 \, \text{mm} \) and \( d = 1 \, \text{cm} \), plot (by hand, or using your favorite software) the real and imaginary parts of the impedance of this circuit versus frequency from 0 to 100 MHz. Explain what is happening.

**EK3.1** Four line charges are arranged to form a square in the plane \( z = 0 \). Each side of the square has a length \( 2a \), and the square is located symmetrically about the origin. Each line charge has linear charge density \( \rho_l \). Find the electric field \( \mathbf{E}(0, 0, z) \) at an arbitrary point along the \( z \)-axis.

![Figure EK3.1: Four line charges arranged in a square.](image)

**EK3.2** A thin flat square occupying \(-a < x < a, \, -a < y < a\) in the plane \( z = 0 \) is uniformly charged with a surface charge density \( \sigma \, \text{C/m}^2 \). By direct integration, find an expression for the electric field at an arbitrary point \((0, 0, z)\) on the positive \( z \)-axis. (Extra Credit: Plot the electric field vs. \( z \), using \( \sigma = 1 \, \text{C/m}^2 \) and \( a = 1 \, \text{m} \).)

Note: Here are two indefinite integrals you will find useful:

\[
\int \frac{dx}{(x^2 + y^2 + z^2)^{3/2}} = \frac{x}{(y^2 + z^2)\sqrt{x^2 + y^2 + z^2}}
\]

\[
\int \frac{dy}{(y^2 + z^2)\sqrt{a^2 + y^2 + z^2}} = \frac{1}{az} \tan^{-1} \left( \frac{ay}{z\sqrt{a^2 + y^2 + z^2}} \right)
\]
**EK3.3** Calculate the electric field along the axis of the disk described in Problem P3.21 of the text if the charge density is not distributed uniformly, but behaves as

\[ \sigma(r) = A(a - r) \]

where \( A \) is a constant, and \( r \) is the distance from the center of the disk. Express your answer in terms of the total charge \( Q \) on the disk, rather than \( A \).

**EK3.4** A pair of uniform line charges in the form of 180° circular arcs of radius \( a \) is arranged in the plane \( z = 0 \) as shown in the figure. One arc (between \( \phi = 0 \)—the positive \( x \)-axis—and \( \phi = \pi \)—the negative \( x \)-axis) carries a total charge \(+Q\), and the other (between \( \phi = \pi \) and \( \phi = 2\pi \)), \(-Q\). By direct integration, obtain an expression for the electric field produced by these arcs at an arbitrary point \((0, 0, z)\) along the \( z \)-axis. Be sure to use symmetry as appropriate to simplify your calculations; otherwise they can become quite messy.

**Figure EK3.4:** Two oppositely charged 180° arcs.

\[ \phi = 0 \text{—the positive } x\text{-axis—and } \phi = \pi \text{—the negative } x\text{-axis) carries a} \]

\[ \text{total charge } +Q, \text{ and the other (between } \phi = \pi \text{ and } \phi = 2\pi \), } -Q. \text{ By} \]

\[ \text{direct integration, obtain an expression for the electric field produced by} \]

\[ \text{these arcs at an arbitrary point } (0, 0, z) \text{ along the } z\text{-axis. Be sure to use} \]

\[ \text{symmetry as appropriate to simplify your calculations; otherwise they can} \]

\[ \text{become quite messy.} \]

**EK3.5** A uniform line charge of density \( Q' \) C/m lies along the \( z \)-axis between \( z = -d/2 \) and \(+d/2\). Use equation (1.39) of the text by Notaroš to obtain an expression for the electric field at an observation point \( P \) that lies along the \( z \)-axis beyond the end of the line charge \((z > d/2)\).

**EK4.1** An electric field is given by

\[ \mathbf{E} = y\mathbf{u}_x + x\mathbf{u}_y \]

Integrate \( \int \mathbf{E} \cdot d\mathbf{l} \) along each of the paths below to determine the potential difference between the points \((x, y, z) = (0, 0, 0) \) and \((1, 2, 0)\) in each case. Path A: Straight line from \((0, 0, 0) \) to \((1, 0, 0)\), then straight line from \((1, 0, 0) \) to \((1, 2, 0)\).
Path B: Straight line from (0, 0, 0) to (0, 2, 0), then straight line from (0, 2, 0) to (1, 2, 0).

EK4.2 A thin rod extends from the point \((-a/2, 0, 0)\) to the point \((a/2, 0, 0)\) along the x-axis. It carries a linear charge density of
\[
\rho_l = A|x|
\]
where A is a positive constant. Find an expression for the potential at an arbitrary point \((0, y, 0)\) on the y-axis. Use this expression then to find \(E_y(0, y, 0)\).

EK4.3 A circular ring of radius \(a\) is uniformly charged with a line charge density \(\rho_l\) and located centered about the origin of the xy-plane. The electric field along the z-axis of the ring is:
\[
\mathbf{E}(0, 0, z) = \frac{2\pi a \rho_l}{4\pi \varepsilon_0} \frac{z}{(a^2 + z^2)^{3/2}} \mathbf{u}_z
\]
By direct integration of \(\mathbf{E}\), find the potential at an arbitrary point \((0, 0, z)\) along the ring’s axis, if the potential reference is at the center of the ring. [You’ll have to look up the integral on your own for this one.]

EK4.4 A potential equal to
\[
V(x, y, z) = 4xy
\]
exists in a certain region of space. Find the electric field associated with this potential, then sketch both the equipotential surfaces and the electric field lines in the xy-plane.

EK4.5 Calculate the potential of the charged disk in problem EK3.3 at an arbitrary point \((0, 0, z)\) along the axis of the disk.

EK4.6 Repeat problem EK4.4 for the potential
\[
V(x, y, z) = 10 \left[ x^2 - \sin(\pi y) \right]
\]

EK4.7 A group of \(N\) not necessarily identical point charges is placed randomly (i.e., not necessarily symmetrically) around the circumference of a circle of radius \(a\) centered about the origin of the xy-plane. You are not given information on the precise locations of the charges on the circle, nor the values of the individual charges, but you are given the value of their total charge \(Q\). Obtain an expression for the potential \(V(0, 0, z)\) at an arbitrary point on the z-axis. From this, obtain an expression for the component \(E_z(0, 0, z)\) at this point. Why is it not possible to find an expression for the other components of the field with the information given?

EK4.8 Consider the group of four line charges arranged in a square as described in problem EK3.1. Determine the potential of this group of charges produced at any point \((0, 0, z)\) along the z-axis.
EK4.9 By some means it has been found that the electric field inside a cylindrical region \((0 < r < a, 0 < z < d)\) is given by \(\mathbf{E} = \hat{r}E_0 \frac{z}{2}\), where \(E_0\) is a constant. Find an expression for the voltage \(V_{0a}\) between the center of the cylinder at \(r = 0\) and the surface \(r = a\).

EK5.1 A nonuniform volume charge density \(\rho(r) = \rho_0 e^{-r/r_0}\) is present in space, with \(r\) being the distance from the origin, and \(\rho_0\) and \(r_0\) being constants. Obtain an expression for the total charge contained in a sphere centered at the origin, if the radius of the sphere is:

(a) \(r_0\);
(b) \(3r_0\);
(c) \(6r_0\).

EK5.2 A spherical “shell” contains a uniform volume charge density \(\rho\) between \(b < r < a\). The charge density is zero when \(r < b\) and when \(r > a\). Use Gauss’ Law and symmetry to determine the electric field due to this charge shell in each of the three regions \(r < b\), \(b < r < a\) and \(r > a\). From your result, find an expression for the electrostatic potential at \(r = 0\) with respect to a reference point at \(r = \infty\).

EK5.3 (a) Given the electric field in problem EK4.9, determine the total charge \(Q_{\text{encl}}(r)\) enclosed inside a circular cylinder of radius \(r\) and length \(d\), where \(r\) has any value between 0 and \(a\).

(b) Show that if the volume charge density is \(\rho(r)\), then

\[
Q_{\text{encl}}(r) = 2\pi d \int_0^r r\rho(r) \, dr
\]

(c) From the results of parts (a) and (b), find an expression for the volume charge density \(\rho(r)\) that must be present with this electric field.

EK6.1 In a certain region of space, the electric field

\[
\mathbf{E} = 10\hat{x} \quad \text{V/m}
\]

has been produced. You possess a thin conducting plate, which you insert into the field in various ways. Which of the following locations for the conducting plate will not disturb the electric field?

(a) \(x = 0\) m.
(b) \(x = 2\) m.
(c) \(y = 3\) m.
(d) \(x + z = 1\) m.
**EK6.2** You have the same field as in problem EK6.1. This time, you possess a very thin straight conducting wire. At which of the following locations can you place the wire, without disturbing the electric field?

(a) the $x$-axis.
(b) the $y$-axis.
(c) the $z$-axis.
(d) the line $y = z, x = 0$.

**EK6.3** You have the same field as in EK6.1. This time, you possess a long, uncharged conducting circular cylinder. You introduce the cylinder into the field without introducing any net charge onto it (i.e., you handle it with insulated gloves), so that the axis of the cylinder is along the $z$-axis.

(a) Sketch roughly what the surface charge density on the cylinder will look like.
(b) Sketch roughly what the total electric field in the vicinity of the cylinder will look like.
(c) Sketch roughly what the additional electric field (that is, the total field you found in part (b) minus the original field before the insertion of the cylinder) will look like.

I don’t want computations, just sketches showing qualitatively correct behavior.

**EK7.1** A slab of dielectric material of thickness $d$ lies between the planes $x = 0$ and $x = d$, with air occupying all the rest of space. An electric field $\mathbf{E} = 50\mathbf{u}_x$ (V/m) is measured in the air just outside the surface $x = d$ of the dielectric. If the relative permittivity of the slab is $\epsilon_r = 9.8$, find the values of $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{P}$ in the slab.

**EK7.2** A conducting sphere of radius $a$ is charged with a total charge $Q$. The sphere is coated out to a radius $b > a$ with a layer of dielectric whose permittivity is $\epsilon$. First find $\mathbf{D}$, then $\mathbf{E}$ everywhere. Then obtain an expression for the potential of the conducting sphere.

**EK7.3** A conducting thin spherical shell of radius $b$ carries no net charge. Inside the shell, there is a spherical layer of dielectric whose permittivity is $\epsilon$, occupying the region $a < r < b$. A point charge $Q$ is placed at the center of the sphere. First find $\mathbf{D}$, then $\mathbf{E}$ everywhere. Then obtain an expression for the potential of the conducting shell with respect to infinity.

**EK7.4** A point charge $Q$ is located at the center of a dielectric sphere of radius $a$ whose permittivity varies with the radial position $r$ as $\epsilon(r) = \epsilon_0 \left(1 + \frac{r}{a}\right)$. Using the generalized Gauss law, find $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{P}$ as functions of $r$. 


EK8.1 Two conducting spherical thin shells have radii \( a \) (conductor 1) and \( b \) (conductor 2), with \( a < b \), and are positioned concentrically. The dielectric is air everywhere. Find expressions for the capacitance coefficients \( C_{11} \), \( C_{12} \), \( C_{21} \) and \( C_{22} \). The “ground” or zero potential reference is at infinity.

EK8.2 Two conducting cylindrical thin shells have lengths \( l \), radii \( a \) (conductor 1) and \( b \) (conductor 2), with \( a < b \), and are positioned concentrically. Assume \( l \gg b \) so that fringing effects at the ends can be neglected. The dielectric is air everywhere. Find expressions for the capacitance coefficients \( C_{11} \), \( C_{12} \), \( C_{21} \) and \( C_{22} \). The “ground” or zero potential reference is a third conducting cylinder of radius \( c < a \).

EK8.3 Two identical conducting spheres of radius \( a \) are located in empty space with their centers separated by a distance \( d \). Assume that \( d \gg 2a \) so that the electrostatic field of the two spheres may be written as the superposition of the field of each sphere in isolation (that is, the field of one sphere does not significantly change the symmetry of the charge distribution on the other). Obtain an expression for the capacitance between these two spheres. If \( a = 2 \text{ mm} \) and \( d = 2 \text{ cm} \), what is the value of this capacitance? At what frequency would the reactance of this capacitance equal 50 \( \Omega \)?

EK8.4 A conducting sphere of radius \( a \) is coated with a dielectric layer between \( a < r < b \). Outside this layer, there is air \( (r > b) \). Obtain a formula for the capacitance of this coated sphere with respect to an infinitely distant “ground”.

![Figure EK8.5: A dielectric-coated conducting sphere.](image)

EK9.1 A conducting sheet of area \( S \) located at \( x = 0 \) carries a uniformly distributed charge \( 2Q \). At \( x = d \) and \( x = -d \) are located two further conducting plates identical to the first and parallel to it, both carrying uniformly distributed charges \(-Q\). The structure is located in air. Neglecting edge effects, use (9.7) to obtain an expression for the total stored electrostatic energy in this system in terms of \( Q \), \( d \) and \( S \).
**EK12.1** A thin insulating “wire” is uniformly charged with a linear charge density \( \rho_l \), and bent into the form of a circular loop of radius \( R \). The loop is spinning at a rate of \( N \) rpm (revolutions per minute). Find the magnetic field \( \mathbf{B} \) at the point about which the loop is centered.

**EK12.2** A line lying along a vector \( \mathbf{d} \) carries a current \( I \) as shown. The line is centered along the \( z \)-axis, so that half the vector \( \mathbf{d} \) lies above the origin, and half below, as shown. The vector from the origin to the observation point is \( \mathbf{R} \), the unit vectors along the directions of \( \mathbf{R} \pm \mathbf{d}/2 \) are denoted \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \), and \( \theta \) is the angle from the \( z \)-axis to \( \mathbf{R} \) as shown.

(a) Show that the magnetic field of this current segment at the observation point is

\[
\mathbf{B} = \frac{\mu_0 I}{4\pi R} \mathbf{u}_y \left[ \frac{R \cos \theta + d/2}{\sqrt{R^2 + dR \cos \theta + d^2/4}} - \frac{R \cos \theta - d/2}{\sqrt{R^2 - dR \cos \theta + d^2/4}} \right]
\]

(b) Show that the result of part (a) can be expressed in the form

\[
\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{\mathbf{d} \times \mathbf{R}}{|\mathbf{d} \times \mathbf{R}|^2} \cdot (\mathbf{u}_1 - \mathbf{u}_2)
\]

which is independent of the coordinate system used, and therefore applies no matter which way the current segment is oriented.

**EK12.3** A thin strip of width \( d \) carries a current \( I \) as in Problem P12.3 of the text. Find the \( \mathbf{B} \) field at the point \((x, y, 0)\) as \( y \to 0 \) while \( y > 0 \). Do the same for \( y \to 0 \) while \( y < 0 \). Does the field vector in either case point in the same direction as in Problem P12.3? Why?
EK12.4 An infinitely long thin wire carrying a DC current $I$ lies along the $z$-axis. Use the Biot-Savart law and symmetry to evaluate the $B$ field at an observation point $(x,0,0)$ along the $x$-axis.

EK12.5 The plane $z=0$ contains a stationary surface current density which flows in the angular direction and depends only on the distance $r$ to the origin of the plane: $J_s = \hat{u}_\phi J_{s\phi}(r)$ (refer to section A1.3.2 of the text for information on the cylindrical coordinate system).

(a) Show that symmetry implies the following properties of the resulting magnetic field just at the surface of the current sheet:

$$B_{z}\big|_{z=0^\pm} = \hat{u}_r B_r(r, z=0^\pm) + \hat{u}_z B_z(r, z=0^\pm)$$

and

$$B_r(r, z=0^-) = -B_r(r, z=0^+)$$

$$B_z(r, z=0^-) = B_z(r, z=0^+)$$

By $z=0^+$, we mean a position just above the current sheet, and by $z=0^-$, one just below it.

(b) Use Ampère’s Law to find an expression for $B_r(r, z=0^+)$ in terms of $J_{s\phi}(r)$. Why is it not possible to find $B_z$ from Ampère’s Law in this problem?

EK13.1 A conducting cylindrical wire of radius $a$ carries a current $I$ in the positive $z$-direction. The wire is coated with a magnetic material of relative permeability $\mu_r$ out to a radius $c$ (so that the thickness of the magnetic layer is $c-a$). From $r = c$ to $r = b$ is air, and from $r = b$ to $r = d$ is located a conducting cylindrical shell carrying a current $I$ in the negative $z$-direction (we have $a < c < b < d$). Use Ampère’s law to determine expressions for the magnetic flux density $B$ and magnetic field intensity $H$ at all points in space, if the current density in each conductor is uniform.

EK13.2 An infinitely long ferromagnetic cylinder of radius $a$ is uniformly permanently magnetized with a magnetization density $M$ directed along the axis of the cylinder. Determine the equivalent current density to this magnetization, and use Ampère’s law to find the resulting $B$ and $H$ both inside and outside the cylinder.

EK14.1 A conducting wire of length $l$ is bent into a circle, and placed in the plane $z=0$, in the presence of a time-varying but spatially uniform magnetic field $B(t) = \hat{z}B_0 \cos \omega t$, where $B_0$ is a constant. Obtain an expression for the induced emf $e_{\text{ind}}$ in the loop. Suppose this same length of wire is formed into a 3-turn loop and placed in the same magnetic field, again in the plane $z=0$. Obtain an expression for the induced emf in this case, observe whether it is larger or smaller than in the first case, and explain why.
EK15.1 A thin conducting wire loop which bounds an area $S$, has an inductance $L$ and resistance $R$ is placed in a spatially uniform but time-varying externally generated $B$ field $B_{\text{ext}}(t)$ oriented perpendicular to the plane of the loop. Show from field theory arguments that the current $i(t)$ induced in the wire loop obeys the differential equation

$$L \frac{di}{dt} + Ri = -S \frac{dB_{\text{ext}}}{dt}$$

Draw a sketch of this system showing the loop and the reference directions of the external $B$ field and the current flow. If the external $B$ field is given in rms phasor form

$$B_{\text{ext}}(t) = \text{Re} \left( \sqrt{2} B_0 e^{j\omega t} \right)$$

where $B_0$ is a constant, show that the rms phasor current induced in the loop is given by

$$I = \frac{-j\omega S B_0}{R + j\omega L}$$

Compute the amplitude of this induced current if $S = 10^{-2}$ m$^2$, $B_0 = 10^{-4}$ T, $L = 1$ $\mu$H, $R = 1$ $\Omega$ and $f = 1$ MHz.

EK15.2 Two conducting wire loops are located in proximity to each other. The self-inductances of the loops are $L_{11}$ and $L_{22}$, while the mutual inductance is $M = L_{12} = L_{21}$ (all positive values). Each loop is accessible to connection to outside circuits via a pair of terminals at a small gap in the loop.

(a) If the gap of the second loop is open-circuited, show that the inductance seen at the terminals of the first loop is $L_{11}$.

(b) If the second loop has no gaps (that is, its gap is short-circuited) and the resistances of the loops are negligible, show that the inductance seen at the terminals of the first loop is

$$L_{\text{eff}} = L_{11} - \frac{M^2}{L_{22}}$$

(c) Explain, in terms of the magnetic fields, why the inductance of the first loop when the second loop is short-circuited is less than it is when the second loop is open-circuited. Could $L_{\text{eff}}$ ever be negative?
EK15.3 A rather complicated calculation can be used to show that the self inductance of a circular wire loop in which the radius of the loop is \( R \) and the radius of the wire is \( a \) is

\[
L \simeq \mu_0 R \left( \frac{\ln \frac{8R}{a}}{a} - 2 \right)
\]

provided that \( R \gg a \). If the loop is made by bending a straight wire of length \( l \) and radius \( a \) into a circle, express the inductance in terms of \( l \) and \( a \), if \( l \gg a \). If that same length of wire is formed into a 3-turn loop, what will be the inductance of the new inductor thus formed? Is the 3-turn loop’s inductance greater or smaller than that of the one-turn loop?

EK17.1 A lightning stroke can be approximately modelled as a line current extending from a cloud to the ground, carrying a transient current which we assume to be of triangular shape as shown in the figure, where the pulse duration is \( T = 0.1 \) ms and the maximum current is \( I_{\text{max}} = 2 \times 10^5 \) A. Assume that the magnetic field of this stroke can be computed as if the stroke were an infinitely long wire carrying this current, and that Ampère’s law can be used to find it.

At a distance of 5 km away from the lightning stroke, there is a computer with a circuit board containing a conducting trace in the form of a rectangular loop 10 cm \( \times \) 20 cm. What must be the orientation of this circuit board in order for the emf \( e(t) \) induced in the loop by the magnetic field of the lightning stroke to be maximum? Compute and plot \( e(t) \). How does your answer change if the circuit board is instead located 50 m away from the lightning stroke?

![Current waveform of a lightning stroke](image-url)
EK18.1 Two resistors (of values $Z_0$ and $2Z_0$) are connected in an otherwise uniform transmission line whose characteristic impedance is $Z_0 = 50 \, \Omega$ as shown.

![Figure EK18.1: Two resistors connected in a transmission line.](image)

(a) A voltage wave is incident from the left side. What is the reflection coefficient of this wave?

(b) If the power in the incident wave is 1 watt, how much power is in the wave transmitted to the line on the right side of the resistors?

(c) Repeat (a) and (b) if the voltage wave is incident from the right side.

EK18.2 A load consisting of a parallel combination of an inductance $L$ and a resistance $R$ is connected at the end of a lossless transmission line whose characteristic impedance is $Z_0$. An incident voltage wave in the form of a unit step function ($v_+ (t) = 1$ for $t > 0$, $= 0$ for $t < 0$) comes towards the load. Find an expression for the load voltage $v_L(t)$ and for the reflected voltage wave $v_- (t)$ resulting from this load.

EK18.3 Repeat problem EK18.2 for the case when the load is a capacitor $C$.

EK18.4 Given a lossless stripline with $a = 1.27 \, \text{mm}$, $b = 5 \, \text{mm}$, $\varepsilon_r = 9.8$ and $\mu_r = 1$ (alumina), find the values of capacitance and inductance per unit length of the line. What is the characteristic impedance of this stripline?

EK18.5 A lossless stripline whose characteristic impedance is $35 \, \Omega$, whose length is $3 \, \text{cm}$ and whose dielectric filling is alumina (see problem EK18.4 for its material parameters), is terminated in a load impedance of $Z_L = 20 - j80 \, \Omega$. Find the input impedance and the VSWR of this line at $f = 1.8 \, \text{GHz}$.

EK18.6 Repeat problem EK18.2 for the case when the load is a capacitor $C$ in series with a resistor $R$.

EK18.7 Repeat problem EK18.2 for the case when the load is a capacitor $C$ in series with an inductor $L$. [Note: This is no longer a first-order circuit, but a second-order one. Its time response generally has two exponential terms, or possibly exponentially damped sines and cosines. Refer to your circuits text if you don’t remember how to handle such circuits with the Laplace transform.]
EK18.8 Repeat problem P18.2 of the text, but assume the conductors are perfect (zero resistance) while it is the dielectric which is imperfect. Find the conductance \( G' \) per unit length which causes this attenuation.

EK18.9 What value of load impedance connected to the transmission line of problem P18.1 of the text would constitute a matched load? If this impedance were connected to the lossy transmission line of problem EK18.8, what would the magnitude of the reflection coefficient be? Assume as in problem P18.2 that \( f = 10 \text{ GHz} \).

EK18.10 A two-wire line as shown in Table 18.1 has \( 2a = 1 \text{ mm}, d = 1 \text{ cm} \), and is surrounded by a dielectric medium with \( \sigma_d = 0, \mu = \mu_0 \) and \( \epsilon = 2\epsilon_0 \). The conductors have zero resistance.

(a) Find the values of the characteristic impedance \( Z_0 \) and wave velocity \( c \) for this transmission line.

(b) A 20 meter length of this line connects an antenna to a television receiver. What is the time delay for a signal to travel from the antenna to the receiver?

EK18.11 A length \( l \) of a certain transmission line is short-circuited at one end, and the impedance is measured looking into the other end, giving a value \( Z_1 \). This same section of line is then terminated by a resistance \( R \) at one end, and an impedance \( Z_2 \) is measured at the opposite end. Obtain an expression for the characteristic impedance of this transmission line in terms of these two measured impedances and the load resistance \( R \).

EK18.12 A coaxial transmission line whose inner conductor has radius \( a = 2 \text{ mm} \) and outer conductor has radius \( b = 7 \text{ mm} \) is filled with a liquid dielectric whose permittivity is \( \epsilon_r \). Resistance and conductance per unit length are assumed to be negligible.

(a) A signal takes 15 nsec to travel from one end of a 2-meter length of this line to the other. Determine the value of \( \epsilon_r \).

(b) Using the value of \( \epsilon_r \) found in part (a), find the reflection coefficient of a load resistance \( R_L = 25 \Omega \) connected to the end of this line.
EK19.1  A cylindrical resistor of radius $a$ and length $l$ as shown below has a uniform current density flowing in the z-direction corresponding to a total DC current $I$. The resistor has resistivity $\rho$, and is long compared to its radius ($l \gg a$) so that fringing effects near the end of the resistor can be ignored.

(a) Find an expression for the current density $\mathbf{J}$ and electric field $\mathbf{E}$ within the resistor.

(b) Find an expression for the static magnetic field intensity $\mathbf{H}$ both inside and outside the resistor.

(c) Compute the Joule power loss in the resistor using eqn. (10.10) of the text. Show that it equals the result expected from circuit theory for a resistor.

(d) Now compute the Joule power loss in the resistor by using instead eqn. (19.43) of the text and Poynting’s theorem. Show that it is the same as the result from part (c).

Figure EK19.1: A cylindrical resistor.

EK19.2  In Example 12.9 on page 197 of the text, suppose the current in the windings of the solenoid varies sinusoidally with time: $i(t) = I_0 \cos \omega t$, and that the magnetic field inside the solenoid is given by eqn. (12.21) with $I$ replaced by $i(t)$. Denote the radius of the solenoid by $a$ so as not to confuse it with the observation point radius $r$.

(a) Using Faraday’s law and symmetry, find the electric field $\mathbf{E}$ at a radius $r$ from the z-axis of the solenoid, for both $r < a$ and $r > a$.

(b) What is the displacement current density inside the solenoid? What is it outside the solenoid?
EK19.3 A parallel-plate capacitor has circular plates of radius $a$, separated by a dielectric of thickness $d$ and permittivity $\epsilon$. A sinusoidally varying voltage $v(t) = V_0 \sin \omega t$ is applied between the plates.

(a) Assuming that $\omega$ is small enough that we may use Gauss’ law to compute the electric field, obtain an expression for $E$ between the plates ($r < a$ and $0 < z < d$), neglecting field fringing effects at the edges of the plates.

(b) Find the displacement current from the E-field you found in part (a).

(c) Using the generalized Ampère’s law (19.4), find the magnetic field $B$ at any point between the plates.

EK19.4 Using the results of problem EK19.3, compute the Poynting vector at the outside edge of this capacitor ($r = a$). Integrate the Poynting vector over this outside edge, and discuss the power flowing into (or out of) the side of the capacitor.

EK19.5 Using the magnetic field of the parallel-plate capacitor found in part (c) of Problem EK19.3, calculate the stored magnetic energy between the plates. Then, using this result together with the expression for current flowing into the capacitor, obtain an expression for the internal self-inductance of this capacitor (without any connecting wires!). Finally, determine an expression for the resonant frequency $f_r$ of this capacitor when this inductance is taken into account. If $d = 1$ mm, $a = 10$ cm and $\epsilon = 10\epsilon_0$, what is the numerical value of $f_r$?

EK19.6 Using the results of Problem EK19.2, obtain an expression for the Poynting vector inside the solenoid. In what direction is it pointing?

EK20.1 Assume that the long thin solenoid described in problem P15.16 has a circular cross section of radius $d$, and is made from a single layer of tightly wound wire of radius $a$, so that its length is related to the wire radius by $b = N(2a)$. Assume that end effects can be neglected, and thus the external inductance of the solenoid is given by the solution of P15.16, with $S = \pi d^2$. If the conductivity of the wire is such that the skin depth is small compared to the wire radius ($\delta \ll a$), and the permeability of the wire is $\mu_0$, show that the ratio of resistance to reactance of this solenoid is given by:

$$\frac{R}{\omega L} = \frac{\delta}{\pi d}$$

independently of the wire radius or length of the solenoid.
EK20.2 A thin, nonmagnetic ($\mu = \mu_0$) strip conductor has a rectangular cross-section of height $a = 1$ mm and width $d = 1$ cm, and a conductivity $\sigma = 3 \times 10^7$ S/m. A time-harmonic current at a frequency of $f = 900$ MHz flows along the conductor.

(a) Verify that the skin depth $\delta$ is small compared to the dimensions of this conductor.

(b) Compute the resistance per unit length $R'$ of this conductor.

EK21.1 A plane wave in free space has the form of a rectangular pulse, so that at $z = 0$ the electric field is $E_x(0, t) = E_0$ when $0 < t < T$, and $E_x(0, t) = 0$ otherwise. It travels in the $+z$-direction, reaching a surface at $z = 10$ m which completely absorbs all electromagnetic energy incident at it. Find the total energy absorbed in a square meter of the surface if

(a) $E_0 = 10$ mV/m and $T = 1$ $\mu$sec;
(b) $E_0 = 1$ kV/m and $T = 1$ nsec.

EK21.2 A television transmitter radiates a total power of 300 kW, which is distributed uniformly over a spherical surface of radius $r$. If the transmitted wave at this surface is locally approximately the same as a time-harmonic plane wave (propagating in air), find the RMS amplitudes of the $E$ and $B$ fields if (a) $r = 30$ km, and (b) $r = 500$ m.

EK22.1 The surface of a half-space of fresh water is covered with a layer of material that is one-quarter wavelength thick at a frequency of $f = 900$ MHz. The layer is to function as a quarter-wave transformer analogous to the transmission-line device described in Example 18.7 in the text. The goal is to have zero reflection of a normally incident plane wave incident from air onto this covered body of water, so as to have maximum transmission of the signal into the water.

(a) If the cover layer material is to be nonmagnetic ($\mu = \mu_0$), what must its relative permittivity be? Find a reasonable material with these properties using information found in electrical engineering or physics handbooks (hard copy or internet), citing your source. Examples of unreasonable materials are ones that are dangerous (plutonium), liquids or gases.

(b) For the material found in part (a), what is the thickness of the cover layer? [Hint: When we say “quarter-wavelength”, what wavelength are we talking about?]

EK24.1 An antenna whose gain is 5 dB radiates a wave towards a large conducting plane surface. The wave reflected from this surface returns and is received by the antenna. If the frequency is $f = 175$ MHz, and the distance from the reflecting surface is $d = 10$ m, find the ratio of the power received by the antenna to the power delivered to the antenna in radiating the original wave. Express this ratio as both an absolute number and in dB.