1. An air-filled rectangular waveguide is fed by a RF source with an internal impedance of 50Ω and a maximum output power of 20dBm. The waveguide has a TE\textsubscript{10} cutoff frequency of 8.X GHz, where X is the first digit of your social security number. The TE\textsubscript{01} cutoff frequency of the waveguide should be 10.Y GHz, where Y is the last digit of your social security number. The source is operating at a frequency equal to

\[ f = \frac{f_{cTE_{10}} + f_{cTE_{01}}}{2} \]

(a) Design the waveguide so that it will have the appropriate cutoff frequencies.
(b) Determine the amount of power the source would transmit into the waveguide at the frequency above if no matched adaptor is used.
(c) Find the length (in cm) of the waveguide that is half wavelength long at the operating frequency.
(d) Determine the amount of attenuation or phase change that would occur if a 10-cm long straight piece of your waveguide was operated at 6GHz and 10GHz.

2. You need to help design a microwave measurement system shown in the figure above using X-band rectangular waveguide. Assume that the waveguide has a dominant mode cutoff at 8GHz and that the next mode cuts off at 12GHz. The waveguide components are filled with air. The
circuit consists of two paths, where the left path has a total length of 30cm and the right path has a total length of 38cm. One path contains a variable phase shifter and attenuator, while the other contains a fixed 10-dB attenuator, a magic tee coupler and a symmetrical coupler in the direct path. Assume that the component parameters do not vary over the waveguide band.

(a) Design the waveguide dimensions and write down the expressions for the dominant mode electric and magnetic field components.

(b) If you want the signal in the right hand path to have the same attenuation as a signal travelling along the left hand path, find the values for the variable attenuator and phase shifter settings.

3. The circuit from the previous problem measures the reflection coefficient of an unknown load. Explain clearly how the circuit works following the two signals on the vertical branches of the coupler, the one reflected from the load and the one coupled from the source directly. Explain the following:

a) What is the function of the adaptor connected to source? Write down the scattering matrix for this two-port device, \( S_{ad} \).

b) What is the function of the isolator? Write down its scattering matrix \( S_i \).

c) What is the role of the coupler? Assume it is a Moreno coupler with a 10-dB coupling coefficient. Write down the scattering matrix of the coupler \( S_c \).

d) Write down the scattering matrix of the fixed attenuator \( S_a \).

e) What is the function of the magic tee? Draw a 3-D sketch of it. If the signals input into the vertical ports are \( a_{top} \) and \( a_{bottom} \), write the expressions for the signals on the other two ports assuming a lossless device.

f) What is the function of the matched load? What range of impedances does it need to cover in order to operate over the entire waveguide band?

g) The RF signal (e.g. at 10GHz) is modulated with a low-frequency (e.g. 1kHz) signal. This is done so that the detector (diode) can measure the signal. Assume that the signal at the output of the diode is a voltage proportional to the input RF power. Explain the relationship between the signal detected at the detector and the two signals at the vertical ports of the coupler.

4. An instrument casing is made of an aluminum box with walls that are 0.05mm thick. In order to provide a cooling mechanism, rectangular holes are made in the box for air flow. The instrument inside the casing is sensitive to electromagnetic interference from outside signals that operate at 5GHz and these signals need to be attenuated by 20dB if they enter the box. Design the holes so that they operate like a very short waveguide section below cutoff.

a) Calculate the appropriate cutoff frequency for the dominant mode.

b) Calculate the attenuation coefficient \( \alpha \) required to provide 20dB of isolation at 5GHz.

c) Find the largest size of the holes that will ensure this attenuation.
Extra credit:

5. You want to make an antenna on a substrate with a ground plane. You want to make sure that there is no power lost to modes in the substrate, i.e. that the antenna radiates the power into the air half-space above it. If the antenna operates at 60GHz and the substrate has a relative permittivity of 12, calculate the required substrate thickness that will ensure that all the power is radiated and nothing is trapped in the substrate in the form of substrate modes. Is there only one possible solution? Show all your work clearly and attach any code you use to compute the answer. Assume the dielectric substrate is infinite in extent and that the dielectric and conductor are perfect.