Experiment 1
The DC Machine

ECEN 4517

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The purpose of this experiment is to become familiar with operating principles, equivalent circuit models, and basic characteristics of a dc machine. Dc machines are most commonly used in control and servomechanism, as well as industrial, applications. The applications range from small permanent-magnet dc motors at a fraction of a Watt in consumer electronics, to large industrial shunt dc machines having a separate field winding. The machine used in this experiment is a representative of an industrial dc motor (or generator) with a rated power of tens of kilowatts.

The basic dc machine with separate field winding contains three sets of input/output terminals, represented schematically here as in Fig. 1.

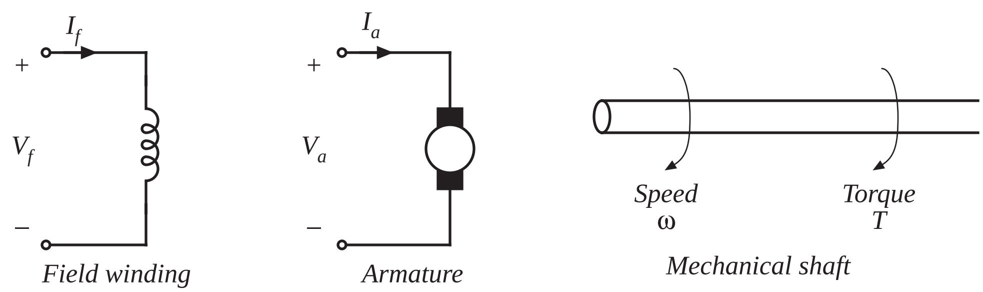

Fig. 1. Symbols used to represent the field winding, armature winding, and mechanical shaft of the dc machine.

The purpose of the machine is to convert electrical energy to mechanical energy (motor action), or vice-versa (generator action). When operated as a generator, it is usually desirable to regulate the dc output voltage V_a. When operated as a motor, it is often desired to control the shaft speed, position, or torque.

In general, the mechanical power of a rotating system is

$$P_{mech} = T\omega = (torque)\ (speed) \tag{1}$$

In MKS units, we have

$$\text{(Watts)} = \text{(Newton-meters)}\ \text{(radians/second)} \tag{2}$$

In the US, we often measure shaft speed in revolutions per minute (rpm), and torque in foot-pounds or ounce-inches. Appropriate unit conversions must then be made.

The "business winding" of any machine is called the armature. This is where the power flows. In a dc machine, the electrical power flowing into the armature is

$$P_{elec} = V_a I_a \quad (3)$$

To the extent that losses within the machine can be ignored (i.e., by approximating the efficiency as 100%), this electrical power is converted into mechanical power:

$$V_a I_a \approx T\omega \quad (4)$$

This is the objective of the machine. For the defined polarities of V_a and I_a, and the defined directions of T and ω shown in Fig. 1, the machine operates as a motor when P_{mech} and P_{elec} are positive, and as a generator when P_{mech} and P_{elec} are negative.

1. Basic relationships in the dc machine

The basic parts of the dc machine are diagrammed in Fig. 2. The stationary part of the machine is called the stator. The field winding is normally placed on the stator. A dc current through this field winding induces a flux ϕ in the machine, which flows through the stator iron, across the air gap, through the rotor iron, across the air gap, and back to the stator iron (Fig. 3). In small dc motors, the field winding is often replaced by permanent magnets on the stator. Such motors have only one electrical port (the armature); the relationships are exactly the same as described here, except that the field ϕ is a fixed value. In machines with a separate field winding, the field winding current I_f is an additional variable that can be used to adjust the machine operation, as discussed below.

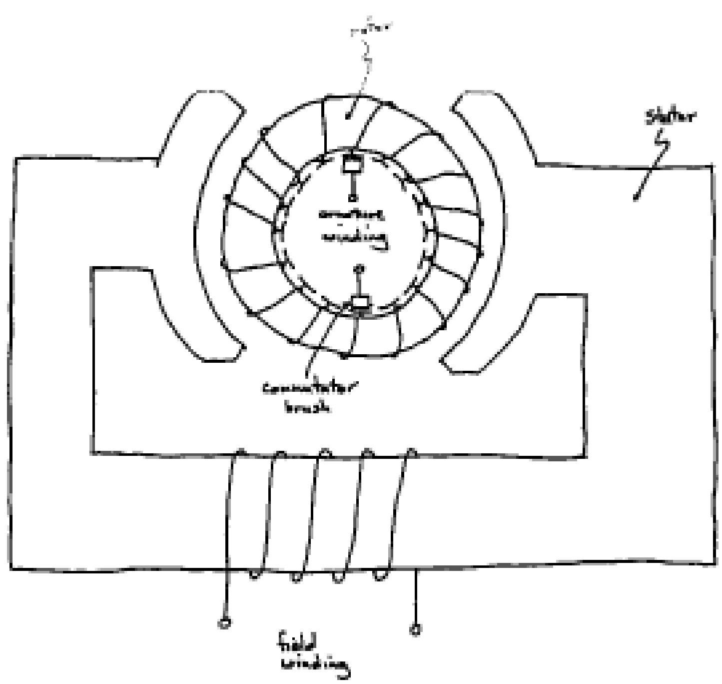

Fig. 2. Basic elements of a dc machine: stator with field winding, rotor with armature winding and commutator.

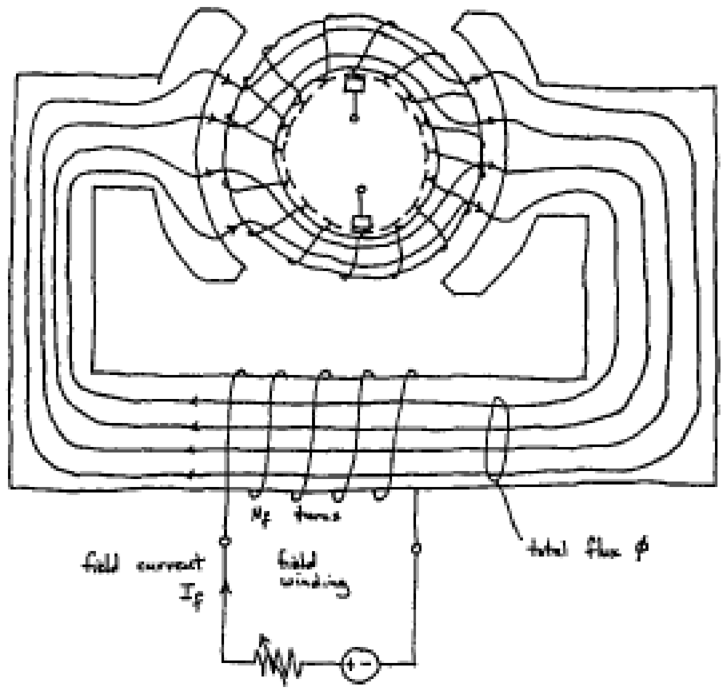

Fig. 3. Field current I_f induces flux ϕ in the machine.

The relationship between the field winding current I_f and the total flux ϕ depends on the *B*-*H* characteristics of the iron and air gaps. The actual relationship is sketched in Fig. 4, and the following linear approximation, Fig. 5, is often useful:

$$\phi = I_f \frac{N_f}{\Re} \tag{5}$$

Here, $N_f I_f$ is the magnetomotive force due to the field winding current, and $\Re$ is the equivalent reluctance of the magnetic path for the flux ϕ. This approximation ignores the hysteresis and saturation of the stator and rotor iron, and it states that the flux ϕ is directly proportional to the field current. This approximation is useful for understanding the behavior of the dc motor. However, it does not fully predict the machine operation, such as the observed behavior of the self-excited generator.

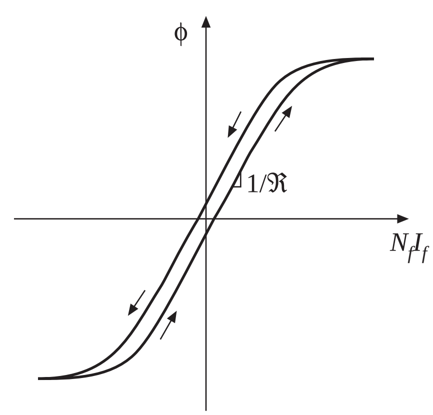

Fig. 4. Relationship between field winding current I_f and total flux ϕ.

Dc machines can be constructed with the field winding connected in series with the armature, in parallel with the armature, or available separately. In cases where the field winding is connected in parallel ("shunt") with the armature (as in this lab experiment), the shunt field windings typically consist of many turns of relatively small wire, and typically $I_f << I_a$. Since series field windings must conduct the armature current, these windings consist of a few turns of much larger wire. In any event, the purpose of the field winding is only to create the flux ϕ in the machine; it does not participate in the electro-mechanical energy conversion.

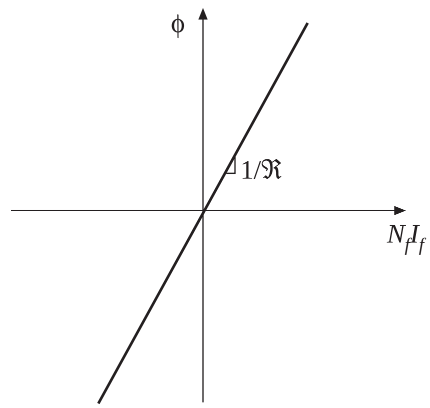

Fig. 5. Approximate linear relationship between I_f and ϕ, which ignores hysteresis and saturation of the stator and rotor iron: $\phi = N_f I_f / \Re$.

The armature winding consists of turns of relatively large wire on the rotor. It is connected via commutator brushes to the stationary frame of reference. As sketched in Fig. 2, the commutator brushes connect to whichever turns of wire are physically at the top and bottom of the rotor. As the rotor turns, the commutator brushes connect to different taps in the rotor armature winding. The effect of this is to rectify the voltage, such that V_a and I_a are dc.

Whenever there is a flux ϕ and the shaft is turning, then a voltage is induced in the armature winding. This voltage E is called the "back EMF," "speed voltage," or "generated voltage." Its origin can be understood by considering Fig. 6.

Let's examine the total flux passing through one of the turns of the armature winding. As sketched in Fig. 6, there are two flux lines passing through the turn when the turn is near the top of the rotor, position a. As the rotor

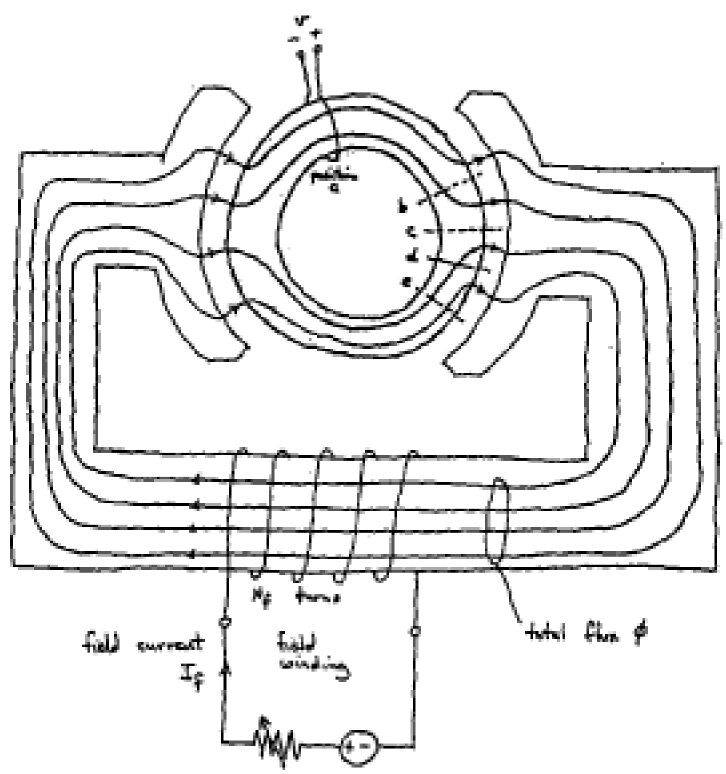

Fig. 6. When the shaft rotates, the flux passing through the armature winding turns changes.

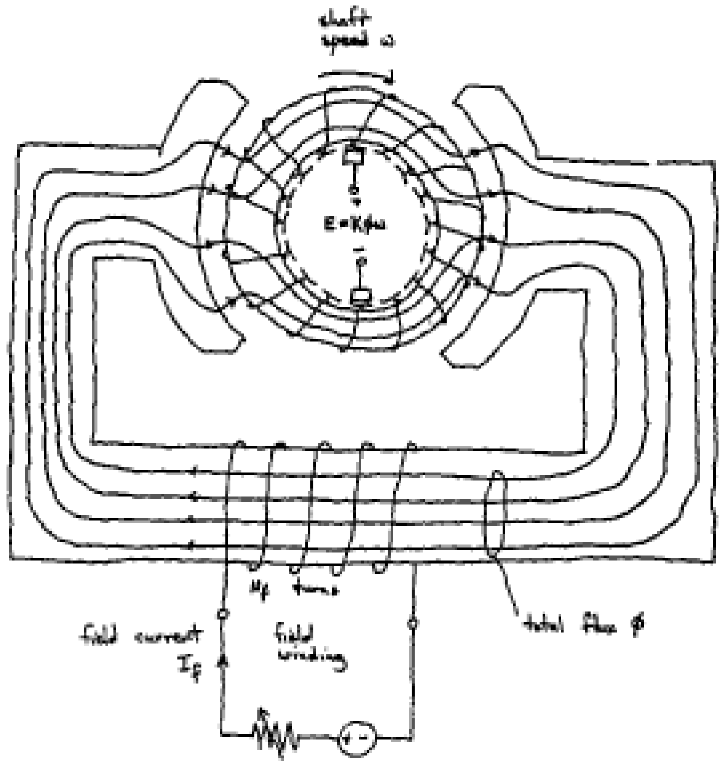

Fig. 7. Hence, whenever the shaft rotates and the field winding is excited, a voltage $E = K\phi\omega$ (called the "back EMF" or "induced EMF") is induced in the armature winding.

turns, the flux decreases to one line (position b), zero lines (position c), one line in the opposite direction (position d) or –1 line, and –2 lines (position e). Thus, as the rotor turns the flux through the armature windings changes.

We know from Faraday's law that when the flux ϕ_{turn} passing through a loop of wire changes, then a voltage is induced:

$$v = \frac{d\phi_{turn}}{dt} \tag{6}$$

Hence, as illustrated in Fig. 7 there is a voltage E induced in the armature winding, proportional to the rate at which the flux in each turn changes. It can be seen that, if the shaft speed is increased, then ϕ_{turn} will change more quickly and E is increased. Also, if the total flux ϕ is increased, then ϕ_{turn} and $d\phi_{turn}/dt$ will be proportionally increased. So E is proportional to the shaft speed ω and the total flux ϕ:

$$E = K\phi\omega \tag{7}$$

where K is a constant of proportionality that depends on the machine construction.

If the armature winding is connected to an external circuit, then a current I_a can flow in the armature winding. As depicted in Fig. 8, this current splits in half: $\frac{1}{2}I_a$ flows

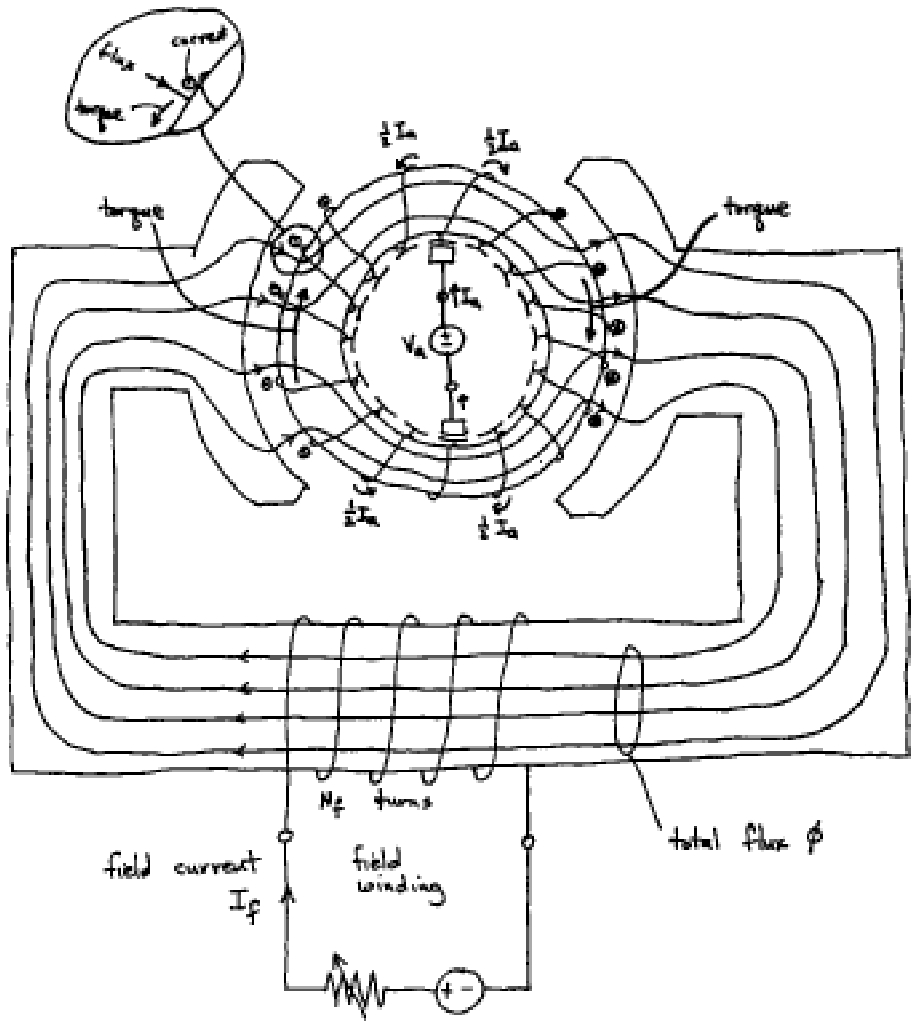

Fig. 8. When armature current I_a flows in the presence of flux ϕ (i.e., in the air gap between the rotor and stator), a torque is produced which tends to cause the rotor to turn.

through the windings on the left side of the rotor and $\frac{1}{2}I_a$ flows on the right. It can be seen from Fig. 8 that, on the right side of the rotor, the conductors lie in the air gap where a magnetic field is present (due to the flux ϕ). When a current flows in the armature winding, there will be a force induced in these conductors, proportional to the cross product of the current and the flux density $\left(\vec{I} \times \vec{B}\right)$. Since the current on the right side flows into the paper and the flux is directed left-to-right, then the force is directed downward. A similar thing happens to the conductors on the left side of the rotor in the air gap, except that the force is directed upward because the current flows out of the paper. As a result of these forces, there is a torque on the rotor, which is proportional to the armature current I_a and to the flux ϕ:

$$T = K\phi I_a \quad (8)$$

The constant of proportionality K in Eq. (8) is identical to the K in Eq. (7). This must be true because the electrical input power is

$$P_{elec} = I_a E = I_a K\phi\omega \tag{9}$$

The mechanical output power is

$$P_{mech} = T\omega = K\phi I_a \omega \tag{10}$$

The dc motor converts the electrical input power to mechanical output power. Equating Eqs. (9) and (10) shows that the two constants of proportionality are the same.

2. An equivalent circuit model for the dc machine

We can construct equivalent circuit models which describe the properties noted in Section 1 above, and which can be solved to find how the dc machine will behave in a given circuit or system.

The field winding is basically an inductor. If we use the linear approximation of Fig. 5, then the inductance of the winding is

$$L_f = \frac{N_f \phi}{I_f} = \frac{N_f^2}{\Re} \tag{11}$$

with

$$\phi = \frac{N_f}{\Re} I_f \tag{12}$$

Usually, the wire used to wind the field winding has significant resistance. We can model this as a series resistor R_f. A suitable equivalent circuit for the field winding is therefore as shown in Fig. 9.

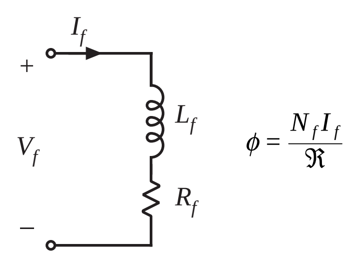

Fig. 9. Field winding equivalent circuit.

The major feature of the armature, as discussed in the previous section, is the back EMF $E = K\phi\omega$. We can model this using a dependent voltage source E. We may also wish to model the Thevenin-equivalent output resistance of the armature, R_a, by placing a resistance in series with E as shown in Fig. 10. Several sources contribute to this resistance, including the resistance of the commutator brushes, the resistance of the copper wire, and an effect known as "armature reaction" in which the armature current causes the air gap total flux to be reduced. In a well-designed machine, the voltage drop $I_a R_a$ is small compared to E under rated conditions, and hence $V_a \approx E$.

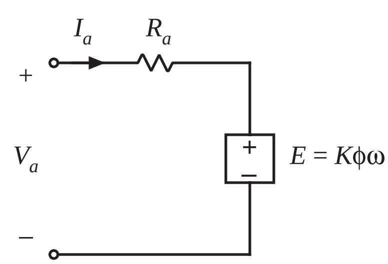

Fig. 10. Armature winding equivalent circuit.

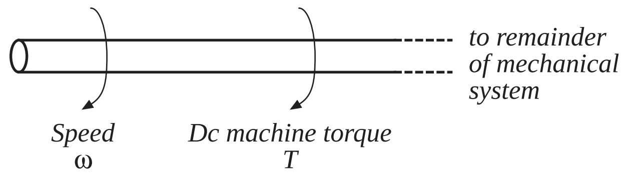

Fig. 11. "Equivalent circuit" for mechanical connection of dc machine shaft.

The last part of the equivalent circuit is the mechanical portion of the system, sketched in Fig. 11. The motor develops torque $T = K\phi I_a$, which pushes in the direction of rotation of the shaft. For generator action, I_a and T change polarities, and the torque opposes the rotation of the shaft. A key point here is that, for a given ϕ, the shaft torque T and the armature current I_a are directly proportional. This is often used in servo applications where it is desired to control the torque rather than the speed: the armature is driven by a current source rather than a voltage source, and the resulting torque is proportional to this applied current.

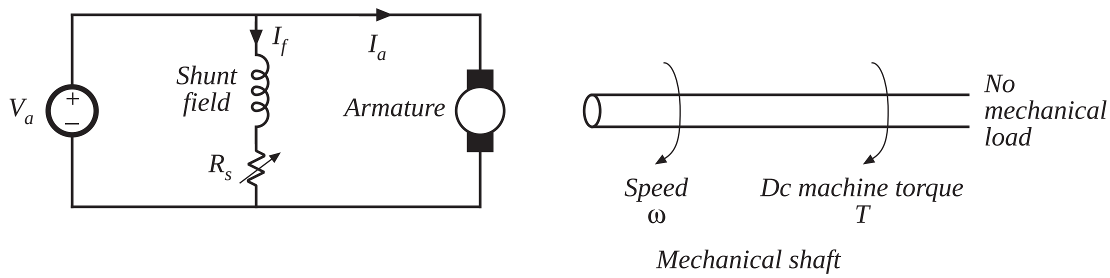

Fig. 12. Dc motor with shunt field winding and no mechanical load.

3. Solution of a simple system containing a dc motor

Let us now consider an unloaded dc motor with shunt field winding, as in Fig. 12. This connection is used in Experiment 1. To understand the behavior of this system, the first thing that we should do is to replace the armature and field with their equivalent circuits, as in Fig. 13.

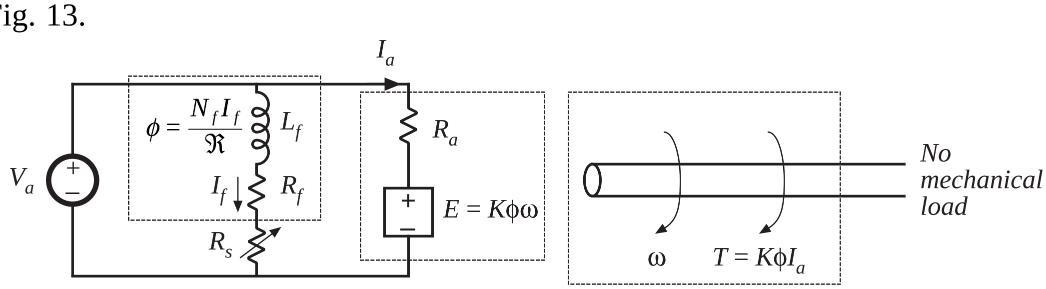

Fig. 13. Equivalent circuit for the network of Fig. 12.

Now, let us solve this circuit, to find the shaft speed ω as a function of the applied armature voltage V_a and field current I_f. Since there is no mechanical load (other than the small friction of the bearings and the resistance of the air to rotation of the shaft), the only torque on the shaft is the motor torque T. Hence, when this torque is positive, then the shaft accelerates according to

$$T = J\,\frac{d\omega}{dt} \tag{13}$$

where J is the moment of inertia of the shaft, and $d\omega/dt$ is the angular acceleration of the shaft. Equation (13) is simply the rotating form of Newton's Law $F = ma$. More generally, if there are other mechanical torques on the shaft, then

$$J\,\frac{d\omega}{dt} = \text{the sum of all torques on the shaft} \tag{14}$$

When the motor is first started, the shaft accelerates. Eventually, the shaft speed reaches some steady-state speed and the system comes to equilibrium. At this steady-state speed, the acceleration $d\omega/dt$ is zero, and hence the torque $T = K\phi I_a$ is also zero. This implies that the armature current I_a is zero in steady-state for this example with no load.

From the equivalent circuit, we can find an expression for I_a:

$$I_a = \frac{V_a - E}{R_a} = \frac{V_a - K\phi\omega}{R_a} \tag{15}$$

If $I_a = 0$, then we must have

$$V_a = E = K\phi\omega \tag{16}$$

We can now solve for ω:

$$\omega = \frac{V_a}{K\phi} \tag{17}$$

Now eliminate ϕ in favor of I_f, using the linear approximation of Fig. 5:

$$\omega = \frac{V_a}{K\left(\frac{N_f}{\Re} I_f\right)} = \frac{V_a}{I_f}\left(\frac{\Re}{KN_f}\right) \tag{18}$$

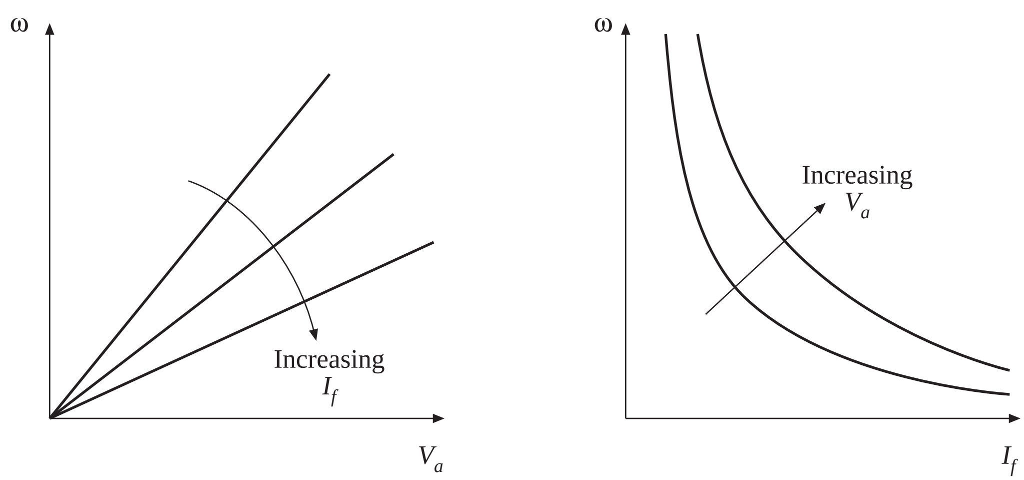

Fig. 14. Relationships between the shaft speed ω, and the armature voltage V_a and field current I_f, for the unloaded dc shunt motor.

So the shaft speed ω is directly proportional to the armature voltage V_a and inversely proportional to the field current I_f, as shown in Fig. 14.

Hence it is possible to control the speed of the dc motor by varying either the armature voltage or the field current. It can also be seen that the field winding must not be disconnected: if $I_f \rightarrow 0$, then the shaft speed tends to a very large value.

REFERENCES

Fitzgerald, Kingsley, and Umans, *Electric Machinery*, Fifth Edition, McGraw-Hill (1990), Chapter 9.

PROBLEMS

1. A shunt dc motor is driving a mechanical load at 1200 rpm with a torque of 10 Newton-meters. The armature resistance is 0.5 Ω, and the dc voltage applied to the armature is 125 volts. Find:

(a) the armature current

(b) the back EMF

(c) the motor efficiency

2. You are given the series-wound dc motor system illustrated in Fig. 15.

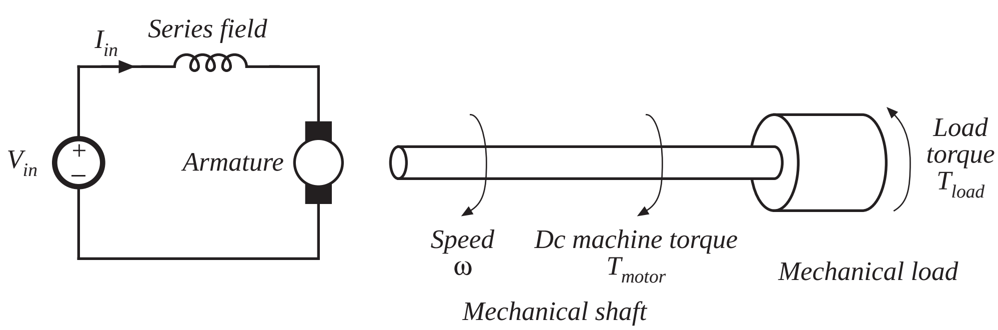

Fig. 15

(a) Sketch the equivalent circuit for this system.

(b) Solve your equivalent circuit, to derive an analytical expression for how the shaft speed ω varies with load torque T_{load} in equilibrium. The input voltage V_{in} and the motor parameters R_f, R_a, N_f, K, and $\Re$ are given. All other quantities are unknown, and should not appear in your equation relating ω and T_{load}.

3 . A dc motor, when operated unloaded from a 250 Vdc source with 0.5 A shunt field current, has an armature current of 1 A. The shaft speed is 1800 rpm. The armature resistance of this machine is 0.1 Ω.

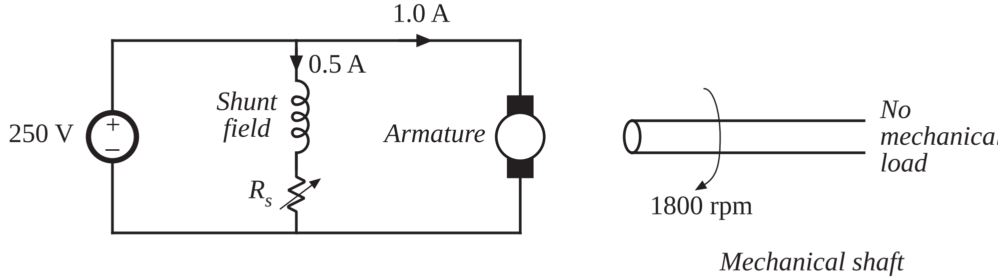

Fig. 16

(a) Even when there is no externally-applied load torque, the motor must still supply mechanical power to the shaft, because of (*i*) friction in the bearings, and (*ii*) windage, or the resistance of the air to rotation of the shaft. Under the conditions described above, find the power loss due to windage and friction, and the effective load torque caused by windage and friction.

(b) It is desired to regulate the shaft speed to be a constant 1800 rpm, regardless of the load torque. This is to be done by controlling the shunt field current I_f. When the mechanical load torque is 100 N-m, to what value should the field current I_f be adjusted to obtain a shaft speed of 1800 rpm?

Experiment 1
Pre-lab assignment
Direct Current Shunt Motors

1. Read sections 1 to 3.

2. Do problems 1 and 2.

3. Read the laboratory procedure.

This assignment is due from each student at the beginning of the lab session.