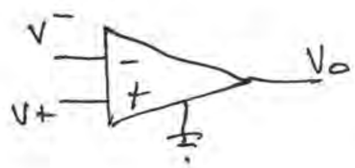
RWE ①

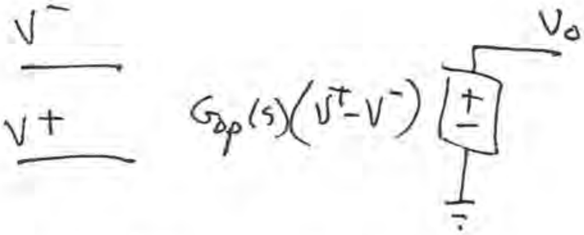
9/10/12
Lecture notes
ECEN 4517

Feedback analysis and compensator design example: Investigation of an op amp lead compensator circuit

The op amp in the parts kit (ALD 2702) is a CMOS rail-to-rail input op amp with 1 MHz gain-bandwidth product.



model $\Rightarrow$



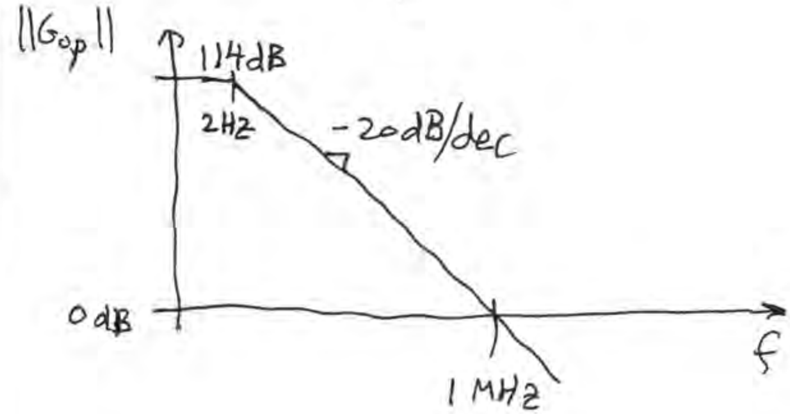
$$G_{op}(s) = G_o \frac{1}{1 + \frac{s}{\omega_{op}}}$$

$$f_{op} = \frac{\omega_{op}}{2\pi} = 2 \text{ Hz}$$

$$G_o = 5 \cdot 10^5 \Rightarrow 114\,\text{dB}$$

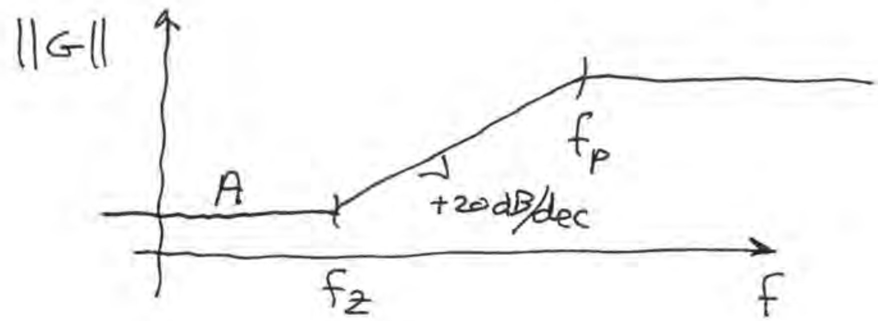
The gain at 1 MHz is

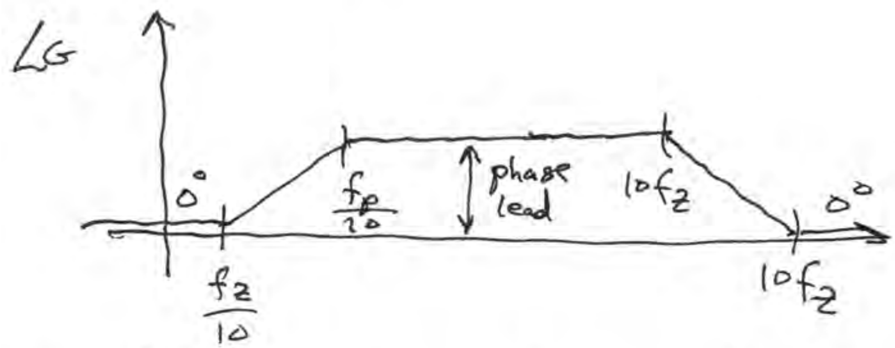
$$(5 \cdot 10^5) \frac{1}{\left\| \cancel{1} + \frac{j\, 2\pi 10^6}{2\pi \cdot 2} \right\|} = \frac{5 \cdot 10^5}{5 \cdot 10^5} = 1$$



Let's consider a lead compensator (PD) having a transfer function

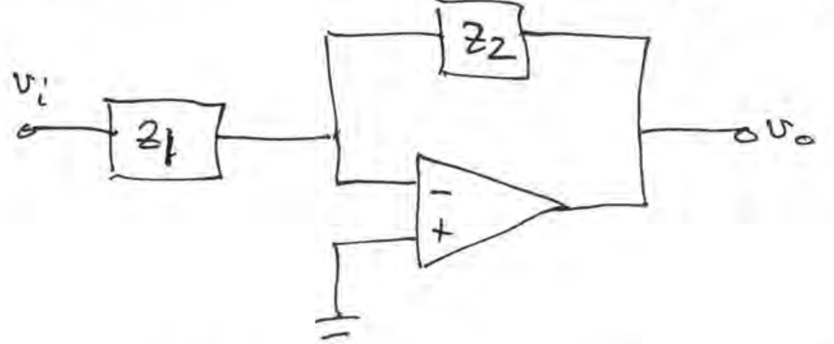
$$G(s) = A \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}$$





Such circuits are sometimes added to feedback loop compensators, to improve phase margin.

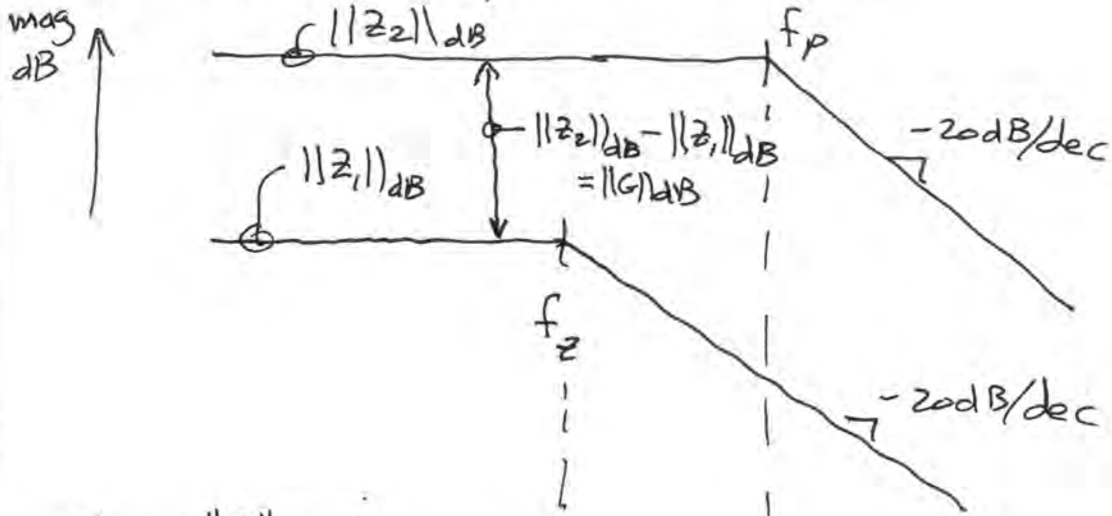
A realization using an inverting amplifier:



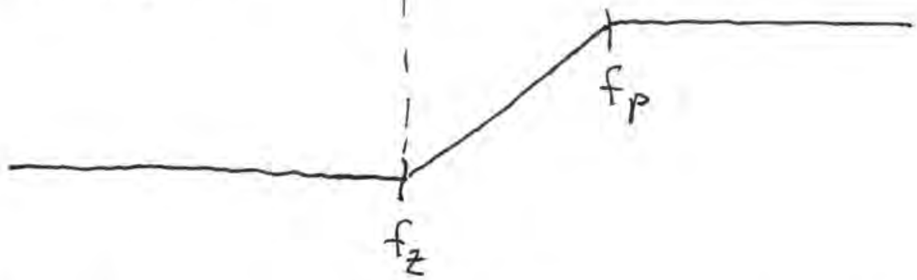
$$G(s) = -\frac{Z_2}{Z_1} = \frac{v_o}{v_i} \qquad \|G\|_{dB} = \left\| \frac{Z_2}{Z_1} \right\|_{dB} = \|Z_2\|_{dB} - \|Z_1\|_{dB}$$

(this also adds a minus sign)

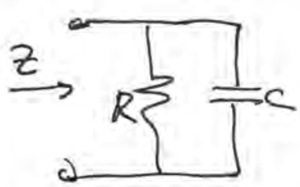
Let's make the impedances do this:

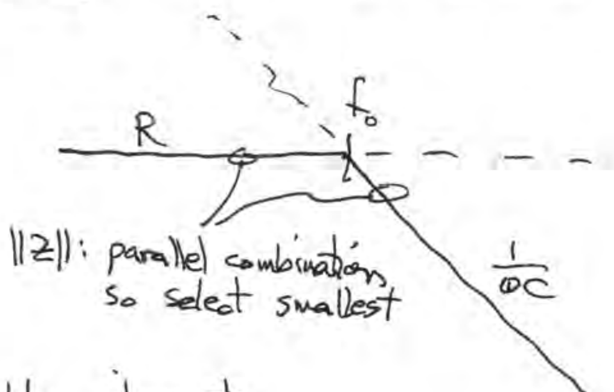


Then $\|G\|_{dB}$ is



Realization of Z_1, Z_2: a parallel R and C will work





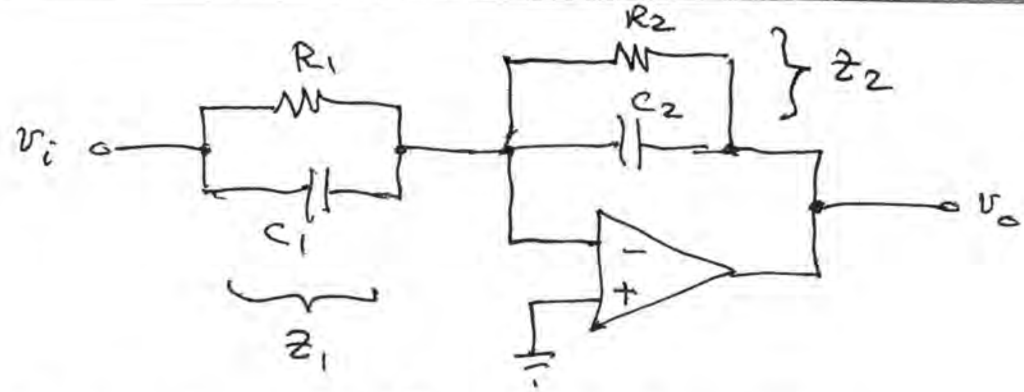
At f_o: asymptotes intersect

$$R = \frac{1}{2\pi f_o C} \Rightarrow f_o = \frac{1}{2\pi RC}$$

Suppose we want $A = 10 \Rightarrow 20$ dB

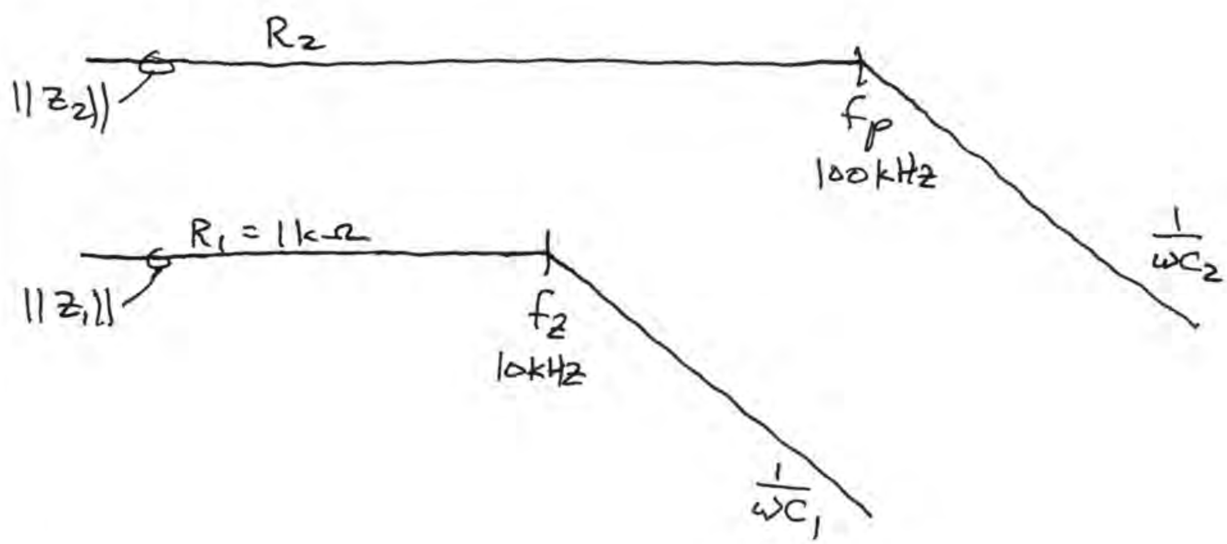
$f_z = 10$ kHz

$f_p = 100$ kHz



Let's choose (arbitrarily) $R_1 = 1\ k\Omega$

Then Z_1 and Z_2 should be:



(a) we want dc gain $A = 10$

At dc, $Z_1 = R_1$ and $Z_2 = R_2$ so $A = \frac{R_2}{R_1}$

$\Rightarrow$ need $R_2 = 10\ k\Omega$

(b) we want $f_z = 10\ kHz$

at f_z, $R_1 = \frac{1}{2\pi f_z C_1}$ so $C_1 = \frac{1}{2\pi f_z R_1}$

$$= \frac{1}{2\pi (10kHz)(1k\Omega)}$$

$$= 0.016\ \mu F$$

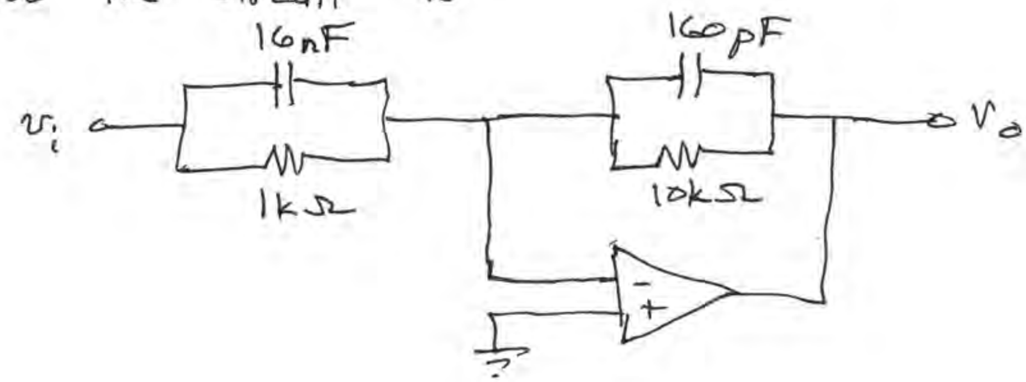
(c) we want $f_p = 100\,kHz$

at f_p, $R_2 = \frac{1}{2\pi f_p C_2}$ so $C_2 = \frac{1}{2\pi f_p R_2}$

$$= \frac{1}{2\pi (100\,kHz)(10k\,\Omega)}$$

$$= 160\,pF$$

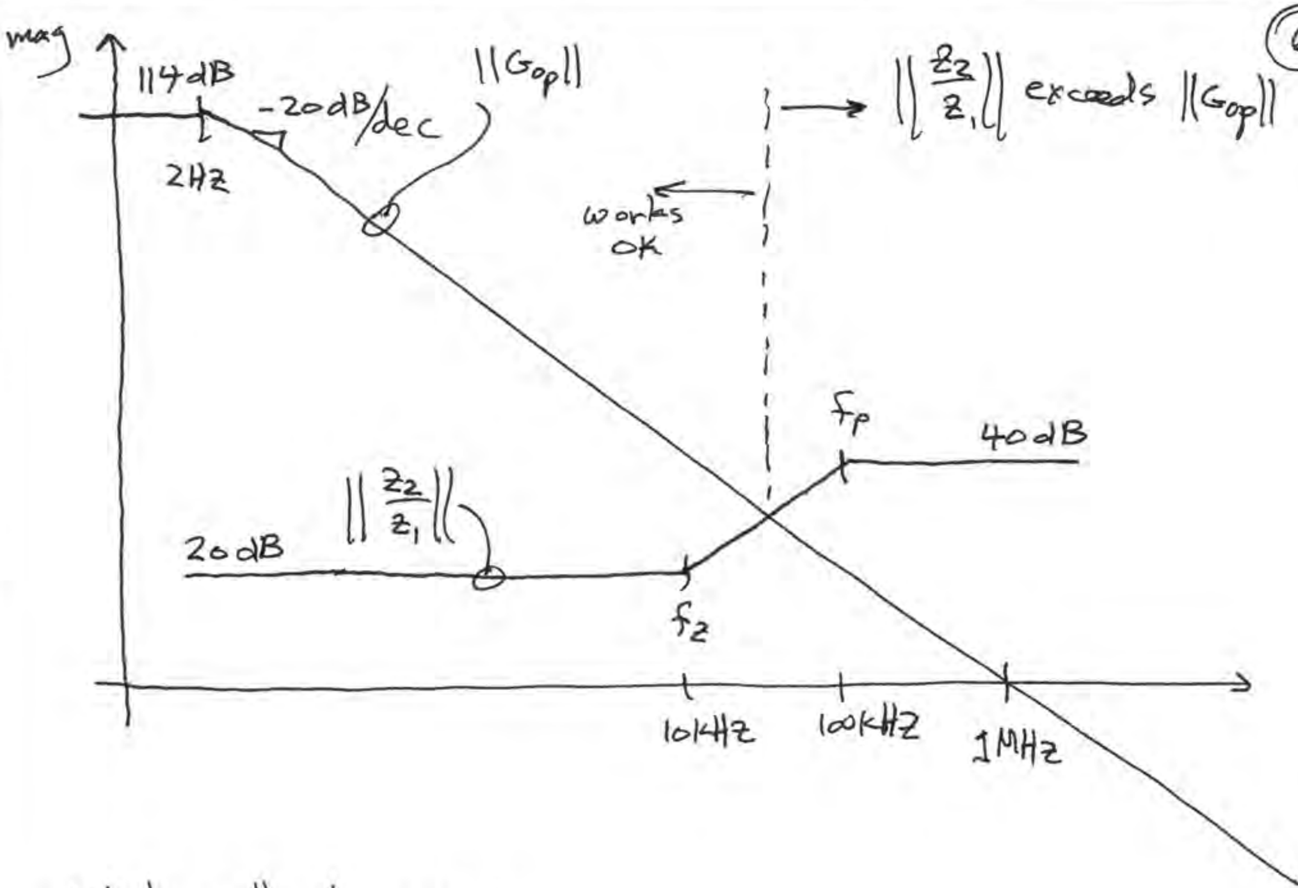
So the circuit is



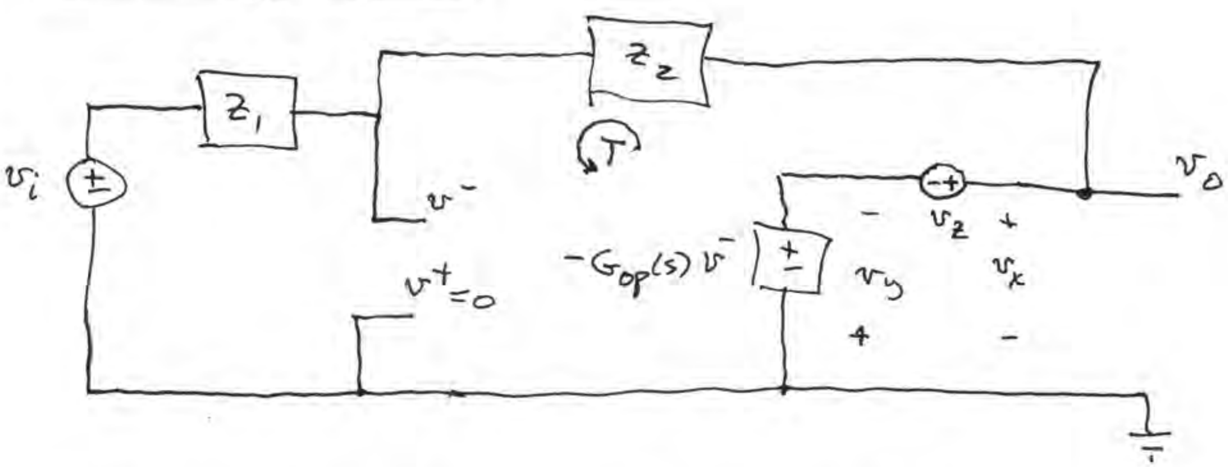
A problem with this design!

we are trying to obtain more gain than the op amp is capable of producing.

The formula $G(s) = -\frac{Z_2}{Z_1}$ is true for ideal op amps with $G_{op} \to \infty$ (ie., for $\|G_{op}\|$ sufficiently large). But at high frequency, G_{op} rolls off.



What really happens:



To find loop gain (or measure loop gain):
inject a voltage v_z at output of op amp
given v_x, find v_y. Loop gain is $T(s) = \frac{v_y}{v_x}$
with $v_i = 0$.

Analysis: with $v_i = 0$, $v^- = \frac{Z_1}{Z_1+Z_2} v_x$ and $-v_y = -G_{op} v^-$

so $v_y = \frac{Z_1}{Z_1+Z_2} G_{op} v_x$ and $T(s) = \frac{Z_1}{Z_1+Z_2} G_{op}(s)$

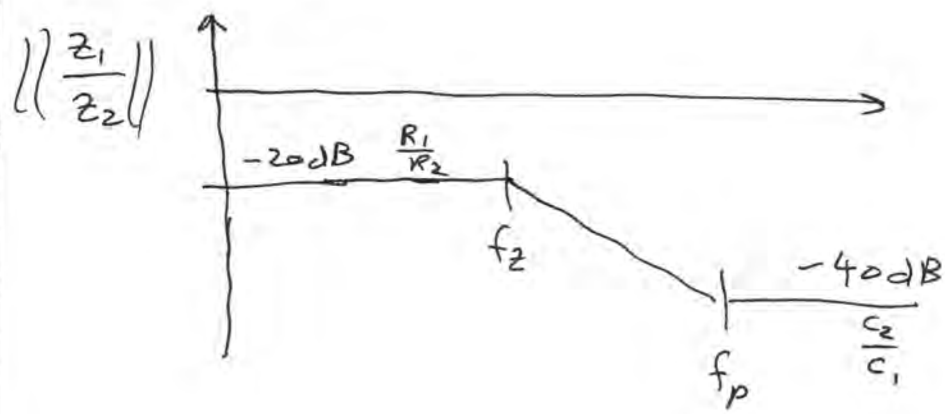
The actual gain is

$$G(s) = \frac{v_o}{v_i} = \underbrace{\left(-\frac{Z_2}{Z_1}\right)}_{\text{ideal gain}} \underbrace{\left(\frac{T}{1+T}\right)}_{\text{effect of finite } G_{op}}$$

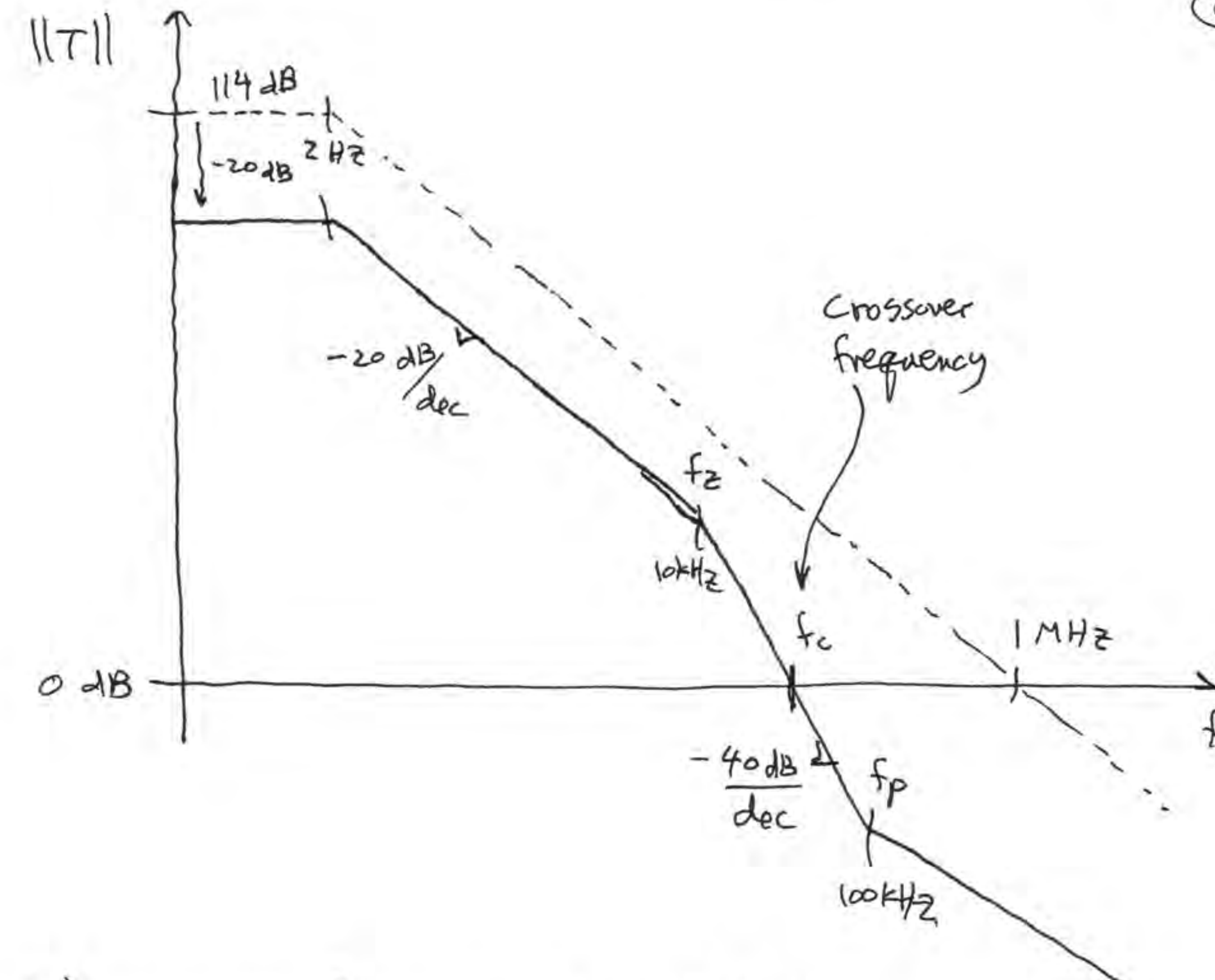
Construct loop gain: $\quad \dfrac{Z_1}{Z_1+Z_2} = \dfrac{1}{1+\dfrac{Z_2}{Z_1}}$

Since $\|Z_2\| \gg \|Z_1\|$ everywhere, this is approximately

$$\frac{1}{\cancel{1}+\dfrac{Z_2}{Z_1}} \cong \frac{Z_1}{Z_2} \qquad \text{which is } \frac{1}{(\text{ideal gain})}$$



So $T \cong \dfrac{Z_1}{Z_2} G_{op}$ is decreased relative to G_{op}



Find crossover frequency where $\|T\| = 1$

$$T = \frac{Z_1}{Z_1 + Z_2} G_{op} \approx \frac{Z_1}{Z_2} G_{op} = \frac{R_1}{R_2} \frac{\left(1 + \frac{s}{\omega_p}\right)}{\left(1 + \frac{s}{\omega_z}\right)} \frac{G_o}{\left(1 + \frac{s}{\omega_{op}}\right)}$$

for $f_z < f < f_p$, $T \cong \frac{R_1}{R_2} \frac{\left(1 + \cancel{\frac{s}{\omega_p}}\right)}{\left(\cancel{1} + \frac{s}{\omega_z}\right)} \frac{G_o}{\left(\cancel{1} + \frac{s}{\omega_{op}}\right)}$

so $\|T\| \cong \frac{R_1}{R_2} G_o \frac{\omega_z \omega_{op}}{\omega^2}$

at $f = f_c$, $\|T\| = 1 = \frac{R_1}{R_2} G_o \frac{f_z f_{op}}{f_c^2}$

$$\Rightarrow f_c^2 = \frac{R_1}{R_2} G_o f_z f_{op}$$

$$f_c = \sqrt{\frac{R_1}{R_2} G_o f_z f_{op}} \cong 30 \text{ kHz}$$

What is the phase margin? (9)

$\angle T$ at $f = f_c$ is

$-90°$ from the pole at $f_{0p} = 2\,Hz$

from the pole at f_z and zero at f_p:

see textbook Fig. 9.16 or Eq. (9.34).

The result is $-\tan^{-1}\left(\frac{\sqrt{10} - \sqrt{0.1}}{2}\right) = -55°$

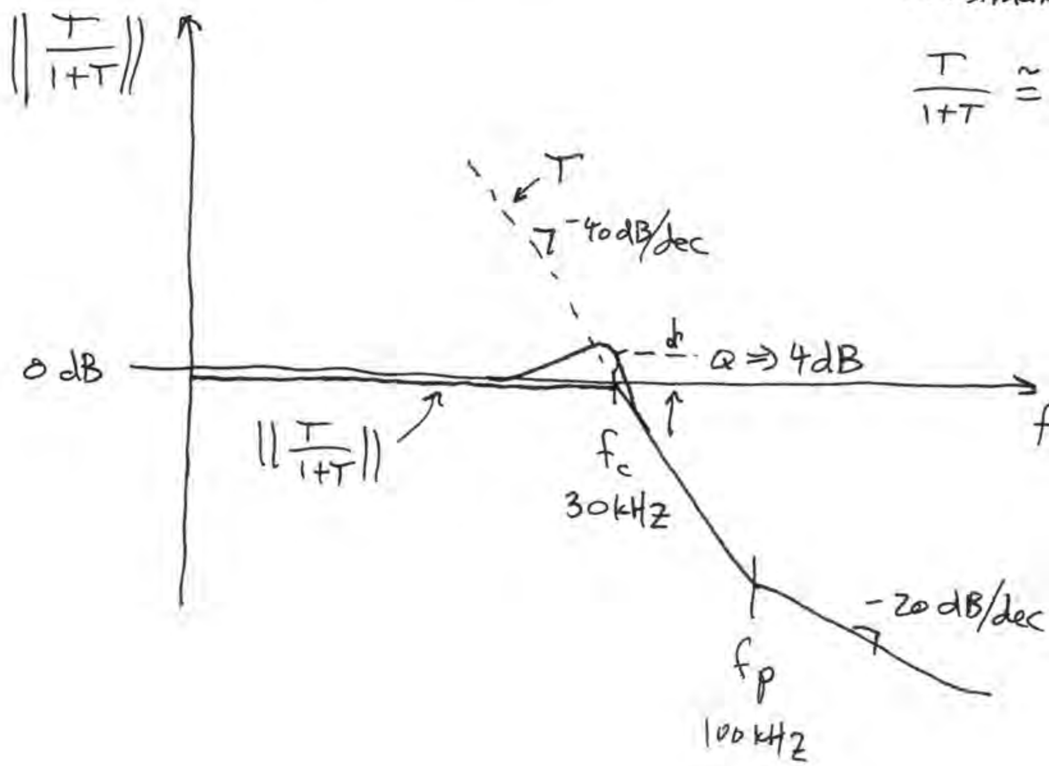
so $\angle T = -90° - 55° = -145°$ at f_c

phase margin $= 180° + \angle T = 35°$

From textbook Fig. 9.13, this leads to closed-loop Q of $1.57 \Rightarrow 4\,dB$

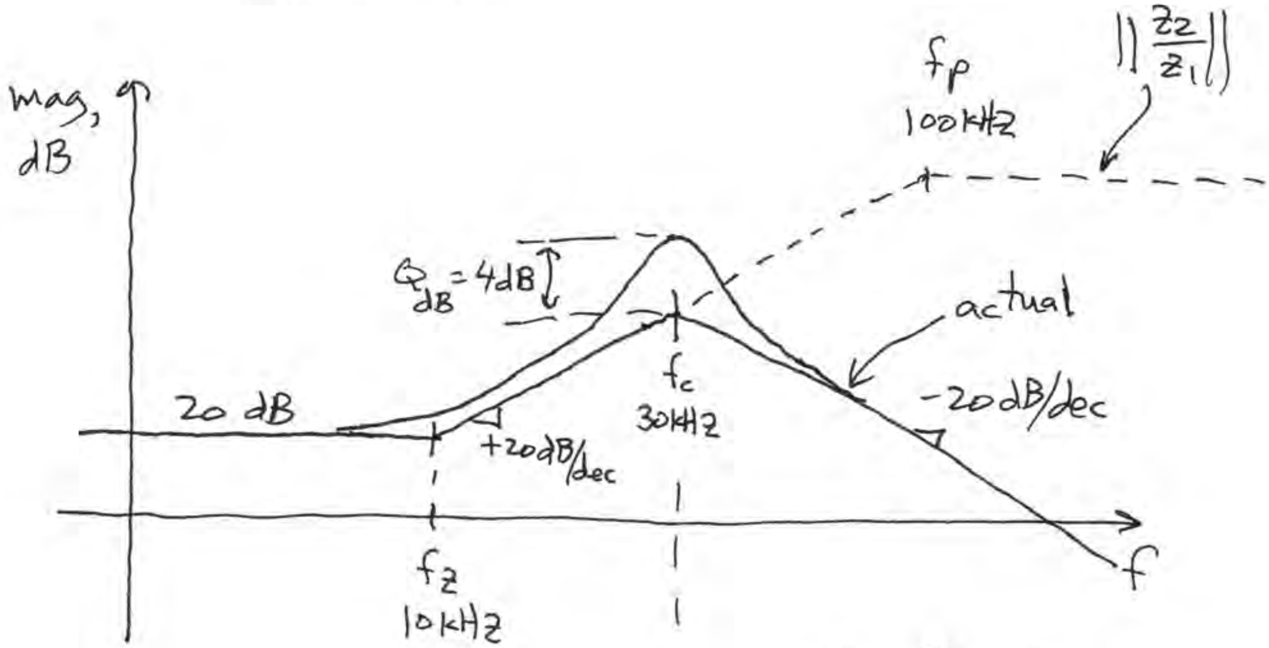
construction of $\frac{T}{1+T}$

$$\frac{T}{1+T} \cong \begin{cases} 1, & \|T\| \gg 1 \\ T, & \|T\| \ll 1 \end{cases}$$



Actual transfer function

$$-\frac{Z_2}{Z_1}\,\frac{T}{1+T}$$



for $f < f_c$,

$$G \cong -\frac{Z_2}{Z_1}$$

for $f > f_c$,

$$G \cong \left(-\frac{Z_2}{Z_1}\right)\left(\frac{T}{1+T}\right)$$

$$= \left(-\frac{Z_2}{Z_1}\right)\left(T\right)$$

$$= \left(-\frac{Z_2}{Z_1}\right)\left(\frac{Z_1}{Z_2}T\right)$$

$$= -G_{op}$$

So the actual transfer function follows the ideal value $\left(-\frac{Z_2}{Z_1}\right)$ as long as $\|G_{op}\| \gg \left\|\frac{Z_2}{Z_1}\right\|$. But when $\|G_{op}\| < \left\|\frac{Z_2}{Z_1}\right\|$ then the actual transfer function follows $(-G_{op})$ instead.