1. Find the input impedance of a 50-Ω line that is quarter-wavelength long, if it is terminated in a (1) short and (2) complex impedance equal to 25-j75 Ω at the frequency of interest. Solve the problem using (a) a formula and (b) a Smith chart. Repeat for a line that is one eighth of a wavelength long at the operating frequency.

2. Write down the formulas and definitions for the:
   (1) characteristic impedance of a transmission line;
   (2) phase velocity of a wave travelling in a transmission line;
   (3) group velocity and explain when this is relevant;
   (4) attenuation constant;
   (5) propagation constant;
   (6) voltage at any point on a line;
   (7) current at any point on the line;
   (8) reflection and transmission coefficient definitions, and expressions for the case when a complex load $Z_L$ is connected at the end of the line;
   (9) impedance at any point along a transmission line; and
   (10) expressions for the capacitance, inductance, resistance and conductance per unit length of a coaxial line filled with a dielectric with relative permittivity $\varepsilon_r$ and of a large but finite resistivity $\rho$, with inner conductor radius $a$, and outer conductor radii $b$ and $c$ made of a metal with a large but finite conductivity $\sigma$.

3. (1) What does a “waveguide mode” mean?
   (2) What is the dominant mode in a rectangular metal waveguide?
   (3) Sketch the E and H field lines for this dominant mode.
   (4) Write down the expressions for the mode impedance, mode propagation coefficient, wavelength, E and H field components, and cutoff frequency for the dominant mode of a rectangular waveguide with cross-section dimensions $a$ and $b$.
   (5) Calculate the mode impedance and the cutoff frequency in a rectangular waveguide for the dominant mode at 10GHz, if the waveguide dimensions are $a=2\text{cm}$ and $b=1\text{cm}$ and the waveguide is filled with air.

4. Assume a coaxial cable has only resistive loss (the dielectric is perfect). In that case, what kind of coaxial cable would you fabricate to minimize the attenuation coefficient?
The conclusion might seem surprising. Explain your conclusion based on electromagnetic field principles.

**ADDITIONAL PROBLEM, ECEN 5634 ONLY**

5. In this problem we examine the 50-Ω choice for the characteristic impedance of a coaxial cable. Use standard notation (inner conductor radius is \(a\), outer radii are \(b\) and \(c\), etc.).

(a) First we look at the power limitation in a coaxial line. Write down the expression for the maximal field in a cable for a given voltage generator of voltage \(V\). Using this expression, find the maximal peak power that is transmitted in a cable as \(P = V^2 / Z_0\) (we are ignoring power matching etc.). Next find the optimal cable dimensions which maximize the peak power. This means you will need to take a derivative of power w.r.t. some dimension and set it to zero. One possible way is to find \(\frac{\partial P}{\partial a} = 0\) and find what ratio of \(b/a\) maximizes the power. Using this optimal ratio, find the optimal characteristic impedance that maximizes peak power transfer. Is it close to 50 \(\Omega\)?

(b) Next, we look at the loss limitation of a line. What is the expression for the attenuation coefficient \(\alpha\) for a lossy line with relatively low loss? Derive the expressions for the resistance and inductance per unit length at DC to find the dependence of \(\alpha\) on the geometrical parameters of the cable. (How does this change at microwave frequencies?) To minimize loss, we set \(\frac{\partial \alpha}{\partial a} = 0\). What do you get in this case for the ratio \(b/a\)? Is it close to 50 \(\Omega\)?

(c) Summarize: what is the optimal characteristic impedance for power handling, and what is the value for lowest loss? What is the peak power for the power-optimized line that has air as the dielectric? (Air breaks down when the electric field magnitude is about 30kV/cm).