Operating Mode Review

- The key to analyzing and later designing for DC biasing in circuits is a solid, intuitive understanding for the operating modes and equations for devices.

- The general procedure is to identify the known "biasing" portions of the circuit, then work step-by-step through the rest of the circuit
  - with each device, use the known conditions to rule out any modes from the plots above
  - then, if more than one possible mode exists, "guess" the mode that is the easiest to prove or disprove → systematically infer which mode each device is operating in of the resulting DC bias currents & voltages.

- We demonstrate this process with a few examples.
Ex #1

- Solve for $V_1 + V_2$ as set of $I_b$, $K_p$, $V_{tn}$, $V_{in}$, $V_{tp}$

- $1^{st}$ note that: $(V_{ds})_{p1} = (V_{gd})_{n1} = 0$
  - Only possible op modes are cutoff and active
  - Since $i_{on1} = i_{p1} = I_b > 0$
    - cannot be cutoff $\Rightarrow$ active!

$\Rightarrow i_{on1} = K_{n1} (V_2 - V_{tn})^2 (1 + 2V_2) = I_b$
$\Rightarrow i_{p1} = K_{p1} (V_{dd} - V_1 - V_{tp})^2 (1 + 2(V_{dd} - V_1)) = I_b$

$\Rightarrow$ Quadratic eqns to solve $V_1 + V_2$. For a quick estimate, we can also freq. approx: $V \approx 0$

$\Rightarrow V_2 = V_{tn} + \frac{\sqrt{I_b}}{\sqrt{K_{n1}}}$
$\Rightarrow V_1 = V_{dd} - V_{tp} - \sqrt{\frac{I_b}{K_{p1}}}$

Ex #2

- Difficult to start w/ $M_2$ since $V_1$ is unknown.
- Start w/ $M_1 + R_b$, which is similar to Ex #1.
- Again, if $I_b > 0$ ($R_b < \infty$, $V_{dd} > V_1$)
  - $M_1$: active
    - Quadratic to solve $I_b$ given $R_b$ & $M_1$
$\Rightarrow V_1 = V_d + \sqrt{\frac{I_b}{K_1}}$
$\Rightarrow I_b = \frac{V_{dd} - V_1}{R_b}$
  - Given desired $I_b$, simple to solve $V_1$, then $R_b$. 

- For $2 \approx 0$
For M2: 
- Given that $V_1 > V_t$ (since $M_{\text{active}}$) 
  $+ V_1 < V_{dd}$. (due to $R_8$)

$\Rightarrow V_{g_2} > V_t \quad \Rightarrow \quad \text{active (M2)}$
$V_{g_2} < V_t \quad \Rightarrow \quad i_{o2} = K_2 (V_1 - V_t)^2 (1 + 2V_{dd}) = I_0$

- However: note that there is a relationship between $I_8$ and $I_0$; since $V_{g_1} = V_{g_2}$

$\Rightarrow i_{o1} = K_1 (V_1 - V_t)^2 (1 + 2V_1) \quad \lambda \neq 0$

$\Rightarrow \quad \frac{i_{o2}}{i_{o1}} = \frac{I_0}{I_0} = \frac{K_2 (1 + 2V_{dd})}{K_1 (1 + 2V_1)} \quad \lambda \neq 0 \Rightarrow \quad \frac{K_2}{K_1} = \frac{(w/l)^2}{(w/l)}$

$\Rightarrow \text{"current-mirror"}, \quad \text{where drain currents are}
\text{scaled by the \((w/l)\) ratios \(\lambda\)}$

\begin{align*}
(1) \quad \text{Both devices are active.} \\
(2) \quad \text{Either} \quad \lambda \neq 0 \quad \text{or} \quad V_{ds_1} = V_{ds_2} \quad \text{(matched drain voltages)} \\
(3) \quad V_{gs_1} = V_{gs_2} \quad \text{(matched gate voltages)} \\
(4) \quad \text{matched parameters:} \quad V_t, \ A'
\end{align*}

Be careful that all of these conditions are true (or good approx) before "blindly" applying the "current mirror" scaling.
Ex #3

- \( M_1 + M_3: V_{gd} = 0V, i_0 = I_0 > 0 \)

\[ \Rightarrow \text{ active} \]

\[ V_1 = V_{gs_1} = V_{th} + \frac{I_0}{K_1} \]

\[ V_2 = V_1 + V_{gs_2} = V_1 + V_1 + \frac{I_0}{K_3} \]

\[ M_4: \text{?} \Rightarrow V_{gs_4} = \text{gate voltage} = V_2 \]

\[ \Rightarrow \text{need more info on } V_3 \text{ on } i_{dy} \]

- Look at \( M_2 \) 1st: \( V_{gs_2} = V_1 > V_2 \) \( \Rightarrow \) active on triode.

- \( \Rightarrow \) depends on \( V_3 \); need to assume active or triode then validate assumption.

- \( \Rightarrow \) assume active, since eqn is simpler to validate.

\[ \Rightarrow V_{dy} = i_{dy} = I_0 \] ; \( M_1 + M_2 \) meet conditions of "current-mirror"

\[ \Rightarrow I_0 = I_0 \cdot \frac{K_2}{K_1} \]

\[ \Rightarrow V_3 = V_2 - V_{gs_4} = V_2 - V_2 - \frac{I_0}{K_4} \]

- Validate assumption: need \( V_{gd_2} < V_2 \) \( \Rightarrow \) depends on device sizes.

- \( \text{if } K_1 = K_2 = K_3 = K_4 \) (all matched)

\[ \Rightarrow I_0 = I_{dy}, V_1 = V_3, V_{gd_2} = 0 \Rightarrow \text{ all active.} \]

Ex #4

- \( \text{w/ ratios shown, } K = \frac{W_1}{L} \)

\[ \Rightarrow \text{ assume all } \frac{W_1}{L} \text{ are matched.} \]

- \( M_1 + M_3: \) same as previous examples.

\[ \Rightarrow \text{ active, } V_1 = V_{gs_1} = V_{th} + \frac{I_0}{K_1} \]

\[ V_2 = \text{Vdd} - V_{gs_2} = \text{Vdd} - V_{gs_1} - \frac{I_0}{K_3} \]
Ex#4 Cont) M₂ + M₄: \( \Omega \to \) again, since \( V₀ \) and \( I₀ \) are unknown, we need to make assumptions on op-modes \( \Rightarrow \) validate

Assume both active: "I-minor" concepts apply: \( (2\pi0) \)

\[
\begin{align*}
M₂: \quad & i₂ = \frac{K₂}{K₁} I₀ = \frac{I₀}{10} \quad \Rightarrow \quad & i₂ = i₄ = I₀ \\
M₄: \quad & i₄ = \frac{K₄}{K₃} I₀ = 10 \cdot I₀
\end{align*}
\]

- Since \( Vₙ₂ > Vₙ₄ \), \( Vₙ₄ > Vₙ₃ \) \( \Rightarrow \) one or both must be triode (cutoff is not an option)
- Since \( Vₙ₂ = Vₙ₃ \) \( \Rightarrow \) \( Vₙ₄ = Vdd - V₀ \) \( \Rightarrow \) If \( V₀ \) is large enough, both cannot be triode.

- This, one is triode, one is active, can find the solution & with one more "guess" \( \Rightarrow \) validate operation.
- Alternatively, consider the following experiment:

\[ Vdd = 10V \]
\[ V₂ \rightarrow Iₘ₄ \]
\[ V₀ \rightarrow I₀₂ \rightarrow Iₘₓ \rightarrow Iₙ₂ \rightarrow V₁ \rightarrow \]
\[ V₁ \rightarrow M₂ \rightarrow I \]

- 1st: \( S₁ \) closed, so \( V₀ = 5V \)
  \[ \Rightarrow \) both \( M₂ \) \& \( M₄ \) active.
  \[ \Rightarrow \] \( Iₙ₂ = Iₙ₄ - I₀₂ = I₀ (10 - \frac{1}{10}) \)

- 2nd: \( S₁ \) open \( \Rightarrow \) does \( V₀ \) incr. or decrea.?
  \[ \Rightarrow \) where does excess current flow
  \[ Iₙ₄ - I₀₂ \]?
* Excess current will flow into parasitic capacitances
  - Positive excess ⇒ Causes $V_o$ to increase.
  - Negative excess ⇒ Causes $V_o$ to decrease.

For Ex#4: Excess is positive ⇒ $V_o$ increases.
- $M_2$: stays active
- $M_4$: moves towards triode as $V_o$ increases (causing $V_{sd4}$ to decrease).
  ⇒ $M_4$: Triode once in triode, $M_4$ becomes resistor,
  $I_o$ depends on $I_{o4} = I_{o2} = \frac{I_B}{10}$.

$\Rightarrow V_o = V_{dd} - V_{sd4}$;
$\quad I_{o4} = K_4 \left[ 2(V_{sg4} - V_{t0})V_{sd4} - V_{sd4}^2 \right]$.

⇒ Solve quadratic for more accurate $V_o$ solution.

If "saturation" currents of $M_2$ & $M_4$ are dramatically different (i.e. $I_{o2} \neq I_{o4}$ if both active), then $V_{sd}/V_{ds}$ of device with larger sat current will be $\approx 0$.

Here: $I_{o4_{sat}} \gg I_{o2_{sat}} \Rightarrow V_{sd4} \approx 0 \Rightarrow V_o \approx V_{dd}$
- Could also neglect $V_{sd4}^2$ term for more accurate solution, but without quadratic.
One more way to visualize Ex#4:

* Plot the I-V curve for $I_0$ vs $V_0$ (load-line plot):

- Use I-V curves for $M_2$ and $M_4$ to find valid operating points.
- Two I-V curves cross @ $V_0 = V_{dd}$.

*M2: Sat portion as expected, $V_0 = V_{dd}$. *

Note: If two "sat" currents are close, then one device will only be "barely" in triode, and $V_{sat}/V_{as}$ of that device is $> 0$ (Vsa term is not negligible) $\Rightarrow$ may need to solve quadratic.

Remember: For two "competing I-sources", the smaller current always "wins" => the larger current device enters triode.

Ex#5:

*Note: $M_1+M_2: V_{gs1} = V_{gs2}$
$\Rightarrow 2 \times 0$ = both active.
$\Rightarrow I_{01} = I_{02} = I_{0/2}$
$\Rightarrow V_3 = -V_{g3} = -V_{g3} = -V_t - \sqrt{\frac{I_{0/2}}{K_{1/2}}}$

Validate: solve $V_1 + V_2$:
need: $V_{gd1} + V_{gd2} < V_t$
Ex. #6:

- New: \( V_{gs} \neq V_{gs2} \)
- \( V_{gs} = -V_s \Rightarrow \) if active, \( V_s \leq -V_t \)
- \( V_{gs2} = V_{dd} - V_s \)

- For large \( V_{dd} \): \( V_{gs2} \gg V_{gs1} \)
  - Most likely: \( M_2: \) active/\( sat \), \( I_2 = I_0 \)
  - \( M_1: \) cutoff, \( I_1 \approx 0 \)

- \( V_s = V_{dd} - V_{gs2} \gg -V_t \)

- Given devices sizes \( +V_{dd} \), \( +I_{dd} \) \( \Rightarrow \) solve to validate assumption.


- Consider inverter:

  - Draw \( V_o \) vs \( V_i \) curve.

  - Label op-modes for \( M_1 \) \& \( M_2 \)

  - Draw approx. curve for \( I_{dd} \) vs \( V_o \).

  - See "Supply Notes" on devices, Section 4, for solution, including small-signal model \& Noise Margin calculation.