Small-Signal Model Review

- Text refs: [Gray]: 1.5-18, 2.10
  [Allen]: ch.3; [Johns]: 1.2-1.3, 2.5
  [Sedra]: ch.5

- Here, we briefly review the small-signal modeling approach to summarize CMOS FET models, followed by gain & speed limitations, comparison to bipolar, subthreshold & short channel effects & latchup.

- Consider a device (nmos) in active/sat:

  \[ i_D = K (V_{GS} - V_t)^2 (1 + 2V_{DS}) = f(V_{GS}, V_{DS}, V_{DS}) \]

  \[ V_t = V_{to} + \gamma (\sqrt{2\Phi} + V_{DS} - \sqrt{2\Phi}) \]

  Due to \( V_t \)

- Consider operation at DC-op-point with small variations in signals around DC-op-point:

  ![Small-Signal Model Diagram]
• To solve circuit operation, can perform partial derivatives on \( i_0 = f(\ldots) \Rightarrow \) This would be very tedious if is not even feasible in a simulator for large circuits.

• Instead: we differentiate the models once, consider only the first order terms \( \Rightarrow \) derive a linearized model that can be solved with algebra only if is accurate as long as we do not deviate "significantly" from dc-op point.

• It is time for more careful notation: let us separate signals into components:

\[
U_s = U_{gs} + U_{g} + \ldots
\]

\( \uparrow \quad \uparrow \quad \uparrow \)

complete DC 1st order Ac term

higher-order terms are neglected.

• Solve 1st order model from Taylor series expansion:

\[
\frac{di_d}{du_{gs}} \bigg|_{\text{dc-op point}} U_{gs} + \frac{di_d}{du_{gs}} \bigg|_{\text{dc-op}} U_{gs} + \frac{di_d}{du_{gs}} \bigg|_{\text{dc-op}} U_{gs} + \ldots
\]

\[\Rightarrow \quad i_d = g_m U_{gs} + g_m U_{gs} + \frac{U_{gs}}{R_{ds}} \]

\[\text{neglect other dependencies for now}\]
Complete nmos ssm in active/sat.

\[ g_m = \left. \frac{\partial i_d}{\partial V_{gs}} \right|_{dc,op} = 2K(V_{gs}-V_t)(1+2V_{bs}) = 2K(V_{gs}-V_t) = 2\sqrt{K}I_0 \]

\[ \frac{1}{r_{ds}} = \left. \frac{\partial i_d}{\partial V_{gs}} \right|_{dc} = K(V_{gs}-V_t)^2 \cdot \lambda \approx \lambda I_0 \]

\[ g_{mb} = \left. \frac{\partial i_d}{\partial V_{bs}} \right|_{dc} = \frac{\partial i_d}{\partial V_t} \cdot \frac{\partial V_t}{\partial V_{bs}} = 2K(V_{gs}-V_t) \cdot \frac{\gamma}{2\sqrt{2}q_p+V_{gs}} = n \cdot g_m \]

\[ n \approx 2 \text{ for example} \]

\[ C_{gs} = C_{ox} \cdot W + \frac{1}{2}C_{ox} \cdot W \cdot L \cdot C_{ox} \quad \text{per unit length of device width.} \]

\[ C_{gd} = C_{ox} \cdot W \]

\[ C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{sb}}{V_{th}}}} \quad \text{dependant on body bias} \]

\[ C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{db}}{V_{th}}}} \quad \text{S/D voltage} \]

\[ C_{ox} = \text{Gate oxide capacitance per unit area.} \quad C_{ox} = \frac{\text{C}_\text{ox}}{W \cdot x \text{o}} \]
nmos, ssm, triode operation:

⇒ use same ssm, with different parameters 
   based on triode equation.

\[ i_{0t} = K \left[ 2(V_{b5} - V_t) V_{b5} - V_{b5}^2 \right] \]

⇒ \[ g_{mt} = \frac{\partial i_{0t}}{\partial V_{b5}} \bigg|_{dc} = 2K V_{b5} \]

⇒ \[ \frac{1}{r_{d5t}} = \frac{\partial i_{0t}}{\partial V_{b5}} \bigg|_{dc} = 2K (V_{b5} - V_t - V_{b5}) \leq \text{resistor} \]

⇒ \[ g_{mb5} = \frac{\partial i_{0t}}{\partial V_{b5}} \bigg|_{dc} = \frac{1}{2} \left( 2K V_{b5} \right) \frac{\partial V_t}{\partial V_{b5}} = \eta g_{mb5} \] \[ \text{generally neglect,} \]

\[ C_{gst} = C_{o1} W + \frac{1}{2} W L \cdot C_{ox} \] \[ \cdot C_{sb} \neq C_{db} \text{ same as active/sat.} \]
\[ C_{gd5t} = C_{o1} W + \frac{1}{2} W L \cdot C_{ox} \]

nmos, ssm, cutoff:

⇒ \[ g_{mc} \to 0, \quad g_{mb5} \to 0, \quad r_{d5c} \to \infty \]
⇒ only caps remain, \[ C_{gs} \approx C_{o1} W, \quad C_{gd} \approx C_{o1} W \]
\[ C_{gb} \approx W L \cdot C_{ox} \text{ = gate-to-body cap.} \]
\[ C_{sb} \neq C_{db} \text{ same as active.} \]

*Note: Nonlinear caps vary with voltage (due to op-mode changes)*
pmos, ssm: same as nmos, with voltage & current polarities switched:

- All parameters are defined the same as nmos in each op-mode. (all positive).

- Note: dc sources are equal to zero in ssm.
  - Volt-sources ⇒ short.
  - Current-sources ⇒ open

Usage Example: Inverter SSM:

- Assume given: M1: Active  M2: Triode

\[ \frac{V_o}{V_{in}} = A_v = \text{small-signal gain} \]

\[ R_o = \text{small-signal output resistance} \]

- Looking for low-freq gain ⇒ ignore caps

- Both M1, M2: Body tied to source ⇒ \( V_{sb} = 0 \)
\[ y = \frac{V_o}{V_{in}} = - (g_m + g_m t_2) \left( R_{ds_1} || R_{ds_2} \right) \]

where: \( g_m = 2k_1 (V_{fN} - V_{fN}) \), \( R_{ds_1} = \frac{1}{2I_{o1}} = \frac{1}{k_1 (V_{fN} - V_{fN})^2} \)

\( g_m t_2 = 2k_2 (V_{oD} - V_{oD}) \), \( R_{ds_2} = \frac{1}{2k_2 (V_{oD} - V_{fN} - V_{oD} - V_{oD} + V_{oD})} \)

Solve \( R_{out} \): Apply test source to output with input set to zero: \( V_{in} \rightarrow 0 \).

\[ R_{out} = \left. \frac{V_o}{I_t} \right|_{V_{in}=0} = \left( R_{ds_1} || R_{ds_2} \right) \]

\[ \approx R_{ds_2} \]