-3dB Bandwidth estimation (in large, multi-stage circuits)

Given \( A(s) = \frac{V_o}{V_i} \),

magnitude response: \( 20 \log |A(j\omega)| \) [dB]

\[ 20 \log |A(j\omega)| \rightarrow \]
\[ 20 \log |A(\omega)| \rightarrow \]

\( BW = f_{bw} \)

By def, \( BW = f_{bw} \) is the solution of

\[ |A(j\omega)| = \frac{|A(\omega)|}{\sqrt{2}} \]

i.e. the frequency where the magnitude response drops by 3dB from the low-frequency gain.

\( 20 \log \sqrt{2} = 3 \text{dB} \)

For design purposes, it is essential to find how circuit parameters affect BW, so that the design can be modified to improve BW.

Such result is given by the Zero-Value Time Constant method that gives a simple BW estimate.
Summary

ZERO-VALUE TIME-CONSTANT METHOD (ZVTC)

\[ f_{BW} = BW \approx f_1 \approx \frac{1}{2\pi a} \]

where \( f_1 \) = dominant-pole frequency, \( f_1 \ll \) all other pole/zero frequencies

\( a \) = coefficient with \( s \) in the denominator of \( A(s) \)

\( a \) can be found from:

\[ a = \sum_{\text{all capacitors}} T_i \]

\( T_i = C_i R_{Di} \) = zero-value time constant for capacitor \( C_i \)

\( R_{Di} \) = resistance "seen" by \( C_i \) when all other caps are zero (open-circuit) and all independent sources are nulled.

So, \( BW \) estimate by the ZVTC method is:

\[ BW = f_{BW} \approx \frac{1}{2\pi \sum_{\text{all caps}} C_i R_{Di}} \]
What if there is no dominant pole?

Consider a two-pole $A(s) = A(0) \frac{1}{(1 + \frac{s}{\omega_1})(1 + \frac{s}{\omega_2})}$

where $f_2 = n f_1$

Find $BW$ by exact calculation:

$$|A(j \omega_{BW})| = \left| \frac{A(0)}{\sqrt{2}} \right| = \frac{|A(0)|}{\sqrt{1 + \left( \frac{\omega_{BW}}{\omega_1} \right)^2 \sqrt{1 + \left( \frac{\omega_{BW}}{n \omega_1} \right)^2}}}
$$

Let $x = \left( \frac{\omega_{BW}}{\omega_1} \right)^2$

$$2 = (1 + x) \left( 1 + \frac{x}{n^2} \right) = 1 + x \left( 1 + \frac{1}{n^2} \right) + \frac{x^2}{n^2}$$

$$x^2 + (n^2 + 1) x - n^2 = 0$$

$$x = \frac{\sqrt{n^4 + 6n^2 + 1} - n^2 - 1}{2}$$

So

$$f_{BW} = f_1 \sqrt{x} = \frac{f_1}{n} \sqrt{\frac{\sqrt{n^4 + 6n^2 + 1} - n^2 - 1}{2}}$$

is the exact result for $BW$.

ZVT method would give:

$$f_{BW} \approx \frac{1}{2\pi a}, \quad a = \frac{1}{\omega_1} + \frac{1}{\omega_2} = \frac{1}{\omega_1} \left( 1 + \frac{1}{n} \right)$$

$$f_{BW} \approx f_1 \frac{n}{n+1}$$
The results give how much error is made using the ZVTC estimate for different $n = f_2 / f_1$:

<table>
<thead>
<tr>
<th>$n$</th>
<th>exact BW</th>
<th>ZVTC estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.64 $f_1$</td>
<td>0.5 $f_1$</td>
</tr>
<tr>
<td>2</td>
<td>0.84 $f_1$</td>
<td>0.67 $f_1$</td>
</tr>
<tr>
<td>5</td>
<td>0.96 $f_1$</td>
<td>0.83 $f_1$</td>
</tr>
<tr>
<td>10</td>
<td>0.99 $f_1$</td>
<td>0.91 $f_1$</td>
</tr>
</tbody>
</table>

Notice that the ZVTC estimate is always pessimistic (conservative) and that the error is at most 22% for $n = 1$ when $f_2 = f_1$. 
Examples of ZVTC method applications

1. Common-source with active load

\[ V_{I} + V_i \]

\[ R_{in} \]

\[ M_1, M_2 \]

\[ V_{DD} \]

\[ V_0 \]

\[ C_L \]

load capacitance

\[ SSM \]

\[ C_2 = C_{db1} + C_{db2} + C_L \]

\[ R_2 = R_{ds1} || R_{ds2} \]

Example numerical values:

\[ G_{m1} = 500 \mu A/V \]

\[ R_{ds1} = R_{ds2} = 400 \, k\Omega \]

\[ R_{in} = 200 \, k\Omega \]

\[ C_{gs1} = 0.27 \, pF \]

\[ C_{gd1} = 0.02 \, pF \]

\[ C_{db1} = C_{db2} = 0.12 \, pF \]

\[ C_L = 0.2 \, pF \]
Compute 2UTC:

* $C_{gs1}: \quad R_{D_{gs1}} \frac{V_{test}}{I_{test}} = R_{in}$

$T_{gs1} = C_{gs1} R_{in} = 54 \text{ns}$

* $C_{2}: \quad R_{D_{2}} = \frac{V_{test}}{I_{test}} = R_{2}$

$T_{2} = C_{2} R_{2} = 88 \text{ns}$

* $C_{gd1}: \quad R_{D_{gd1}} = \frac{V_{test}}{I_{test}} = R_{in} + R_{2} + \frac{g_{m1} R_{2} R_{in}}{|A(0)|}$

$V_{test} = R_{in} I_{test} + R_{2} (I_{test} + g_{m1} R_{in} I_{test})$

$T_{gd1} = C_{gd1} (R_{in} + R_{2} + g_{m1} R_{in} R_{2}) = 408 \text{ns}$
\[ f_{BW} \approx \frac{1}{2\pi \sum T_i} = \frac{1}{2\pi \left( T_{g_{01}} + T_2 + T_{gd1} \right)} = \]
\[ = \frac{1}{2\pi \left( 54\text{ns} + 88\text{ns} + 408\text{ns} \right)} = 290\text{kHz}. \]

Note that \( T_{gd1} \) from \( C_{gd1} \) is the major contributor to the BW limitation.

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2. Possible improvements:

* cascode gain stage:

Find resistance \( R_4 \) looking into the source of \( R_4 \):

(neglect body effect of \( M_4 \))
\[ V_{\text{test}} = R_{ds4} (i_{\text{test}} - g_{m4} V_{\text{test}}) + R_{ds2} i_{\text{test}} \]

\[ V_{\text{test}} \left( 1 + g_{m4} R_{ds4} \right) = i_{\text{test}} \left( R_{ds4} + R_{ds2} \right) \]

\[ R_4 = \frac{V_{\text{test}}}{i_{\text{test}}} = \frac{R_{ds2} + R_{ds4}}{1 + g_{m4} R_{ds4}} \]

If \( g_{m4} R_{ds4} \gg 1 \) and \( R_{ds2} \approx R_{ds4} \)

\[ R_4 \approx \frac{2}{g_{m4}} \]

Use the same \textit{w} values and \( g_{m4} = g_{m1} \).

Now, we can compute all time constants:

\[ T_{gs1} = R_{in} C_{gs1} = 54 \text{ ns} \]

\[ T_{gd1} = C_{gd1} \left( R_{in} + \frac{2}{g_{m4}} + g_{m1} \frac{2}{g_{m4}} R_{in} \right) = 12 \text{ ns} \]
\[ T_4 = C_4 R_4 , \quad C_4 = C_{db_1} + C_{sb_4} + C_{gs_4} = 0.51 \text{ pF} \]

\[ T_4 = 2.1 \text{ ns} \]

\[ T_2 = C_2 R_2 , \quad C_2 = C_{db_4} + C_{gs_4} + C_{db_2} + C_L \]

\[ C_2 = 0.46 \text{ pF} \]

\[ R_2 \approx R_{ds_2} = 400 \text{ k}\Omega \]

\[ T_2 = 184 \text{ ns} \]

Note that the gain is \[ -g_m R_2 = -200 \text{, twice as much compared to the basic CS amplifier.} \]

So, \[ BW \approx \frac{1}{2\pi \sum T} = \frac{1}{2\pi (252 \text{ ns})} = 632 \text{ kHz}. \]

In this example, cascode has 2x gain and about 2x BW of the simple CS amplifier.

The main BW limitation is now the output time constant \[ T_2 = C_2 R_2. \]

Further improvements:

- Use a source follower to reduce the output resistance if \( C_L \) is large.
- Use a source follower at the input to reduce \( T_{gs_1} \).