ECEN4827/5827 lecture 26

- HW8 due via D2L by 10am MT, Friday Nov. 1
- Today:
  - Continue analysis of frequency responses
  - BW estimation using zero-value time-constant (ZVTC) method
CS amplifier frequency responses

Assume DC bias such that $M_1, M_2$ are AS

$V_{O+V_o} = +V_{DD}$ = square good $+$ Cgs + Cgd $+$ Cgb $+$ signal

$M_1$

$M_2$

$M_3$

$R_{in}$

$C_{gs1}$

$C_{gd1}$

$C_{gb1}$

$C_{gd2}$

$C_{gb2}$
CS amplifier magnitude response and BW

\[ 20 \log |A(j\omega)| \text{[dB]} \]

- Single-pole response
- Dominant pole

\[ f_{p1} \approx 300 \text{ kHz} \]

\[ f_{p1} \approx f_{p2} \]

\[ f_{p2} \gg f_{p1} \]

\[ f_{p2} \approx 16 \text{ kHz} \]

Bandwidth: \( 20 \log |A(j\omega)| \text{[dB]} \)

\[ f_{BW} = \frac{\omega_p}{2\pi} \]

\[ B(s) = \frac{B(o)}{1 + \frac{s}{\omega_p}} \]

\[ f_{BW} = f_p = \frac{\omega_p}{2\pi} \]
Dominant pole approximation

\[ A(s) = A(0) \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2} = A(0) \frac{1 - \frac{s}{\omega_z}}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})} \]

Dominant pole: \( f_{p1} << f_{p2}, f_z \) (\( f_{p1} << \) other pole and zero frequencies).

\[ BW \approx f_{p1} \approx \frac{1}{2\pi a_1} \text{ [Hz]} \]

\[ A(s) = A(0) \frac{1 - \frac{s}{\omega_z}}{1 + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + \frac{1}{\omega_{p1}\omega_{p2}} s^2} \]

\[ a_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \approx \frac{1}{\omega_{p1}} \] (\( \omega_{p1} << \omega_{p2} \))

\( a_1 \) can be found without computing complete \( A(s) \)!
Zero-value time constant (ZVTC) method

Note: Powerpoint crashed so annotations were lost.

Many thanks to Arielle Blum who kindly provided the lecture notes in the pages that follow this page.
Zero-value time constant (ZVTC) method

**Exact:** \( a_i = \sum C_i R_{pi} \)

- BW \( \approx f_{pi} \approx \frac{1}{2\pi f_{pi}} \)
- This is based on dominant pole approximation

**ZVTC Method**

Small-signal, "incremental"

- \( R_{pi} = \) resistance seen by \( C_i \)
- When all other caps are zero
  - \( C_j = 0; i \neq j \)
- Often very easy to find
- Design-oriented result because it provides insight into where the BW limitation is coming from. Tell that the resistance associated.

The dominant pole approximation doesn't always hold, s.t. \( f_{pi} \approx \omega \)
However, still holds value.

\[
A(s) = \frac{1}{SC} + R = \frac{1}{1+a_iS}
\]

Value of capacitor multiplied by resistance seen

- \( a_i = C_1 R_1 + C_2 (R_1 + R_2) + C_3 (R_1 + R_2 + R_3) \)
- \( C_3 \) dominates here, you can see that \( C_3 \) see the largest capacitance

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ZVTC method applied to the CS amplifier

\[ q_1 = \tau_{gs1} + \tau_{gd1} + \tau_2 \]

\[ \tau_{gs1} = C_{gs1}(R_{og1}) + C_{gd1}(R_{ogd}) + C_2(R_{o2}) \]
\[ R_{\text{gs}1} = V_{\text{test}} = R_{\text{in}} \]
\[ g_m \cdot V_{\text{gs}1} \]

\[ \tau_{\text{gs}1} = R_{\text{in}} C_{\text{gs}1} = 54 \text{ns} \]

\( (200 \text{ k}\Omega)(0.27 \text{ pF}) \)
\[ R_{oa} = \frac{V_{test}}{I_{test}} = R_2 \]

\[ C_2 = \frac{C_2}{R_2} = 88 \text{ ns} \]
\[ R_{gd} = \frac{V_{\text{test}}}{i_{\text{test}}} \]

\[ V_{g1} = R_{\text{init}} \cdot V_{\text{test}} \]

\[ R_{\text{init}} \cdot i_{\text{test}} + R_{2} (1 + g_{m} R_{\text{in}}) i_{\text{test}} = 0 \]

\[ V_{\text{test}} = R_{\text{gd}} R_{\text{in}} + R_{2} + g_{m} R_{\text{gd}} R_{\text{in}} = 20 \text{mA} \]

\[ V_{g1} = g_{m} \cdot \frac{R_{\text{gd}}}{g_{m}} (R_{\text{in}} + R_{2} + g_{m} R_{\text{in}}) \]

\[ (0.02 \text{pF})(200 \text{K} + 200 \text{K} + 100 \text{K}) = 400 \text{ns} \]

By far, contributes the MOST to time.
**CS dominant pole estimation using ZVTC**

\[ a_1 = \gamma_2 s^2 + \gamma_3 s + \gamma_0 \]

\[ \text{BW} \approx f_p, \omega = \frac{1}{2 \pi \left( \gamma_2 s + \gamma_3 s + \gamma_0 \right)} = 290 \text{kHz} \quad \text{[estimate]} \]

Compare to \( \text{BW} = 300 \text{kHz} \), numerically computed.

The numerically computed value aligns with the estimate of using the dominant pole - approximation.

Don't assume individual capacitors correspond to discrete poles! \( \gamma_0, \gamma_2, \gamma_3 \), ZVTC's.

\( f_{p1} = \frac{1}{2 \pi \gamma_0} \quad \times \quad \text{NOT TRUE IN GENERAL} \)

\( f_{p2} = \frac{1}{2 \pi \gamma_2} \quad \times \quad \text{only special cases when we can decouple capacitors} \)

\( f_{p3} = \frac{1}{2 \pi \gamma_3} \quad \times \)
$C_{gd}$ is a "Miller" Capacitor

\[ i_{\text{miller}} = sC_{\text{miller}} (v_i - v_2) \]
\[ = sC_{\text{miller}} (v_i - AV_i) \]
\[ = sC_{\text{miller}} (1-A) v_i \]

Common-Source Amplifier

- Capacitor appears to be much larger than it actually is due to the gain stage of the amplifier.
- Large positive gain $\Rightarrow$ RHP pole - unstable.
Possible improvements? While keeping $A(0)$ same

How can we possibly apply this theory to increase BW?

Given

$V_i + v_i$

$R_{in}$

$C_{gs1}$

$C_{gh1}$

$M_1$

$M_2$

$M_3$

$V_{DD}$

$V_O + v_o$

$C_L$

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