Middlebrook’s Feedback Theorem*

\[ G = G_\infty \frac{T}{1+T} + G_o \frac{1}{1+T} \]

\[ G_o = direct\ transmission\ through\ feedback\ path. \]

\[ G_o = \frac{v_o}{v_i} \mid v_x \rightarrow \text{null} \]

\[ T = \text{loop\ gain} = \frac{v_x}{v_i} \mid v_i = 0 \]

\[ G_\infty = \text{ideal\ closed\ loop\ gain} \]

\[ = \frac{v_o}{v_i} \mid v_x \rightarrow \text{null} \]

*For derivation, see notes posted on the course website*
Feedback Theorem Application Examples

\[ G = \frac{v_v}{v_{ref}} = \frac{G_\infty}{1+T} + G_v \frac{1}{1+T} \]

\[ T = \frac{v_x}{v_{ref}} \bigg|_{v_{ref}=0} = g_m \frac{(R_1+R_2)R_2}{1+g_m((R_1+R_2)R_2)} \cdot \frac{R_2}{R_1+R_2} \cdot A_0 \]

\[ G_\infty = \frac{v_0}{v_{id}v_{y} \to 0} = \frac{L_1+L_2}{R_2} \]

\[ G_v = \frac{v_0}{v_{id}v_{x} \to 0} = 0 = \frac{v_{g_m}}{R_2+R_1} \cdot \frac{R_2}{R_2+R_1} \]

\[ \text{null and null} \]

\[ v_{ref} \]

\[ v_x \]

\[ v_y \]

\[ v_{id} \]
Feedback Theorem Application Examples

Mistake in the lecture: a current source $i_{\text{test}}$ should have been used to find $R_{\text{out}}$

$$R_{\text{out}} = \frac{v_{\text{test}}}{i_{\text{test}}} = R_{\text{out},\infty} \frac{T}{1+T} + R_{\text{out},0} \frac{1}{1+T} = \frac{1}{g_m} \frac{1}{1+T}$$

$$R_{\text{out},\infty} = \frac{v_{\text{test}}}{i_{\text{test}}} \bigg|_{v_y \rightarrow 0} = 0$$

$$R_{\text{out},0} = \frac{v_{\text{test}}}{i_{\text{test}}} \bigg|_{v_x \rightarrow 0} = \frac{1}{g_m} \| R_0 \| (R_1 + R_2) \approx \frac{1}{g_m}$$
Feedback Theorem Application Examples

Assumptions: all biased in M's, low-frequency, no caps.

\[ \frac{v_o}{v_i} = G = G_{\infty} \frac{T}{1 + T} + G_o \frac{1}{1 + T} \]

Done!

\[ T = \frac{v_y}{v_x} \big|_{v_i = 0} = (-g_m R) \frac{R_1}{R_1 + R_2} (-g_m R) \]

Assume: \( R \ll R_o \)
\( R \ll R_2 \)

\[ = (g_m R)^3 \frac{R_1}{R_1 + R_2}. \]
Feedback circuits based on op-amps with single-pole open-loop response $A(s)$

$$v_0 A(s) = \frac{v_0}{v(+) - v(-)} = A(0) \frac{1}{1 + \frac{s}{\omega_p}}$$

$f_p$ = dominant pole.

$f > f_p$ : $A(s)$ behaves as:

$$A(0) \frac{1}{\frac{s}{\omega_p}} = \frac{A(0)\omega_p}{s}$$

\[
\begin{align*}
\text{Gain-bandwidth product} 
&= A(0)f_p \\
&= \boxed{\text{GBW}}
\end{align*}
\]

What is $BW$ of $A(s)$?

$BW = f_p$ [Hz].

Low-frequency gain = $A(0)$.

**Property of the op-amp itself.**