There are several reasons why feedback is applied in electronic circuits:

- Circuit characteristics, such as gain, can be precisely controlled, and made relatively independent of wide variations in active device parameters;

- Circuit characteristics can be made relatively independent of operating conditions such as supply voltages or temperature;

- Signal distortion, which is a result of the nonlinear nature of active devices, can be significantly reduced;

- Frequency response and the gain/bandwidth trade-off can be controlled.

Feedback is a powerful tool that allows us to design high-performance circuits using components with relatively poor or uncertain parameter values. Two types of feedback are applied in electronic circuits: positive and negative. In positive-feedback circuits, devices are usually driven up to their extreme operating modes (such as cut-off or saturation of bipolar transistors). Applications include voltage comparators, flip-flops, oscillators and timing circuits. In negative-feedback circuits, devices are usually in their linear operating modes (such as active mode of bipolar transistors). Applications include amplifiers, linear voltage regulators and filters.

The purpose of this part of the notes is to present basic tools for analysis and design of feedback circuits. In Section 1, we first look at simple op-amp applications with negative and positive feedback. A simple and general approach to evaluate and design circuits with negative feedback is based on the concepts of loop-gain and ideal closed-loop gain introduced in Section 2.

In this part of the notes, we neglect dynamic response limitations in electronic circuits. Effects of feedback on the frequency response, including: gain-bandwidth trade-offs, stability limits, and compensation techniques will be discussed later.

1 Examples of negative and positive feedback circuits

Consider the operational amplifier in Fig. 1. In this section, we will assume that the op-amp has ideal characteristics: its open-loop gain is very large:

\[ A_{OL} = \frac{v_o}{v_e} \to \infty, \]
its input resistance is very large,
\[ R_{\text{in}} = \frac{v_e}{i_e} \to \infty, \]  
(2)
and its output resistance is very small \( R_{\text{out}} \approx 0 \). Fig. 1(b) depicts a simple op-amp model that includes the parameters \( A_{\text{OL}}, R_{\text{in}} \) and \( R_{\text{out}} \). This model is a crude, but in many cases adequate approximation to actual op-amp characteristics, as long as the op-amp output voltage \( v_o \) stays within limits determined by the op-amp internal structure and the supply voltages. The op-amp outputs cannot exceed the supply voltages \(+V_{\text{CC}}\) and \(-V_{\text{EE}}\). The actual limits (positive \( V_{O+} \) and negative \( V_{O-} \)) are usually more restrictive, but close to the supply voltages:
\[ -V_{\text{EE}} < V_{O-} \leq v_o \leq V_{O+} < +V_{\text{CC}}. \]  
(3)

The op-amp is used in simple feedback applications as shown in Fig. 2: the input \( v_i \) is applied to one of the op-amp inputs, while the output \( v_o \) is fed back to the other input via two resistors, \( R_a \) and \( R_b \). In Fig. 2(a), the output is returned to the \(-\) input, while in Fig. 2(b), the output is returned to the \(+\) input. Our goal is to determine the properties of the two closed-loop application circuits, i.e., to determine how the output \( v_o \) depends on the input \( v_i \). Before we find \( v_o(v_i) \) for the two circuits in Fig. 2, consider first what happens qualitatively in the two circuits when the input voltage is nulled, as shown in Fig. 3. With no input, \( v_i = 0 \), one would expect that \( v_o = 0 \), but in one of the two circuits this is not the case.

Even with zero input, small time-varying noise signals do exist in any circuit. Suppose that there is a very small positive voltage change at the outputs of the op-amps in Fig. 3(a) and
This is indicated by the arrow pointing upwards. Because of the feedback connection, this small output voltage change is returned back to the amplifier input through the voltage divider $R_a$, $R_b$. A positive change in the output will in both circuits result in a positive change at the input where the output signal is fed back. A change at the input will produce a change in the output voltage, negative in the circuit of Fig. 3(a), and positive in the circuit of Fig. 3(b). Because the initially assumed change produces opposite results when propagated through the feedback loop, the behavior of the two circuits is very different. In the circuit of Fig. 3(a), the feedback produces a signal that opposes and tends to cancel the originally assumed change. As a result, the steady-state signal output with zero signal input is indeed zero. Furthermore, since the op-amp open-loop gain $A_{OL}$ is very large, the op-amp input voltage $v_e$ between the + and the − inputs must be equal to zero, with or without the input voltage $v_i$ applied. This is a characteristic of a negative feedback circuit. In the circuit of Fig. 3(b), however, the feedback produces a change that tends to amplify the originally assumed change. As a result, the output $v_o$ in the circuit of Fig. 3(b) will move away from zero until it hits the upper limit $v_o = V_O^+$. This is a characteristic of a positive feedback circuit. It is important to note that the circuit of Fig. 3(b) may also end up in the negative saturation limit, $v_o = V_O^-$. In fact, based on the available information, it is not possible to say whether $v_o = V_O^+$ or $v_o = V_O^-$, but in any case the output is saturated at one of the limits, away from zero, although no input voltage is applied. It is also important to note that in the positive feedback circuit, the op-amp input voltage $v_e$ is not zero.

We can now determine the closed-loop $v_o(v_i)$ characteristics of the two application circuits. In the circuit of Fig. 2(a), because of the negative feedback, and because the op-amp open-loop

![Figure 2: Two op-amp application examples.](image)
Figure 3: Feedback loops in the examples of Fig. 2.

gain is very large, the op-amp input is zero, \( v_e = 0 \), which yields:

\[
v_e = v_i - v_1 = v_i - v_o \frac{R_a}{R_a + R_b} = 0, \quad (4)
\]

\[
A_{CL} = \frac{v_o}{v_i} = \frac{R_a + R_b}{R_a} \cdot (5)
\]

Therefore, as long as the output is within the saturation limits, the circuit is a non-inverting amplifier with the closed-loop gain equal to \((R_a + R_b)/R_a\). The complete \(v_o(v_i)\) characteristic, including the saturation limits, is shown in Fig. 4(a).

In the circuit of Fig. 2(b), because of the positive feedback, and because the op-amp open-loop gain is very large, the output is always in one of the two saturation limits, \( v_o = V_O^+ \) or \( v_o = V_O^- \), depending on both \( v_i \) and \( v_e \). Suppose that initially \( v_i \) is a large negative voltage so that \( v_o = V_O^- \). The + input is positive at \( v_1 = V_O^+ R_a/(R_a + R_b) \). The output stays in the positive saturation limit as long as \( v_e = v_1 - v_i \) is greater than zero. Once \( v_i \) exceeds \( v_1 \), \( v_e \) becomes negative, and because the op-amp has very large open-loop gain, the output jumps abruptly to \( V_O^- \). As a result, \( v_1 \) jumps to the negative value \( v_1 = V_O^- R_a/(R_a + R_b) \).

The output stays at the negative saturation limit as long as the op-amp input \( v_e = v_1 - v_i \) is negative. The input voltage must drop below \( v_i \) for the output to jump back to \( V_O^+ \). The complete characteristic of the positive-feedback circuit is shown in Fig. 4(b). The characteristic is distinctly different from the amplifier characteristic in Fig. 4(a): the output jumps from one saturation limit to the other at two different points, depending on the previous state of the output. The characteristic with hysteresis is useful in voltage-comparator applications where the circuit can be used to decide a logic-level (high or low) depending on the analog voltage input. The hysteresis provides noise immunity in the decision process.
2 Analysis of negative feedback circuits

Consider again the negative-feedback amplifier of Fig. 2(a). Assuming ideal op-amp characteristics, we have found that the closed-loop gain is given by (5). Suppose now that the op-amp open-loop gain $A_{OL}$ is finite. Keeping the assumptions about ideal input and output resistances, we can find the closed-loop gain $A_{CL} = v_o/v_i$ as follows:

\[ v_e = v_i - v_1 = v_i - \frac{R_a}{R_a + R_b} v_o \],  
\[ v_o = A_{OL} v_e \],  
\[ v_o = A_{OL} \left( v_i - \frac{R_a}{R_a + R_b} v_o \right) \].

Solve for $v_o/v_i$,

\[ A_{CL} = \frac{v_o}{v_i} = \frac{A_{OL}}{1 + A_{OL} \frac{R_a}{R_a + R_b}} \],

or,

\[ A_{CL} = \frac{R_a + R_b}{R_a} \frac{A_{OL} \frac{R_a}{R_a + R_b}}{1 + A_{OL} \frac{R_a}{R_a + R_b}} = A_{T \to \infty} \frac{T}{1 + T} \],

where

\[ A_{T \to \infty} = \frac{R_a + R_b}{R_a} \]

is the ideal closed-loop gain, and

\[ T = A_{OL} \frac{R_a}{R_a + R_b} \]
is the *loop-gain* of the feedback circuit. The final expression (10) for the feedback circuit of Fig. 2(a) introduces two important concepts in the analysis of feedback circuits: *loop gain* $T$ and *ideal closed-loop gain* $A_{T \to \infty}$.

In general, the loop gain shows how much the signal is amplified when propagated through the feedback loop. The concept of “propagation through the loop” was used in Section 1 to examine qualitatively whether the feedback is positive or negative. $T$ is the quantity that tells us not only whether the feedback is negative ($T > 0$) or positive ($T < 0$) but also how “strong” the feedback is.

Referring to the example of Fig. 3(a), assuming a small change in $v_o$, and propagating this change through the loop, we have that the change is first attenuated by the resistive voltage divider,

$$v_1 = \frac{R_a}{R_a + R_b} v_o,$$

and then amplified through the op-amp,

$$v_o = -A_{OL} v_1$$

Thus the initially assumed change in the output voltage is amplified through the feedback loop by the total gain equal to

$$\left( \begin{array}{c} v_1 \\ v_o \\ v_1 \end{array} \right) = \left( \frac{R_a}{R_a + R_b} \right) (-A_{OL})$$

The loop gain is the negative of this total gain, so that for negative feedback we have $T > 0$,

$$T = \left( \frac{R_a}{R_a + R_b} \right) (A_{OL})$$

When the feedback is very strong ($T \to \infty$), the closed-loop gain assumes the ideal value $A_{T \to \infty}$. In the example of Fig. 2(a), this is the value found in (5), where we assumed that the op-amp had infinite open-loop gain.

### 2.1 Finding the loop gain $T$ and the ideal closed-loop gain $A_{T \to \infty}$

The expression

$$A_{CL} = A_{T \to \infty} \frac{T}{1 + T},$$

which was derived for a particular circuit example, is very useful and important because it *holds in general*. Although feedback circuits can be analyzed using any standard circuit-analysis techniques (for example writing and solving a system of nodal equations), the analysis of feedback circuits based on (17) has a number of advantages:

1. In most cases, both $A_{T \to \infty}$ and $T$ can be obtained by inspection or with very little amount of (messy) algebra.

2. $A_{T \to \infty}$ shows what the circuit is ideally supposed to do, it reveals the function and the purpose of the circuit, and is therefore valuable in understanding the circuit configuration and its properties. The factor $T/(1 + T)$ shows how close the actual circuit performance is to the ideal.
3. The expression \( A_{T \to \infty} \frac{T}{1 + T} \) leads directly to the circuit design: one constructs the feedback so that \( A_{T \to \infty} \) has the desired value, and then makes sure that \( T \) is large enough (in the frequency range of interest and subject to stability conditions discussed later) so that \( A_{CL} \approx A_{T \to \infty} \).

4. The loop gain \( T \) is the main quantity used to determine frequency-response and stability of feedback circuits, which will be discussed later.

The approach based on (17) requires knowledge of the ideal closed loop response \( A_{T \to \infty} \), and the loop gain \( T \). We now discuss how to determine \( A_{T \to \infty} \) and \( T \) for an arbitrary feedback circuit. A general block diagram of a feedback circuit is shown in Fig. 5.

If the circuit is nonlinear, as is essentially always the case, we assume that the steady-state dc operating point has been found, and that the block diagram of Fig. 5 corresponds to the linear small-signal model of the feedback circuit.

The input is \( u \), and the output is \( y \). At some point, the feedback loop contains a gain block where the output behaves as a voltage source \( K u_e \), controlled by the input \( u_e \). The other parts of the feedback circuit may contain other gain stages or passive components.

For the purpose of finding \( A_{T \to \infty} \) and \( T \), a test signal \( v_t \) is introduced in series with the controlled voltage source \( K u_e \). The test source is needed only to produce the signal \( v_x \) that is propagated through the loop in order to find the loop gain \( T \). It is not a part of the feedback circuit. With the signals defined as in Fig. 5, we have:

- The loop gain shows how much the signal \( v_x \) is amplified when propagated through the loop (while the input signal is nulled, \( u = 0 \)).

\[
T = \left. \frac{v_y}{v_x} \right|_{u=0} \quad (18)
\]
Note that \(v_x\) is the signal that enters the loop, and \(v_y\) is the signal that comes back through the loop. The polarity of \(v_y\) is selected so that \(T > 0\) corresponds to negative feedback.

- The ideal closed loop gain shows the gain of the circuit when the loop gain is very large, i.e., when \(u_e = v_y = 0\),

\[
A_{T \to \infty} = \left. \frac{y}{u} \right|_{v_y = 0}
\]  

(19)

In op-amp applications, one usually gets \(A_{T \to \infty}\) simply by solving the circuit where all op-amps are assumed ideal. An ideal op-amp has infinite gain, and when negative feedback is applied around the op-amp, the voltage between the (+) and the (−) input terminals is nulled. Therefore, the ideal \(A_{T \to \infty}\) can be obtained starting from the condition that \(v(+) = v(−)\) at the op-amp inputs. For a real op-amp, the open-loop gain is finite, and \(v(+) \neq v(−)\). The difference \(v_e = v(+) - v(−)\) can be considered an error signal for the feedback circuit, and the ideal response \(A_{T \to \infty}\) corresponds to zero error. In general, to determine \(A_{T \to \infty}\) for an arbitrary feedback circuit, it is necessary to identify the error signal \(u_e\) in the circuit. Once the location of the error signal is identified, nulling the error yields the ideal response \(A_{T \to \infty}\).

- Finally, the closed-loop gain is the gain of the original circuit where the test source is zero,

\[
A_{CL} = \left. \frac{y}{u} \right|_{v_y = 0} = A_{T \to \infty} \frac{T}{1 + T}
\]  

(20)

The setup of Fig. 5 and the results (18-20) are the basis of our approach to feedback circuit analysis and design.

### 2.2 Examples

As an example, consider another simple op-amp application shown in Fig. 6(a). Very large open-loop gain of the op-amp (equivalent to very large \(T\)), and negative feedback imply that the op-amp input is \(v_e = 0\). As a result, the − input is at the ground potential ("virtual ground") and the ideal closed-loop gain is (by inspection):

\[
A_{T \to \infty} = \frac{R_b}{R_a}
\]  

(21)

The circuit functions as an inverting amplifier with gain set by \(R_a, R_b\).

If the open loop gain of the op-amp is finite, the closed loop gain can be found using the general feedback results. The setup for finding the loop gain is shown in Fig. 6(b). Note that the input is zero, \(v_i = 0\), and that the test source is inserted in series with the op-amp output, which behaves as a controlled voltage source \(A_{OL}v_e\). The loop-gain can be found as follows:

\[
T = \frac{v_y}{v_x} = \frac{v_1}{v_x} \frac{v_y}{v_1} = \frac{R_a}{R_a + R_b} A_{OL}.
\]  

(22)
Figure 6: Another feedback circuit example: inverting amplifier.

It is interesting that the expression for the loop gain is exactly the same as for the non-inverting amplifier of Fig. 2(a). Finally, the closed-loop gain is

$$A_{CL} = A_{T \to \infty} \frac{T}{1 + T} = \frac{R_b}{R_a} \frac{A_{OL}}{1 + \frac{R_b}{R_a + R_b}}.$$

Fig. 7 shows another negative-feedback circuit example. Note that in this case the feedback loop is closed through an active device $Q$. The ideal (large-signal) response is obtained assuming that the op-amp has very large gain so that $v_e = 0$, i.e. $v(+) = v(-) = 0$:

$$I = I_D = V_g/R,$$

$$V_o = -V_{GS} = -\sqrt{K}T - V_t.$$

The ideal small-signal response is obtained by using the small-signal model for the MOS device (assume $r_{ds} \to \infty$),

$$v_y/R = i = i_d = g_m v_{gs} = -g_m v_o,$$

$$A_{T \to \infty} = \frac{v_o}{v_y} = -\frac{1}{g_m R}.$$

The loop-gain $T$ is found as shown in Fig. 7(b): the input $v_g$ is nulled (shorted to zero), and the test source $v_t$ is inserted in series with the op-amp output which behaves as a controlled voltage source $A_{OL}v_e$. The device is replaced with the small-signal model.

$$T = \frac{v_y}{v_x} = \frac{v_y}{v(-)} \frac{i_d}{v_x} = (A_{OL})(-R)(-g_m) = g_m R A_{OL}.$$

The fact that the loop gain $T = g_m R A_{OL}$ is positive is an indication that the feedback is indeed negative. The small-signal response of the feedback circuit can now be determined using the general result (17).
Figure 7: A feedback circuit example with an active device $Q$ in the feedback loop.