Summary of the Hodgkin-Huxley model

The Hodgkin-Huxley model of the process by which action potentials are generated in the giant axon of the squid lies at the basis of most neuronal models. Here is a brief summary of the equations and assumptions which went into the model.

The mathematical model is based upon the equivalent circuit for a patch of cell membrane. In your text, this is Figure 9-5. The two variable conductances G_K and G_{Na} shown in the diagram represent the average effect of the binary gating of many potasssium and sodium channels, and the constant "leakage conductance" G_L represents the effect of other channels (primarily chloride) which are always open. Each of these is associated with an equilbrium potential, represented by a battery in series with the conductance.

The net current which flows into the cell through these channels has the effect of charging the membrane capacitance, giving the interior of the cell a membrane potential V_m relative to the exterior. From basic circuit theory, we know that the current which charges a capacitor is equal to the capacitance times the rate of change of the voltage across the capacitor. Ohm's law gives the current through each of the conductances, resulting in the equation

$$C_m \frac{dV_m}{dt} = G_{Na}(E_{Na} - V_m) + G_K(E_K - V_m) + G_L(E_L - V_m) + I_{inject}. \qquad (1)$$

Here, an additional term I_{inject} has been added to describe any currents which are externally applied during the course of an experiment. In principle, all that is needed in order to find the time course of the membrane potential is to solve this simple differential equation.

The hard part was to model the time and voltage dependence of the Na and K conductances. As you know from reading Chapter 9, their solution was to perform a series of voltage clamp experiments measuring both the total current and the current when the Na conductance was disabled. This enabled them to calculate the K current and, from these currents and the known voltages, calculate the values of the two conductances. By performing the experiments with different values of the clamp voltage, they were able to determine the time dependence and equilibrium value of the conductances at different voltages. Figure 9-6 in the text shows some typical results for the behavior of the K and Na conductances when the clamping voltage is stepped to several different values and then released. From these measurements they were able to fit the the K conductance to an equation of the form

$$G_K = \overline{g}_K n^4, \qquad (2)$$

where n is called the "activation state variable" and has a simple exponential dependence governed by a single time constant, τ_n:

$$n(t) = n_\infty(V) - (n_\infty(V) - n_\infty(0))e^{-t/\tau_n}. \qquad (3)$$

$n_\infty(V)$ is called the "steady state activation", i.e. the value reached by n when it is held at the potential V for a long period of time. Hodgkin and Huxley were able to fit the voltage dependence of n_∞ and τ_n to an analytic function of voltage involving exponentials. In the interest of brevity, we will not give the expressions for $n_\infty(V)$ and $\tau_n(V)$ here. However, the plot of $n_\infty(V)$, shown further below, reveals that it is a monotonically increasing function of V, reaching a maximum value of 1.

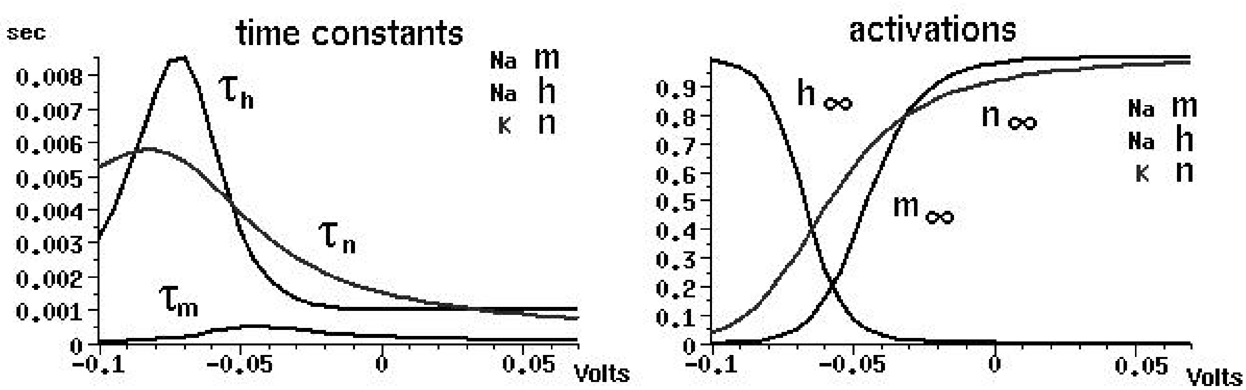
If we are describing the time course of action potentials rather than the behavior during a voltage clamp, n_∞ and τ_n are changing along with the changing membrane potential, so we can't use this equation for n. Instead, we use use a differential equation which has this solution when V is constant,

$$dn(V)/dt = (n_\infty(V) - n(V))/\tau_n(V). \tag{4}$$

Their fit for the Na conductance was a little different, because they found that at a fixed voltage, the conductance rose with time and then decreased (as shown in Figure 9-6), so they had to fit it to a product

$$G_{Na} = \overline{g}_{Na} m^3 h. \tag{5}$$

Here, m is the activation variable for Na, and h is called the "inactivation state variable", since it becomes smaller when m (and the membrane potential) becomes larger. m and h obey equations just like the ones for n, but with different voltage dependences for their steady state values and time constants. These voltage dependences are shown in the following plot, derived from Hodgkin and Huxley's fit to their experimental results:



We now have all that was needed by Hodgkin and Huxley to reconstruct the action potential. For a given injection current I_{inject}, Eq. 1 is solved for V_m, using Eqs. 2 and 5 for the conductances. These two equations must be solved simultaneously with Eq. 4 and the two analogous equations for m and h. These last equations make use of the voltage dependent quantities shown in the plot.

This plot shows that although the time constants vary with voltage, the time constant for the Na activation variable m is about an order of magnitude less than that for the Na inactivation and the K activation throughout the entire range. This means that during an action potential, when the voltage is high and m is large, and h is supposed to be small, it will take a while for h to decrease. Also, it will take n a while to become large and contribute to the opposing K current.

As we will see in a later lecture, the behavior of these quantities is the key to understanding the time course of the action potential, as well as the phenomenon of the "refractory period" following the action potential.