Parts (a) and (b) are solved in the same way as in problem p2-15. For part (c), first let $\theta_A$ be arbitrary. Then inserting (** *) into (**) gives

$$2Y_c^2 = \frac{\csc^2 \theta_A}{Z_{cA}^2}$$

From (*) and the identity $\csc^2 \theta = 1 + \cot^2 \theta$ we have

$$Y_c^2 = \frac{1}{Z_{cA}^2} - \frac{1}{Z_{cB}^2}$$

or

$$\frac{1}{Z_{cB}^2} = Y_c^2 (2 \sin^2 \theta_A - 1)$$

Note incidentally that for $Z_{cB}$ to be real and finite, we must have (to within an additive integer multiple of $\pi$) $\frac{\pi}{4} < \theta_A < \frac{3\pi}{4}$. This disqualifies $\theta_A = \frac{\pi}{4}$ as a possible length, so part (c) of the problem as specified has no solution. Nevertheless, let us pursue the solution for other possible values of $\theta_A$. With the expressions

$$Z_{cA} = \left| \frac{\csc \theta_A}{Y_c \sqrt{2}} \right|, \quad Z_{cB} = \frac{1}{Y_c \sqrt{2 \sin^2 \theta_A - 1}}$$

found for $Z_{cA}$ and $Z_{cB}$, we find from (*) that

$$\cot \theta_B = \pm \frac{\sqrt{2} \cos \theta_A}{\sqrt{2 \sin^2 \theta_A - 1}}$$

where, by (*), $\cot \theta_B$ must be positive if $\cot \theta_A$ is negative, and vice versa. If, for example, $d_A$ were just a little bit greater than $\lambda_A/8$, then $\cot \theta_B$ would be large and negative, meaning that $\theta_B$ would be a little bit less than $\pi$, and thus $d_B$ a little bit less than $\lambda_B/2$. Therefore, even with these relaxed conditions, a reduction in the length $d_A$ will require a lengthening of $d_B$ by comparison with the standard quarter-wavelength design. It turns out that the bandwidth of this hybrid coupler will be reduced due to the longer length of the B arms.