For TE modes,

\[ U_{E0} = \frac{1}{4} \int_{S_0} \epsilon E_m \cdot E_m^* \, dS = \frac{\omega^2 \mu^2 c}{4k_c^4} \int_{S_0} \nabla_T H_{mz} \cdot \nabla_T H_{mz}^* \, dS \]

by (5.50). But (as in the solution of problem 5-9),

\[ \int_{S_0} \nabla_T H_{mz} \cdot \nabla_T H_{mz}^* \, dS = \int_{S_0} [\nabla_T \cdot (H_{mz}^* \nabla_T H_{mz}) - H_{mz}^* \nabla^2_i H_{mz}] \, dS \]

by (B.6),

\[ = \oint_C H_{mz}^* \nabla_T H_{mz} \cdot u_n \, dl + k_c^2 \int_{S_0} H_{mz} H_{mz}^* \, dS = k_c^2 \int_{S_0} |H_{mz}|^2 \, dS \]

by (B.16), (5.43) and (5.47). Thus,

\[ U_{E0} = \frac{\omega^2 \mu^2 c}{4k_c^2} T_H \]

where we have abbreviated \( T_H = \int_{S_0} |H_{mz}|^2 \, dS \), which is real and positive.

Likewise, we have

\[ U_{M0} = \frac{1}{4} \int_{S_0} \mu \mathcal{H}_m \cdot \mathcal{H}_m^* \, dS = \frac{1}{4} \int_{S_0} \mu \mathcal{H}_{mT} \cdot \mathcal{H}_{mT}^* \, dS + \frac{1}{4} \int_{S_0} \mu H_{mz} H_{mz}^* \, dS \]

\[ = \frac{\mu |\gamma|^2}{4k_c^2} \int_{S_0} \nabla_T H_{mz} \cdot \nabla_T H_{mz}^* \, dS + \frac{\mu}{4} \int_{S_0} H_{mz} H_{mz}^* \, dS = \frac{\mu}{4} \left( \frac{|\gamma|^2}{k_c^2} + 1 \right) T_H \]

by (5.50) and the results above. Then

\[ U_{M0} - U_{E0} = \frac{\mu T_H}{4} \left( 1 + \frac{|\gamma|^2 - k_c^2}{k_c^2} \right) = \frac{\mu T_H \gamma^2 + |\gamma|^2}{4k_c^2} = \frac{\mu T_H \alpha^2}{2} > 0 \]

if the mode is cutoff \( (\gamma = \alpha > 0) \).

The procedure is nearly identical for TM modes, and only the highlights of the derivation are quoted here:

\[ U_{E0} = \frac{\epsilon}{4} \left( \frac{|\gamma|^2}{k_c^2} + 1 \right) T_E; \quad U_{M0} = \frac{\omega^2 \mu^2 c}{4k_c^2} T_E \]

where \( T_E = \int_{S_0} |E_{mz}|^2 \, dS \). Thus,

\[ U_{M0} - U_{E0} = -\frac{\epsilon T_E \alpha^2}{2} < 0 \]

if the mode is cutoff \( (\gamma = \alpha > 0) \).