Trace the ray reflections from an initial ray at an angle $\theta$ to the $x$-axis as shown, following the path $ABCDEFGHI$.

It is readily verified that none of these 8 rays produces any rays in new directions when reflected from any portion of the wall. The total longitudinal $H$-field from these 8 rays is:

$$H_z = H_1 e^{-jk_c(x \cos \theta + y \sin \theta)} \quad (FG)$$

$$+ H_2 e^{-jk_c(x \cos \theta - y \sin \theta)} \quad (EF)$$

$$+ H_3 e^{-jk_c(-x \cos \theta + y \sin \theta)} \quad (AB)$$

$$+ H_4 e^{-jk_c(-x \cos \theta - y \sin \theta)} \quad (DE)$$

$$+ H_5 e^{-jk_c(x \sin \theta + y \cos \theta)} \quad (CD)$$

$$+ H_6 e^{-jk_c(x \sin \theta - y \cos \theta)} \quad (HI)$$

$$+ H_7 e^{-jk_c(-x \sin \theta + y \cos \theta)} \quad (BC)$$

$$+ H_8 e^{-jk_c(-x \sin \theta - y \cos \theta)} \quad (GH)$$

The boundary condition is $0 = \partial H_z / \partial n |_{wall} = \mathbf{u}_n \cdot \nabla H_z |_{wall}$. Identify the incident/reflected pairs at $y = 0$ (where $\mathbf{u}_n = \mathbf{u}_y$) and enforce the boundary condition $0 = \partial H_z / \partial y |_{y=0}$:

$$EF \rightarrow FG : \quad jk_c \sin \theta e^{-jk_c x \cos \theta} (H_2 - H_1) \Rightarrow H_2 = H_1$$

$$DE \rightarrow AB : \quad H_4 = H_3$$

$$HI \rightarrow CD : \quad H_6 = H_5$$

$$GH \rightarrow BC : \quad H_8 = H_7$$

Likewise, at $x = 0$,

$$AB \rightarrow FG : \quad H_3 = H_1$$

$$BC \rightarrow CD : \quad H_7 = H_5$$
While from the second and third equations,

\[ \text{We now have} \]

\[ H_1 = H_2 = H_3 = H_4 \]

and

\[ H_5 = H_6 = H_7 = H_8 \]

and the expression for \( H_z \) can be reduced to the following form:

\[ H_z = H_1 \left( e^{-jk_c x \cos \theta} + e^{jk_c x \cos \theta} \right) (e^{-jk_c y \sin \theta} + e^{jk_c y \sin \theta}) + H_5 \left( e^{-jk_c x \sin \theta} + e^{jk_c x \sin \theta} \right) (e^{-jk_c y \cos \theta} + e^{jk_c y \cos \theta}) \]

The remaining part of the boundary is \( y = a - x \); on this segment the unit normal is \( \mathbf{u}_n = -(\mathbf{u}_x + \mathbf{u}_y)/\sqrt{2} \). Identifying pairs of incident and reflected plane waves and applying the boundary condition, we obtain:

\[ AB \rightarrow BC : \]

\[ jk_c (\cos \theta - \sin \theta) H_1 e^{-jk_c [-x \cos \theta + (a-x) \sin \theta]} + \]

\[ jk_c (\sin \theta - \cos \theta) H_5 e^{-jk_c [-x \sin \theta + (a-x) \cos \theta]} = 0 \]

\[ CD \rightarrow DE : \]

\[ jk_c (\cos \theta + \sin \theta) H_1 e^{-jk_c [-x \cos \theta - (a-x) \sin \theta]} + \]

\[ jk_c (-\sin \theta - \cos \theta) H_5 e^{-jk_c [x \sin \theta + (a-x) \cos \theta]} = 0 \]

\[ HI \rightarrow EF : \]

\[ jk_c (-\cos \theta + \sin \theta) H_1 e^{-jk_c [x \cos \theta - (a-x) \sin \theta]} + \]

\[ jk_c (-\sin \theta + \cos \theta) H_5 e^{-jk_c [x \sin \theta - (a-x) \cos \theta]} = 0 \]

\[ FG \rightarrow GH : \]

\[ jk_c (-\cos \theta - \sin \theta) H_1 e^{-jk_c [x \cos \theta + (a-x) \sin \theta]} + \]

\[ jk_c (\sin \theta + \cos \theta) H_5 e^{-jk_c [-x \sin \theta - (a-x) \cos \theta]} = 0 \]

Canceling the common factors (including the \( x \)-dependencies):

\[ H_1 e^{-jk_c a \sin \theta} = H_5 e^{-jk_c a \cos \theta} \]

\[ H_1 e^{jk_c a \sin \theta} = H_5 e^{-jk_c a \cos \theta} \]

\[ H_1 e^{jk_c a \sin \theta} = H_5 e^{jk_c a \cos \theta} \]

\[ H_1 e^{-jk_c a \sin \theta} = H_5 e^{jk_c a \cos \theta} \]

From the first two of these equations,

\[ 1 = e^{2jk_c a \sin \theta} \]

while from the second and third equations,

\[ 1 = e^{2jk_c a \cos \theta} \]

Thus, we have

\[ k_c a \sin \theta = m\pi \]

\[ k_c a \cos \theta = n\pi \]

where \( m \) and \( n \) are integers. We square and add to eliminate \( \theta \):

\[ k_c^2 a^2 = (m^2 + n^2)\pi^2 \]
or
\[ k_c = \frac{\pi}{a} \sqrt{m^2 + n^2} \]
Also from the equations resulting from the boundary condition on the diagonal, we have:
\[ H_5 = H_1 e^{jk_c a (\sin \theta + \cos \theta)} = (-1)^{m+n} H_1 \]
and substituting into the equation for \( H_z \), we find
\[ H_z = 4H_1 \left[ \cos \frac{n\pi x}{a} \cos \frac{m\pi y}{a} + (-1)^{m+n} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{a} \right] \]
When \( m = n = 0 \), we have \( H_z = \text{constant} \), and this generates a nontrivial field only at zero frequency, as indicated in Appendix H. Thus we have \( m, n = 0, 1, 2, \ldots \); \( m \) and \( n \) not both equal to zero for the nontrivial TE modes. The lowest-order TE mode is the TE_{10} (or what is the same, TE_{01}), for which \( k_c a = \pi \), and
\[ H_z = 4H_1 \left[ \cos \frac{\pi y}{a} - \cos \frac{\pi x}{a} \right] \]
Note that the cutoff wavenumbers of these modes are a subset of those for the square waveguide of side \( a \), and that their fields are the sum or difference of the TE_{mn} and TE_{nm} modes of the square guide.