There are two ways to solve this problem: one simple but basically mathematical, and the other more physically intuitive, but somewhat harder to make rigorous.

Method A: Put
\[ \mathbf{E} \times \mathbf{H} = -\nabla T \Phi \times \mathbf{H} = \Phi \nabla T \times \mathbf{H} - \nabla T \times (\Phi \mathbf{H}) = -\nabla T \times (\Phi \mathbf{H}) \]
by (8.11), (B.7) and (8.4). Now integrate \( \frac{1}{2} \mathbf{E} \times \mathbf{H} \cdot \mathbf{u}_z \) over the cross section \( S_0 \):

\[ P_{\text{osc}} = \frac{1}{2} \int_{S_0} \mathbf{E} \times \mathbf{H} \cdot \mathbf{u}_z \, dS = -\frac{1}{2} \int_{S_0} \nabla T \times (\Phi \mathbf{H}) \, dS = -\frac{1}{2} \oint_{C_g} \Phi \mathbf{H} \cdot d\ell + \frac{1}{2} \oint_{C_1} \Phi \mathbf{H} \cdot d\ell \]
by Stokes’ theorem (B.14) (note the directions of \( d\ell \) in the figure; Stokes’ theorem requires it to obey the right-hand rule relative to \( \mathbf{u}_z \)).

But \( \Phi = \Phi_1 \) on \( C_1 \), and \( \Phi = 0 \) on \( C_g \), while \( \oint_{C_1} \mathbf{H} \cdot d\ell = I_1 \). Thus,

\[ P_{\text{osc}} = \frac{1}{2} \Phi_1 I_1 \]
as demanded. The proof of

\[ P_{\text{av}} = \frac{1}{2} \text{Re}(\Phi_1 I_1^*) \]
follows in a similar way.
**Method B:** Sketch $\mathcal{E}$ and $\mathcal{H}$ field lines of the TEM mode as shown below.

These sets of lines form an orthogonal coordinate system $(u_1, u_2)$ along the $\mathcal{E}$ and $\mathcal{H}$ lines respectively. Define the coordinates so that $u_1 = 0$ on $C_g$ and $u_1 = \Phi_1$ on $C_1$, i.e., $u_1 = \Phi$. Then we have

$$P_{osc} = \frac{1}{2} \int_{S_0} \mathcal{E} \times \mathcal{H} \cdot u_z \, dS = \frac{1}{2} \int_{S_0} (\mathcal{E} \times \mathcal{H}) \cdot (d\ell_1 \times d\ell_2)$$

by (B.3) and (B.4), and where $C(u_1)$ is the coordinate line for a given value of $u_1$. But $\mathcal{E} \cdot d\ell_2 = 0$ by construction and $\mathcal{E}$ does not depend on $u_2$ (on a line of constant $u_1$), so we have

$$P_{osc} = \frac{1}{2} \int_{u_1=0}^{\Phi_1} (\mathcal{E} \cdot d\ell_1) \int_{C(u_1)} (\mathcal{H} \cdot d\ell_2) = \frac{1}{2} \left( -\Phi_1 \right) (-I_1) = \frac{1}{2} \Phi_1 I_1$$

as desired. Again, the result for $P_{av}$ follows similarly.

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$^1$These special coordinates are sometimes called harmonic coordinates.