Homework Assignment #1  

Due 27 Jan 09

1. Question 1.12, p. 23

2. Question 7.2, p. 163

3. Question 7.3, p. 163

4. Question 7.7, p. 164

5. a. Calculate the reflectance R(\(\lambda\)) for a particular metal using the Drude model with the following parameters: 
\(\varepsilon\infty = 1\)
\(\hbar \omega_p = 15.0 \text{ eV}\)
\(\tau = 0.5 \times 10^{-14} \text{ sec}\)

Plot R(\(\lambda\)) for 1 \(\mu\text{m} < \lambda < 1 \text{ nm}\) (use a logarithmic scale). Indicate the plasma wavelength \(\lambda_p\).

b. Why is the natural frequency of oscillations of electrons (in a plasma) the same as the frequency below which transverse electromagnetic waves are reflected?

6. What is the velocity of light in a metal in the vicinity of \(\omega = \omega_p\)? Hint: for \(\omega > \omega_p\) the answer is not > c.

7. Consider a bounded metal film of thickness \(d\) extending from \(-d/2 \leq z \leq d/2\). Assume the dielectric constant is well described by the Drude model, and the imaginary part may be neglected.

a. Using the appropriate boundary conditions at the surfaces, show that the dispersion relation for plasma oscillations is

\[\omega = \omega_p \left( 1 \pm \exp\left(-k_z d\right) \right)^{1/2}\]

and plot the dispersion relation. Hint: You might want to set up a plasmon potential \(\phi(x,z)\) that, in the metal film, decays symmetrically in \(z\) about \(z = 0\) and is composed of odd and even function, and that decays exponentially outside the film. A surface plasmon travels in the \(x\)-direction, and decays exponentially in the \(z\)-direction such that for \(\phi\) (or \(V\)) defined by Eq. A-7 inside the metal film \(\phi = f(x) \cdot \left[\exp(k_z z) \pm \exp(-k_z z)\right]\), and outside the film
(z > d/2) \( \phi = f(x) \cdot \exp(-k_z z) \cdot g(z) \). Then just use continuity of \( \mathbf{D} \), the formula for Drude model dielectric constant etc., to solve.

b. In the limit of short wavelength, show that this result reduces to the result for surface plasmons in a semi-infinite plasma. Explain why these results should be the same.

c. In the limit of long wavelengths, there are two characteristic dispersion relations. State what these are, and describe the electron motion for each of these two types of plasmons. Justify your description.

8. a) For the optical beam coupled with a prism as shown, find the coupling angle \( \theta \) for the optical excitation of surface plasmons when \( \hbar \omega = 2 \text{ eV} \), \( \hbar \omega_p = 15 \text{ eV} \), and \( n_p = 1.5 \).

Hints: The x-component of the optical beam wavevector \( k_x = \frac{\omega n_p}{c} \sin \theta \).