1 Introduction

Fourier optics is a powerful tool for the coherent manipulation of optical fields, for the spatial frequency analysis of images, and for the construction of real time correlators. One property that makes optics an exciting tool for frequency analysis is that, in free-space propagation of light, one can see the effects of lenses and filters on a light beam as the beam propagates through the system to the far field.

In this experiment you will set up a system of lenses that allow the display and modification of the Fourier spectrum of two-dimensional objects. One can then use a variety of filters from simple opaque objects to computer generated holograms that will selectively block certain spatial frequencies of the original object. These devices can be used either to modify the characteristics of the object or to perform various computations to extract information out of the object. By using complex transparencies in the Fourier plane, implemented in a dynamic organic holographic material, you will be able to build matched spatial filter correlators for object recognition and location.

2 Background in Fourier Optics

Fourier transformations can be generalized from the common 1-D temporal case to the 2-dimensional case required for the frequency analysis of images and even to the 3-D case required to understand 3-D propagation and diffraction in $\vec{k}$-space.

2.1 Review of 2-D Fourier Theorems

The 2-dimensional spatial Fourier transform is defined as

$$G(u, v) = G(\vec{u}) = \int g(x, y)e^{-i2\pi(ux + vy)}dx dy = \mathcal{F}_{xy}\{g(x, y)\} = \int g(\vec{x})e^{-i\vec{k} \cdot \vec{x}}d\vec{x}$$

and the inverse spatial Fourier transform

$$g(x, y) = g(\vec{x}) = \int G(u, v)e^{i2\pi(ux + vy)}du dv = \mathcal{F}^{-1}_{uv}\{G(u, v)\} = \int G(\vec{k})e^{i\vec{k} \cdot \vec{x}}d\vec{k}$$

Note that this final form readily generalizes to $N$ dimensions.

**Linearity**

$$af(x, y) + bg(x, y) \leftrightarrow aF(u, v) + bG(u, v)$$
Anisotropic Scaling
\[ f(\alpha x, \beta y) \longmapsto \frac{1}{|\alpha \beta|} F(u/\alpha, v/\beta) \]

Isotropic 2-D Scaling
\[ f(\alpha x, \alpha y) \longmapsto \frac{1}{|\alpha|^2} F(u/\alpha, v/\alpha) \]

2-D Shift Theorem
\[ f(x + x_0, y + y_0) \longmapsto e^{-i2\pi(ux_0 + vy_0)} F(u, v) \]

Modulation by a 2-D linear phase factor
\[ e^{i2\pi(u_0x + v_0y)} f(x, y) \longmapsto F(u - u_0, v - v_0) \]

Parseval’s Theorem
\[ \int_{-\infty}^{\infty} |g(x, y)|^2 dxdy = \int_{-\infty}^{\infty} |G(u, v)|^2 dudv \]

Convolution Theorem
\[ \int f(x', y')g(x - x', y - y')dx' dy' \longmapsto F(u)G(u) \quad (1) \]
\[ f(x, y)g(x, y) \longmapsto \int F(u', v')G(u - u', v - v')du' dv' \quad (2) \]

Correlation
\[ \int f(x', y')g^*(x' + x, y' + y)dx' dy' \longmapsto F(u, v)G^*(u, v) \quad (3) \]
\[ \int f(x', y')f^*(x' + x, y' + y)dx' dy' \longmapsto |F(u, v)|^2 \quad (4) \]

Fourier Integral
\[ \mathcal{F}\{\mathcal{F}^{-1}\{g(x, y)\}\} = \mathcal{F}^{-1}\{g(x, y)\} = g(x, y) \quad (6) \]
\[ \mathcal{F}\{\mathcal{F}\{g(x, y)\}\} = g(-x, -y) \quad (7) \]

Rotation Operator and Fourier theorem \( \mathcal{R}_\theta \{ \} \) operator that rotates image by \( \theta \) (CCW, RH) about origin
\[ f \longmapsto F \]
\[ \mathcal{R}_\theta \{ f \} \longmapsto \mathcal{R}_\theta \{ F \} \]
\[ f(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta) \longmapsto F(u \cos \theta - v \sin \theta, u \sin \theta + v \cos \theta) \]
\[ g(r, (\theta - \alpha)) \longmapsto G(\rho, \phi - \alpha) \]
Projection Slice Theorem \[ \int f(x, y)dy = p_0(x) \] 0 degree projection of realn domain

\[
F(u, v) = \mathcal{F}_{2D}\{f(x, y)\} = F(r \sin \theta, r \cos \theta) = \tilde{F}(r, \theta)
\]

\[
S_0(u) = \mathcal{F}_{1D}\{p_0(x)\} = \tilde{F}(r, 0) \quad 0 \text{ degree slice of Fourier plane}
\]

Arbitrary angle \( \theta_0 \)

\[
p_{\theta_0}(x') = \int f(x' \cos \theta_0 - y' \sin \theta_0, x' \sin \theta_0 + y' \cos \theta_0)dy'
\]

\[
S_{\theta_0}(u') = \mathcal{F}\{p_{\theta_0}(x')\} = \tilde{F}(u', \theta_0) = F(u' \cos \theta_0, u' \sin \theta_0) = R_{-\theta_0} \left\{ \mathcal{F}\{p_0 \{ R_{\theta_0} \{ f(x, y) \} \} \} \right\} \bigg|_{v=0}
\]

\[
f ** R_{\theta_0} \{ \delta(x) \cdot 1(y) \} = F \cdot R_{\theta_0} \{ 1(u) \cdot \delta(v) \}
\]

2.2 Diffraction and Propagation as a linear system

Suppose we know the optical field \( E_i(x, y) \) on some plane, say the slide of a slide projector, and we want to know what is \( E_0(x', y') \) on some other plane? The Rayleigh-Sommerfeld integral represents the optical field on the output plane at coordinates \( \vec{r}' = (x', y') \) as an integral over the input plane coordinates \( \vec{r} = (x, y) \) as

\[
E_o(x', y') = E_o(\vec{r}') = \int \int_S E_i(\vec{r}) \frac{ie^{ikR}}{\lambda |\vec{R}|} \cos(\hat{n}, \vec{R})dS
\]

When a plane wave laser passes through the slide, spherical wavefronts known as Huygens wavelets, \( \frac{e^{ikR}}{\lambda |\vec{R}|} \), are launched from each transparent point on the slide in proportion to the local transmission, \( E_i(\vec{r}) \), across the surface of the transparency, \( S \). The diffraction has an additional 90° phase shift as well as an obliquity factor, \( \cos(\hat{n}, \vec{R}) \) due to the polarization property of radiating dipoles. The distance, \( R \), from each image point to each output point can be represented...
Figure 1: Geometry for optical diffraction as a linear system.

\[ R = \sqrt{(z' - z)^2 + (x' - x)^2 + (y' - y)^2} \approx (z' - z) \left( 1 + \frac{(x' - x)^2 + (y' - y)^2}{2(z' - z)} \right) \]

(8)

\[ \approx (z' - z) + \frac{(x' - x)^2 + (y' - y)^2}{2(z' - z)} \]

(9)

using the approximation \( \sqrt{1 + \epsilon} \approx 1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8} \). Additionally, for small angles we can approximate \( \cos(\hat{n}, \vec{R}) \approx 1 \) good to 5% accuracy for \( \theta < 18^\circ \), and use within the phase factor the more accurate approximation \( R \approx z + \frac{(x' - x)^2 + (y' - y)^2}{2z} \) while in the less critical amplitude factor use \( R \approx z \). This gives the Frenel integral recognizable as a linear system for describing optical diffraction

\[
E_o(x_o, y_o) = -ie^{ikz} \frac{1}{\lambda z} \int \int_A E_i(x_i, y_i) e^{i \frac{k}{\lambda z}(x_o - x_i)^2 + (y_o - y_i)^2} dx_i dy_i
\]

(10)

\[
= \int \int_A E_i(x_i, y_i) h(x_o - x_i, y_o - y_i) dx_i dy_i
\]

(11)

\[
= \frac{e^{ikz}}{i\lambda z} e^{i \frac{k}{\lambda z}(x_o^2 + y_o^2)} \int \int_A E_i(x_i, y_i) e^{i \frac{k}{\lambda z}(x_i^2 + y_i^2)} e^{-i \frac{k}{\lambda z}(x_o x_i + y_o y_i)} dx_i dy_i.
\]

(12)

Where the 2-D spatial impulse response of the spatially invariant convolutional integral can be represented as a separable impulse response or as a circularly symmetric impulse response \( (\rho^2 + x^2 + y^2) \) as is appropriate for the natural symmetries of space

\[
h(x, y) = \frac{e^{ikz}}{i\lambda z} e^{i \frac{k}{\lambda z}(x^2 + y^2)} = e^{ikz} \left[ \frac{1}{\sqrt{\lambda z}} e^{i \frac{k}{\lambda z} x^2} \right] \left[ \frac{1}{\sqrt{\lambda z}} e^{i \frac{k}{\lambda z} y^2} \right] \frac{e^{ikz}}{i\lambda z} e^{i \frac{k}{\lambda z} \rho^2}.
\]

(13)

Since this convolution can be instead represented as a product with the transfer function of free space for propagation through a distance \( z \)

\[
H(u, v) = e^{ikz} e^{i \sqrt{k^2 - k_x^2 - k_y^2}} \approx e^{ikz} e^{i \pi \lambda z (u^2 + v^2)}
\]

(14)
Figure 2: Far-field Fraunhoffer diffraction from a square aperture producing a 2-D sinc.

where $k = 2\pi/\lambda$ for wavelength $\lambda$ and $k_x = 2\pi u$ and $k_y = 2\pi v$. The convolutional integral can be simplified in the Fourier domain by using the 2-D convolution theorem

$$F_{x_0y_0}\{E_o(x_o, y_o)\} = H(u, v)F_{x_iy_i}\{E_i(x_i, y_i)\} \quad (15)$$

**Far-field Fraunhoffer approximation**  For large propagation distances and small apertures

$$z \gg k\frac{x_{i\text{max}}^2 + y_{i\text{max}}^2}{2}$$

In which case we can approximate over the entire aperture $e^{i\frac{k}{2z}(x_{i\text{max}}^2 + y_{i\text{max}}^2)} \approx e^{i\epsilon} \approx 1$. For example for HeNe laser $\lambda = 632.8 \mu m$,

$$x_{i\text{max}} \approx 2.5cm \implies z > 1.6km \quad (16)$$

$$x_{i\text{max}} \approx 100\mu m \implies z > 5cm \quad (17)$$

In this regime we can make the Fraunhoffer approximation

$$E_o(x_o, y_o) = \frac{-ie^{ikz}}{\lambda z} \int \int_A E_i(x_i, y_i)e^{i\frac{k}{2z}(x_o-x_i)^2+(y_o-y_i)^2}dx_idy_i \quad (18)$$

$$= \frac{e^{ikz}}{i\lambda z}e^{i\frac{k}{2z}(x_o^2+y_o^2)} \int \int_A E_i(x_i, y_i)e^{-i\frac{k}{2z}(x_o x_i+y_o y_i)}dx_idy_i \quad (19)$$

So that far-field propagation can be represented as a Fourier transform using $f_x = \frac{x_o}{\lambda z}$ and $f_y = \frac{y_o}{\lambda z}$.

For example consider a square wave illuminated by a normally incident monochromatic plane wave producing an input field

$$E_i(x_i, y_i) = \Pi \left( \frac{x_i}{X} \right) \left( \frac{y_i}{Y} \right) \quad (20)$$

This field propagates into the far-field as seen in Fig. 2 producing a far-field diffraction pattern

$$E_o(x_o, y_o) = \frac{e^{ikz}}{i\lambda z}e^{i\frac{k}{2z}(x_o^2+y_o^2)}X\text{sinc}(Xf_x)Y\text{sinc}(Yf_y) \quad (21)$$

The detected far-field intensity pattern is proportional to the modulus-squared of the field amplitude.

$$I_o(x_o, y_o) = |E_o(x_o, y_o)|^2 = \frac{X^2Y^2}{\lambda^2z^2}\text{sinc}^2\left( \frac{Xx_o}{\lambda z} \right)\text{sinc}^2\left( \frac{Yy_o}{\lambda z} \right) \quad (22)$$
\[ \Delta(x, y) = \Delta_0 - R_1 \sqrt{1 - \frac{x^2 + y^2}{R_1^2}} + R_2 \sqrt{1 - \frac{x^2 + y^2}{R_2^2}} \]
\[ R \sqrt{1 - \frac{x^2 + y^2}{R^2}} \approx R - \frac{x^2 + y^2}{2R} \]

Figure 3: The geometry of a spherical lens introduces a spatial quadratic phase factor based on the surface radius of curvatures \( R_1 \) and \( R_2 \).

Figure 4: Object placed adjacent to the lens and Fourier transform with a residual quadratic phase factor appears at the back focus.

Figure 5: Fourier diffraction pattern analysis showing how the FT of the letters produce identifiable diffraction patterns.

2.3 Fourier transform with a lens

A spherical lens made of glass of index of refraction \( n \) with a geometry as illustrated in Fig. 3 has a quadratic phase transmission function

\[ t(x, y) = e^{ikn\Delta_0} e^{-ik(n-1)\frac{x^2+y^2}{2} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)} \]
\[ = e^{-i\frac{k}{2f}(x^2+y^2)} \]

The focal length, \( f \), of the lens is found as

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Such a lens can be used in either spatial Fourier transforming or imaging systems.

When an object is placed adjacent to the lens, as illustrated in Fig. 4, and illuminated by a plane wave, the field just after the lens is given by the product of the transmission functions

\[ E_i(x_i, y_i) = A(x_i, y_i)t_i(x_i, y_i) = A(x_i, y_i)e^{-i\frac{k}{2f}(x^2+y^2)} \]

(25)
Propagating a distance \( f \) to the back focal plane produces a spatial output proportional to the scaled Fourier transform of the object

\[
E_o(x, y) = \frac{1}{\lambda F} e^{i \frac{2 \pi}{\lambda F} (x^2 + y^2)} \int \int A(x_i, y_i) e^{-i \frac{2 \pi}{\lambda F} (x_i^2 + y_i^2)} e^{-i \frac{2 \pi}{\lambda F} (x x_i + y y_i)} dx_i dy_i. \tag{26}
\]

Note the cancelation of the compensating phase factors in the integral. The residual quadratic phase factor can be canceled with an additional lens placed just before the output Fourier plane, implementing the Multiply-Convolve-Multiply approach to Fourier transformation. Alternatively the quadratic phase factor can be eliminated by squaring, producing an output intensity proportional to the scaled Fourier power spectrum of the object.

\[
I_o(x, y) = \frac{1}{\lambda^2 F^2} \| \mathcal{F} \{ A(x_i, y_i) \} \|_u = \frac{x}{\lambda F}, v = \frac{y}{\lambda F} \|^2 \tag{27}
\]

The output spatial frequency variables of the Fourier transform along the \( x \) and \( y \) direction are proportional to the spatial frequency (measured in cycles per mm) are \( u = \frac{x}{\lambda F} \) and \( v = \frac{y}{\lambda F} \).

Finally, the more common optical Fourier transform system is implemented as shown in Fig. 5 with the object placed a distance \( F \) before the lens of focal length \( F \), so that a distance \( F \) after the lens yields a phase flat scaled Fourier transformation at the back focal plane without any residual quadratic phase factor. This can be shown as a chirp Convolve-multiply-Convolve transformation to yield

\[
E_o(x, y) = \frac{1}{\lambda F} \int \int A(x_i, y_i) e^{-i \frac{2 \pi}{\lambda F} (x x_i + y y_i)} dx_i dy_i. \tag{28}
\]

Which is an ideal complex Fourier transformation scaled by \( \frac{1}{\lambda F} \) with the physical spatial coordinates in the output proportional to spatial frequency plane, and we can define a normalized set of output coordinates \( u = x / \lambda F \) and \( v = y / \lambda F \) which are spatial frequency variables measured in cycles/mm.

### 2.4 Coherent Two-lens 4F imaging system for Spatial Filtering

Consider an input object whose spectrum we wish to modify or filter. As an example think of a screen mesh written as

\[
t(x, y) = \text{comb} \left( \frac{x}{\Delta} \right) \ast \Pi \left( \frac{x}{W} \right) \ast \Pi \left( \frac{y}{W} \right) \ast \Pi \left( \frac{x}{L} \right) \ast \Pi \left( \frac{y}{L} \right)
\]

The Fourier transform at the back focal plane of a lens of focal length \( F_1 \) when the object is illuminated by a plane wave of wavelength \( \lambda \) is

\[
E_-(x', y') = \frac{1}{i \lambda F_1} \mathcal{F}_{2D} \{ At(x, y) \} \big|_{u = x' / \lambda F_1}, v = y' / \lambda F_1
\]

\[
= \frac{1}{i \lambda F_1} \Delta \text{comb}(\Delta u) \text{w sinc}(wu) \ast \Delta \text{comb}(\Delta v) \text{w sinc}(wv) \ast \text{L sinc}(uL) \ast \text{L sinc}(vL)
\]

This 2D array of impulses (bed of nails) is weighted by the FT of the transmission gap \( w \) and blurred by the sinc due to the finite size of the screen. We filter this Fourier plane with a vertical slit centered at DC and of width \( \delta' = \frac{1}{\Delta} = \frac{\delta}{\lambda F} \) chosen to just pass one order.

\[
E_+(x', y') = E_-(x', y') \Pi \left( \frac{x}{\delta} \right)
\]

\[
= \frac{1}{i \lambda F_1} \Delta \text{comb}(\Delta v) \text{w sinc}(wv) \ast \text{L sinc}(uL) \ast \text{L sinc}(vL)
\]
Apertures in the Fourier Plane

Figure 6: 4F spatial filtering system showing how a vertical slit in the Fourier plane removes the horizontal off-axis diffraction orders, so that re-Fourier transforming with a lens reproduces the vertical structure of the screen but has eliminated all horizontal structure.

A second lens of focal length $F_2$ is used to Fourier transform again in order to produce an output Image with magnification $m = \frac{-F_2}{F_1}$

$$I(x'', y'') = \left| \frac{1}{i\lambda F_2} \mathcal{F}\{E_+ (x', y')\} \right|^2 = \frac{-1}{\lambda^2 F_1 F_2} \text{comb}\left( \frac{y}{m\Delta} \right) * \Pi \left( \frac{y}{mw} \right) \Pi \left( \frac{x}{mL} \right) \Pi \left( \frac{y}{mL} \right)$$

This only displays a variation in the vertical direction that passed through the vertical slit because the spatial frequencies needed to represent horizontal variations have been blocked by the slit.

3 Preparation

Read the sections on Fourier optics in your favorite coherent optics book.

- Saleh and Teich, Photonics, Ch.4
- Hecht and Zajac, Optics, Addison-Wesley 1976, chapter 11.
- W.T. Cathey, Fourier Optics and Holography.
4 Prelab

1. A wire screen with .1mm cells, and wire thickness of $25\mu m$ is illuminated by a $0.6328\mu m$ laser and the diffracted light is Fourier transformed with a lens with a focal length of $F=250mm$. Remember, when the grating is illuminated the wires are opaque and the space in between passes light. Sketch and dimension the Fourier plane. What size slit should be used to remove all but the first order diffraction in the $x$ direction, and all orders in the $y$ direction. Which diffraction maxima is missing for this particular ratio of wire thickness to wire spacing?

2. Sketch the Fourier transform of the letters A,E,W,F and O. Are all these letters distinguishable by their Fourier spectra?

3. Sketch the autocorrelation of a circle $\circ$ with itself, and a disk $\bullet$ with itself, and of the crosscorrelation of the circle with the disk, what does this illustrate about edge enhanced matched spatial filters?

4. [NOT REQUIRED] Correlators in general, and optical matched spatial filters in particular, are quite sensitive to the change in scale of an object with respect to the scale of the reference with which the Van der lugt filter was recorded. An interesting approach to overcoming this problem is to place the input object transparency on a translation stage in the converging beam of a Fourier lens that is illuminating the matched filter, and translate the transparency forwards and backwards while examining the output plane and searching for the best correlation peak, which corresponds to the scale compensated correlation. Analyze this system, and show that it indeed performs a scale compensating correlation, at some position along the optical axis for the input transparency, while at other positions a scale mismatched crosscorrelation is produced.
5 Set Up

This section has been added in order to help the person setting up the lab experiment and the student who will be doing the lab. This lab set up is intended to be done once and then left alone in order to reduce the amount of work done by the students.

Using the diagram in Figure 7 as a guide, align the HeNe laser with the table as usual using the multiple iris method.

Spatial filter the beam and collimate the expanding output with a lens that produces a beam at least 40 mm in diameter (leave room before the spatial filter for ND filters and polarizers. Set up the two lens telescopic imaging system shown in the bottom of Figure 7. Leave room for the removable mirrors and the object. The object should be placed one focal length before the first FT lens, and the image should appear one focal length beyond the second FT lens, and the separation between the two lenses should be the sum of their focal lengths. Have your TA show you some of the tricks that can be used to ensure this alignment, (auto-collimation condition, speckle size maximization). Make sure that the imaging system produces a sharp image of an object placed in the input plane. Align the mirror and the camera so that the Fourier transform plane is imaged on the CCD array. Make sure the spot on the CCD is as sharp as possible.

Set up the path length matched holographic interferometer that uses the dynamic photoanisotropic optical media (DPOM) as a Vander Lugt filter as shown in the upper part of Figure 8. Using a Fourier transform lens a distance F beyond the DPOM film plate, focus the reference beam to a spot on the output observation plane. Make sure that the DPOM film plate is perpendicular to the object beam, to minimize depth of field requirements on the Fourier plane, and that the Fourier transform lens is exactly one focal length before the film. The object should be placed one focal length before the FT lens on a stage with both translation and rotation.

5.1 Materials and Equipment

- Doubled YAG or Argon laser
- HeNe laser
- Dynamic Photoanisotropic Material
- CCD Camera
- ND Filters
- Polarizer
- half wave plate
- 2 Aperture stops
- 6 Positive (100 - 250 mm) lenses
- 5 Mirrors
- Beam (non-polarizing) splitter
- Kinematic mount
- High resolution IC mask
- translation and rotation stage mount
6 Procedure


(a) Temporarily place the variable slit in the raw beam, close it down all the way and describe (and sketch) the pattern observed on a card 10cm behind the slit as you slowly open it. Do you see a \( \text{sinc}^2 \) pattern? · · · · · · 2.5

(b) Curl a piece of paper into a 10cm diameter half cylinder, and place the axis of the cylinder at the slit position, what do you see on the paper, and what is the difference between this and the pattern seen on the flat card? · · · · · · 2.5

(c) Why do you see this pattern if we are not in the Fourier plane of a lens? · · · · · · 2.5

(d) Replace the variable slit with the fixed slit and make the measurements on the diffraction pattern that allow you to estimate the slit width. What is the slit width and how does it compare to your expectation? · · · · · · 2.5

2. Fourier Transformation [20]

Figure 7: 4F optical system for spatial filtering

(a) Z-fold the laser onto the rail (leave room before the spatial filter for ND filters and a shutter) and align it with the center of the rail using irises. (Hopefully this is still set up properly) Spatial filter the beam and collimate the expanding output with a lens that produces a beam at least 40mm in diameter (leave room before the spatial filter for ND filters and a shutter). Set up the two-lens 4-F telescopic imaging system shown in Figure 7. The object should be placed one focal length before the first FT lens, and the image should appear one focal length beyond the second FT lens, and the separation between the two lenses should be the sum of their focal lengths and should be checked using the collimation tester. Have your TA show you some of the tricks that can be used to ensure this alignment, (collimation tester, autocollimation condition, speckle size maximization, or Shack-Hartman wavefront sensor) and describe the procedure that you used. Make sure that the imaging system produces a sharp image of an object placed in the input plane.

\{ label the spatial filter, lens focal lengths · · · · · · 3;
\}

\{ describe your procedure to align for collimation and to find the image plane · · · · · · 6.\}

(b) Place the wire mesh, object A, in the input plane, and describe (and sketch) what you see in the Fourier plane, you may wish to look at a magnified image of the Fourier plane on the far wall using a short focal length objective lens, or just place the CCD or digital camera focal plane in the Fourier plane in order to frame grab
an image. (CAREFUL: the optical viewfinder is not eye safe!!!) What is the wire thickness/spacing ratio for this mesh (and how do you infer it)? · · · · · · 5

(c) Translate the object and DESCRIBE the effect on the Fourier plane. Is it translationally invariant (at least as far as the eye can see)? Rotate the input transparency, and describe the effect on the Fourier plane. Is the Fourier plane rotationally invariant? · · · · · · 3

(d) Place the object after the first Fourier transform lens and move it along the optical axis, and DESCRIBE (and explain) the effect on the Fourier plane. · · · · · · 3

3. Spatial Filtering of Periodic Objects[10]
Still using the periodic wire screen mesh (object A) place the symmetrically opening variable slit in the Fourier plane, with the slit vertical, and centered on the DC spot.

(a) DESCRIBE your procedure for making sure the slit is actually in the Fourier plane, and not in front of or behind it, and DESCRIBE how you centered the slit in the Fourier plane. · · · · · · 5

(b) Describe what happens to the image as you open the slit slowly (hint: what happens if you include only DC, DC and 1st orders, and more higher orders). Rotate the slit by 90 degrees and perform the same operation. Note your observations, and sketch the output for both cases, or frame grab and print your experimental results, making sure to label the key features. · · · · · · 5

4. Spatial Filtering [10]
Insert one of the block letter objects into the input plane of the Fourier spatial filtering system. Carefully align a microscope slide with a small opaque dot in the center of the Fourier plane or use a tiny wire stretched across the Fourier plane to block out all of the low spatial frequency components.

(a) How critical is the transverse and longitudinal alignment of this high pass spatial filter, and how critical would these alignments be for a 25µ opaque DC block? (Hint: Estimate the DC spot size and compare it with the DC block size.) Observe and DESCRIBE the output plane. · · · · · · 6

(b) What processing operation has been performed on the input object, and why would this be useful? · · · · · · 4

5. Computer Generated Holograms [OPTIONAL]
Image the Fourier transform plane with a high magnification on the far wall or onto a CCD or digital camera focal plane.

(a) Replace the input object with one of the CGH masks from the B series and observe the Fourier plane and the magnified image and DESCRIBE (and sketch) what you observe. · · · · · · 2

(b) Replace with one of the CGH masks from the C series and DESCRIBE (and sketch) what you observe in the Fourier plane. · · · · · · 2
(c) Compare the two types of CGHs under the magnification of the loop and describe the differences in the masks. They are both Lohmann style Fourier transform holograms with one important difference, can you tell what the difference is in the encoding algorithm? · · · · · · 2


Figure 8: Van der Lugt holographic matched spatial filter system using dynamic photoanisotropic media (DPOM) in a path length matched configuration.

(a) Set up the path length matched holographic interferometer that uses the dynamic photoanisotropic optical material (DPOM) as a Van der Lugt filter as shown in Figure 8. Choose a high resolution object such as an IC mask mounted on a translation and rotation stage. Using a Fourier transform lens a distance $F$ beyond the DPOM, focus the reference beam to a spot on the output observation plane. The DPOM plate should be perpendicular to the object beam, to minimize depth of field requirements on the Fourier plane, and the Fourier transform lens should be exactly one focal length before the film. The object should be placed one focal length before the first FT lens. Alternatively, the object can be placed less than $F$ before the lens or even after the lens in the converging beam to investigate scale selectivity of the correlation. \{Label the important parameters (focal length, beam interaction angle, etc.), briefly describe the critical alignments (how to find the exact FT plane and image plane, etc.) · · · · 14 \}

(b) Measure the spatial distribution of power in the DPOM film plane, and adjust the reference beam power to be uniform and essentially equal to the information bearing
wings of the object Fourier transform. When recording, do not worry about saturating the DC spot, you actually would like to avoid recording any fringes in the low spatial frequency region of the hologram in order to form a high pass spatial filter (or edge enhanced hologram), because such a hologram has very good recognition and discrimination capabilities as compared with an all pass spatial filter (prelab problem 3). Record a hologram for a few seconds. Now block the reference beam, and reilluminate the DPOM with the Fourier transform of the object. Observe the output screen and locate the correlation peak. You will find that the correlation peak decays as the hologram is read out due to erasure of the DPOM.

7. Polarization holography [30]

(a) Put a half wave plate in the reference beam, to rotate the polarization by 90 degrees with respect to the object beam. Now try to record a hologram with these orthogonally polarized beams. \{How do you make sure that the beams are truly orthogonally polarized? \ldots \ldots \ldots 2\}\n
(b) Do you see any diffraction from the DPOM and if so what is the polarization of the diffracted light? \ldots \ldots 4

(c) Insert a polarizer to block unwanted scatter from the DPOM, and see if the holographic diffraction has been polarization switched to pass through the polarizer. Is the quality of the correlation peak data improved? \ldots \ldots 4

(d) Do the polarization holograms appear to decay at the same rate as the intensity holograms? Why or why not? Try recording the hologram for different amounts of time, and measure the erasure time as a function of the recording time. Logarithmic variation of the recording time can span more range quickly (eg 1,10,100s). \{Record the original data, plot it, and briefly describe it. \ldots \ldots 6

(e) Use the CCD camera to observe the fine structure of the correlation peak. \{You can print out the image you observe on the CCD. } \ldots \ldots 10

i. Translate the input object, and observe the correlation peak. Is the correlator space invariant? eg does the peak move with the object? How does the angle between object and reference influence this space-invariance?

ii. Rotate the input object and observe the correlation peak. Is the correlation rotation invariant? What is the rotational sensitivity of the correlator?

iii. If you placed the object in the converging beam, you can slide the object along the rail after recording to investigate the scale sensitivity of the correlation. What is the scale sensitivity of the correlator?

iv. Block off most of the input object and use just a small piece of the object as the input. What is the effect on the correlation peak (please explain)?

(f) Illuminate the hologram with the reference beam, to produce a reconstruction of the edge enhanced object which is stored in the hologram as the matched spatial filter template. If sufficient diffraction efficiency is available to see the reconstruction in the image plane, and the polarizer cleans up the image sufficiently, use the CCD or digital camera to take a picture of your edge enhanced reconstruction as well as the original input object imaged through the hologram. Explain the result. \ldots \ldots 4