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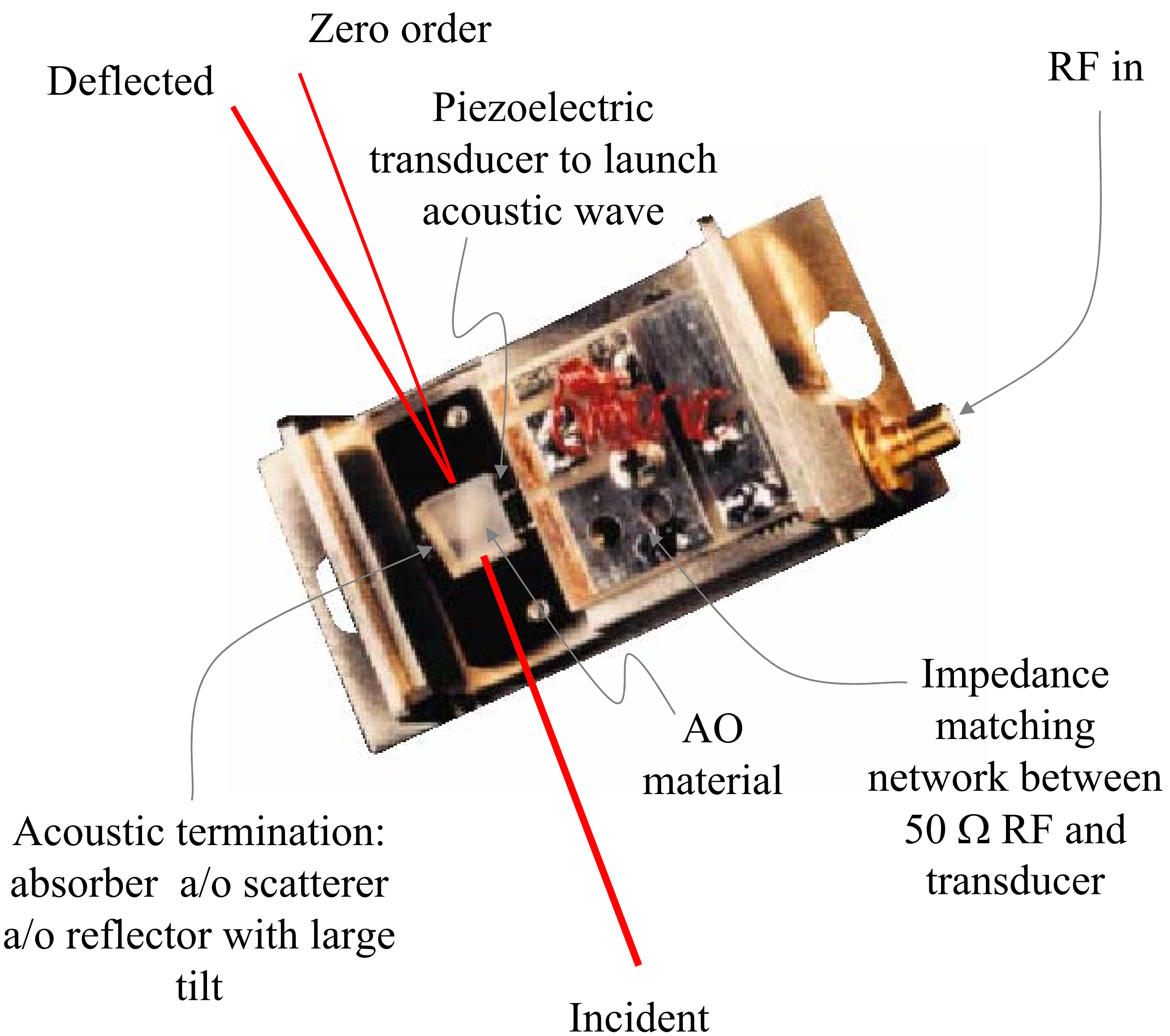
# Acoustooptic basics

## Overview




The RF signal drives a piezoelectric transducer which launches a traveling acoustic wave in the material. Through the elastooptic effect, this acoustic wave induces a traveling volume index grating which can efficiently diffract an optical wave.

# Acoustic wave equation
## Analogous to EM w/ 3 polarizations

Starting with the fundamental first-order DEs

$$\nabla \cdot \overline{\overline{T}} = \rho \frac{\partial \vec{V}}{\partial t} - \vec{F}$$

$$\nabla_s \vec{V} = \overline{\overline{\overline{s}}} : \frac{\partial \vec{T}}{\partial t}$$

| | |
|---|---|
| $\overline{\overline{T}}$ | Stress tensor |
| $\rho$ | Density |
| $\vec{V}$ | Velocity field of the material |
| $\vec{F}$ | Applied force |
| $\nabla_s$ | Symmetric part of the gradient |
| $\overline{\overline{\overline{s}}}$ | 4th order compliance tensor (= $c^{-1}$ stiffness tensor) |

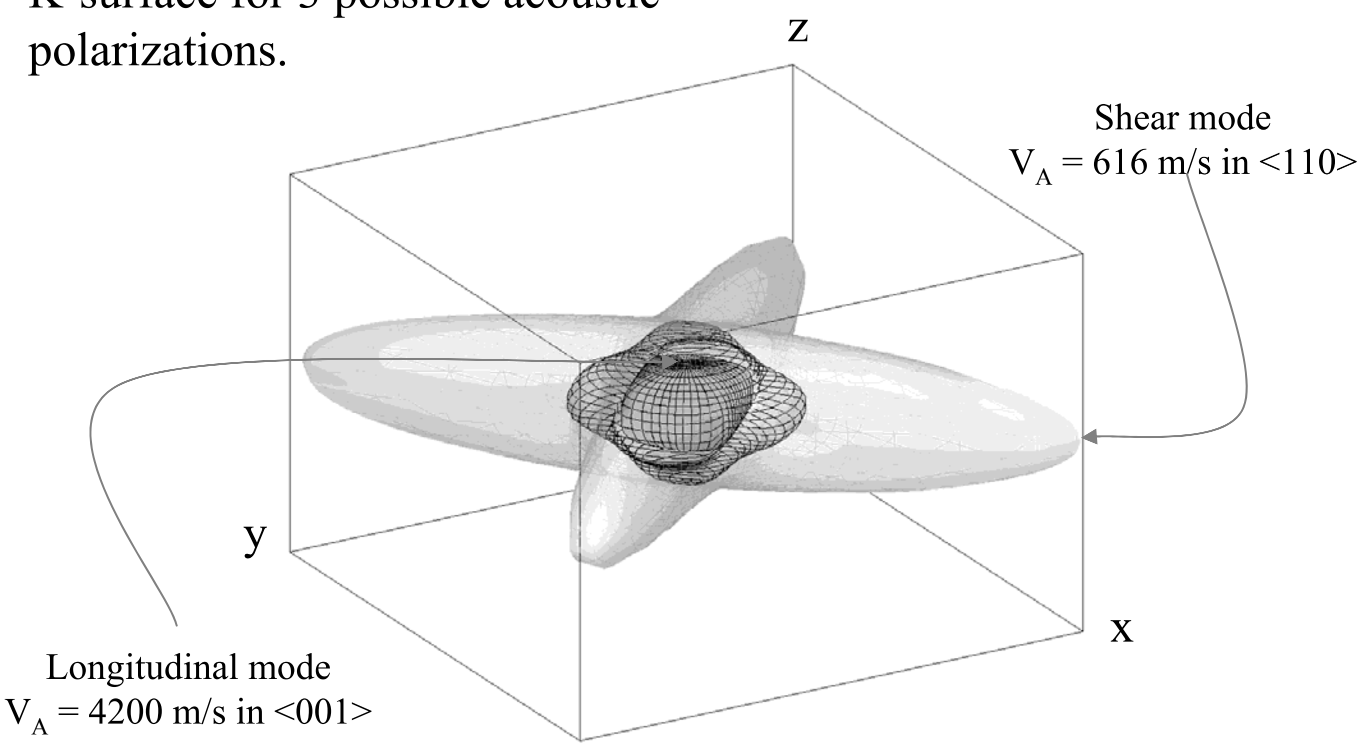
we can derive the second-order wave equation for acoustics:

$$\nabla \cdot \overline{\overline{\overline{c}}} : \nabla_s \vec{V} - \rho \frac{\partial^2 \vec{V}}{\partial t^2} = -\frac{\partial \vec{F}}{\partial t}$$

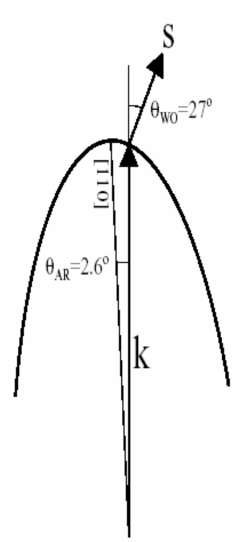
Since there is no acoustic equation analogous to Gauss' law, this wave equation supports longitudinal modes (3 polarizations).

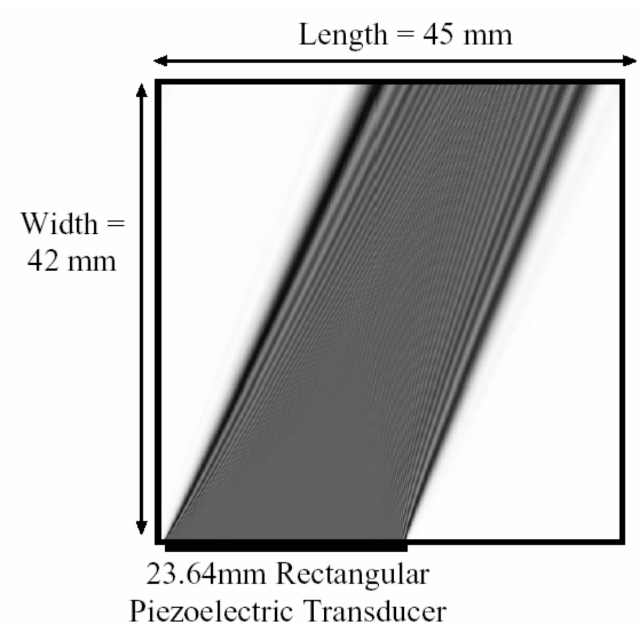
# Acoustic propagation
## in TeO2, a common AO material

K-surface for 3 possible acoustic polarizations.




Highly aniostropic k-surface causes large walkoff (27°) and excess diffraction (50x).



# AO interaction
## in 4 easy steps

$$\overline{\overline{S}}(\vec{r},t)=\frac{1}{2}W\,\hat{S}\cos\left(\Omega t-\vec{K}\cdot\vec{r}\right)$$

A traveling strain wave

$$\Delta\overline{\overline{\varepsilon}}=-\overline{\overline{\varepsilon}}_o\,\overline{\overline{\overline{\overline{p}}}}\,\overline{\overline{S}}\,\overline{\overline{\varepsilon}}_o$$

induces a dielectric perturbation via the elastooptic effect

$$\left(\Delta n\right)_{ij}=-\frac{n^3}{2}p_{ijkl}S_{kl}$$

which can be related to an index change

$$\left(\Delta n\right)_{ij}=-\frac{n^3}{2}W\,p_{ijkl}\hat{S}_{kl}\cos\left(\Omega t-\vec{K}\cdot\vec{r}\right)$$

resulting in a *moving* index grating.

Conservation of momentum gives *direction* of possible diffraction:

$$\vec{k}_d=\vec{k}_i\pm m\vec{K}$$

$m$ = # phonons destroyed or created

Conservation of energy gives *frequency* of possible diffraction:

$$\omega_d=\omega_i\pm m\Omega$$

Same $m$ as for momentum conservation!

# Direction of diffraction
## Bragg diffraction via k-space

$$\vec{k}^{\,0} + \vec{K}^{G} = \vec{k}^{\,1}$$ Conservation of k.

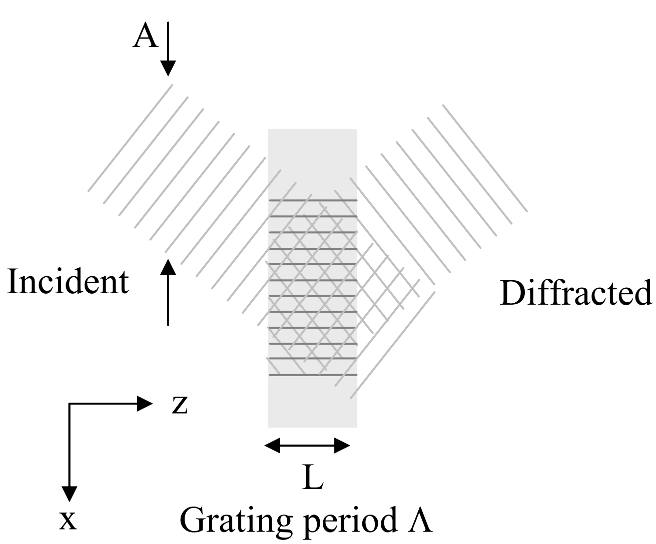
$$k_x^0 + K_x^G = k_x^1$$ Transverse to optical boundary

$$\frac{2\pi}{\lambda_0}\sin\theta^0 + \frac{2\pi}{\Lambda} = \frac{2\pi}{\lambda_0}\sin\theta^1$$ via Snell's law (all quantities *external* to material)

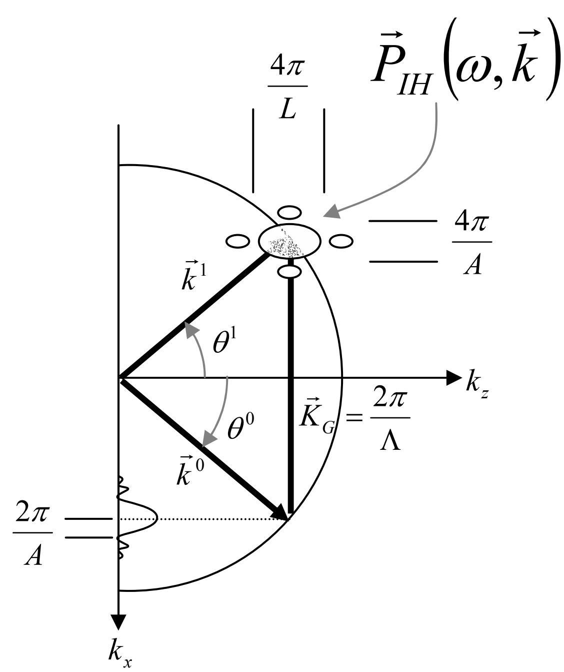
$$-\sin\theta^0 = \sin\theta^1 = \frac{\Lambda}{2\lambda_0}$$ Isotropic Bragg matching.

K-space picture including finite beams and finite transducer length:

**Real space**




**Fourier space**



# Efficiency of diffraction
## in an isotropic material

At Bragg incidence (no momentum mismatch), coupled-modes gives:

$$\eta = \sin^2\left(\frac{\Delta\phi}{2}\right) = \sin^2\left(\frac{k_0 \Delta OPL}{2}\right) = \sin^2\left(\frac{k_0 \Delta n\, L}{2\cos\theta}\right)$$

*OPL*=optical path length

From the previous slide, we know that:

$$\Delta n = -\frac{n^3 p\, S}{2}$$

effective elastooptic coefficient *p*, strain magnitude *S*

The acoustic energy density is given by:

$$\frac{\rho V^2 S^2}{2} \quad \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$$

material density $\rho$, acoustic velocity *V*

and thus the average energy flow (=the acoustic power) is given by

$$P_a = VHL\frac{\rho V^2 S^2}{2} \quad [\mathrm{W}]$$

transducer area $A = H\,L$

so the peak diffraction efficiency (@ Bragg matching) is given by

$$\eta = \sin^2\left(\frac{\pi}{\lambda_0 \cos\theta}\sqrt{\frac{M_2 L}{2H}P_a}\right) \approx \left(\frac{\pi}{\lambda_0\cos\theta}\right)^2 \frac{M_2 L}{2H} P_a$$

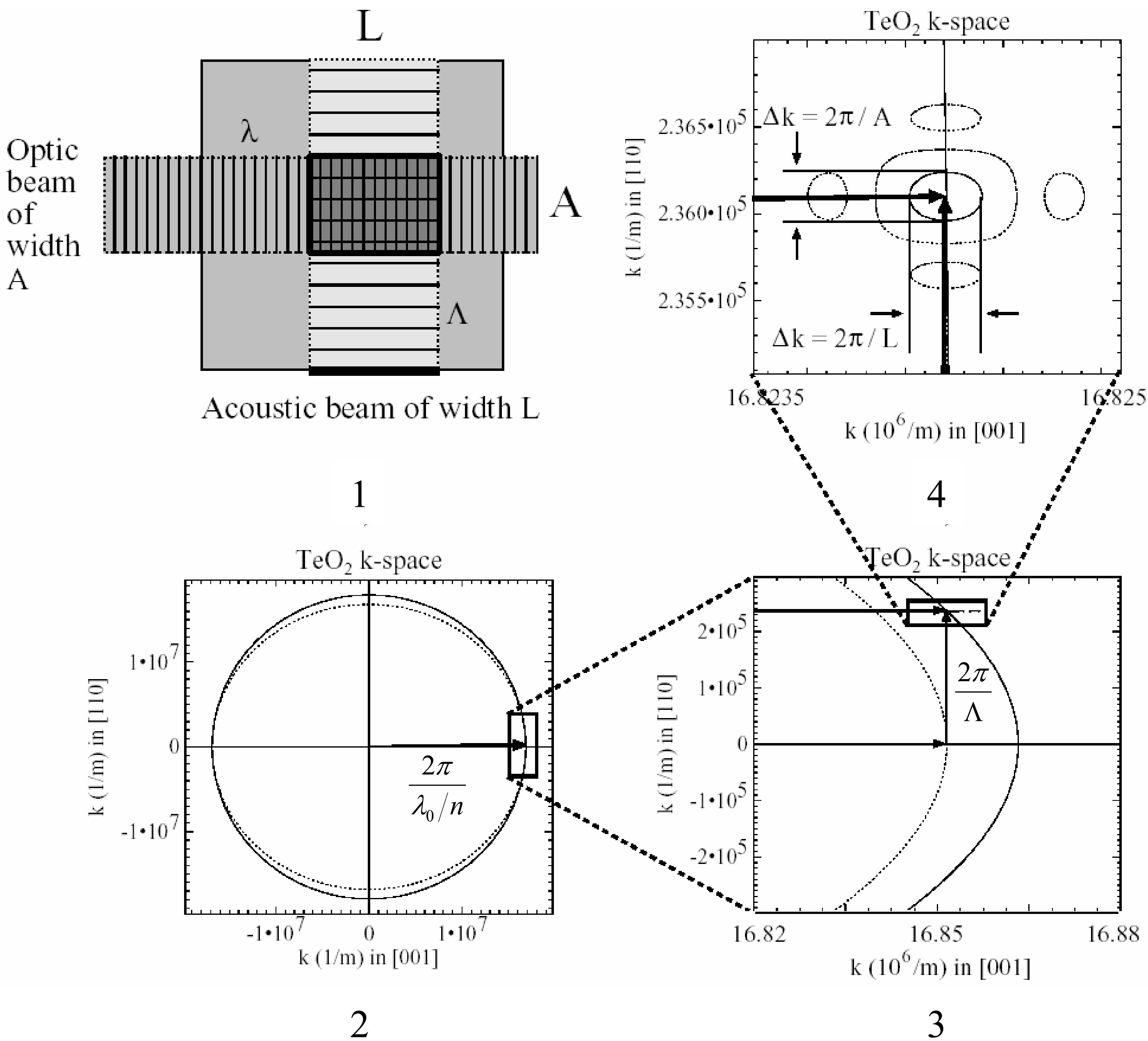
for low efficiency

$$M_2 \equiv \frac{n^6 p^2}{\rho V^3}$$

Acoustooptic figure of merit

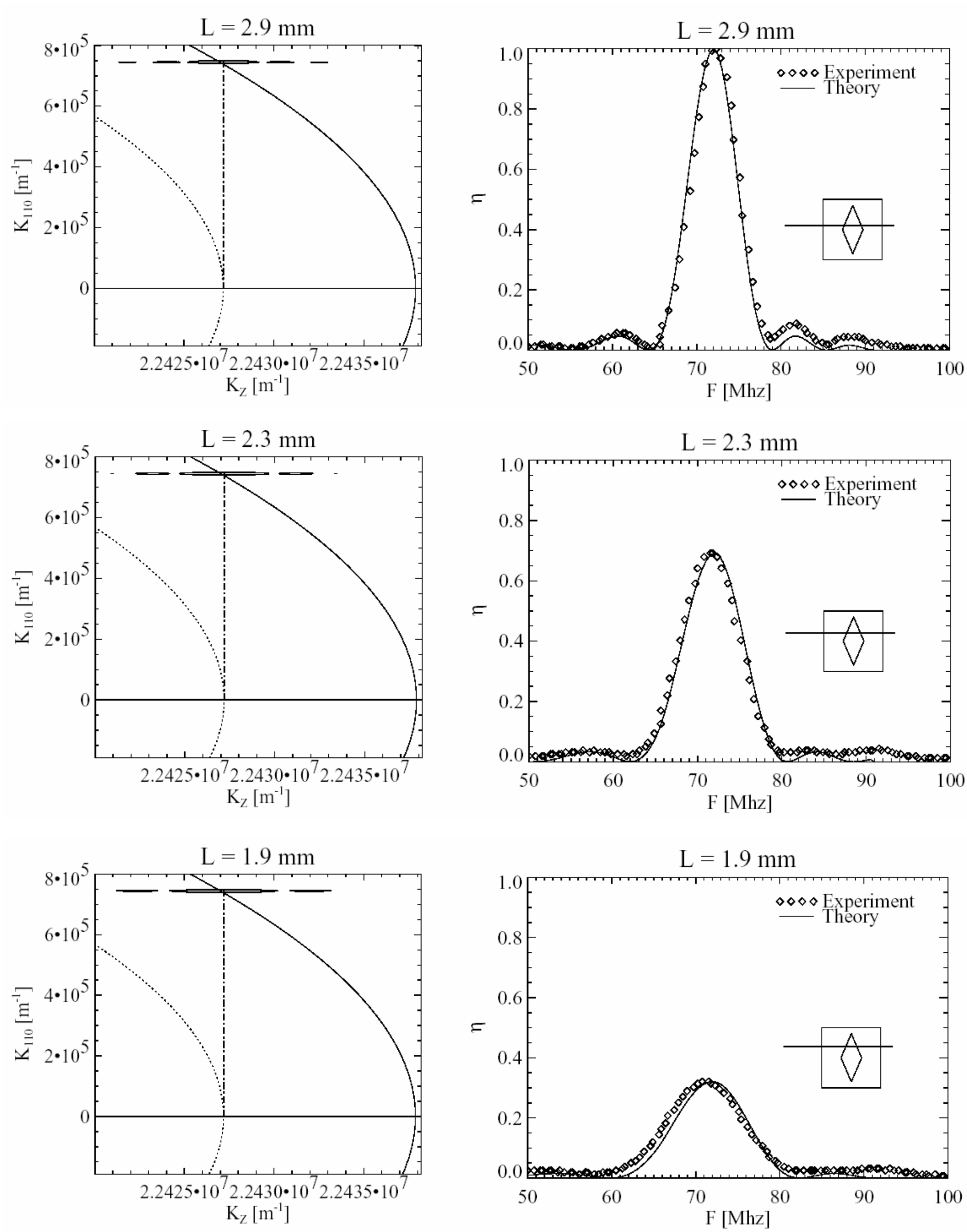
# Example

## $TeO_2$ AO deflector (backwards)



# Measured band shapes

## vs. transducer length, L



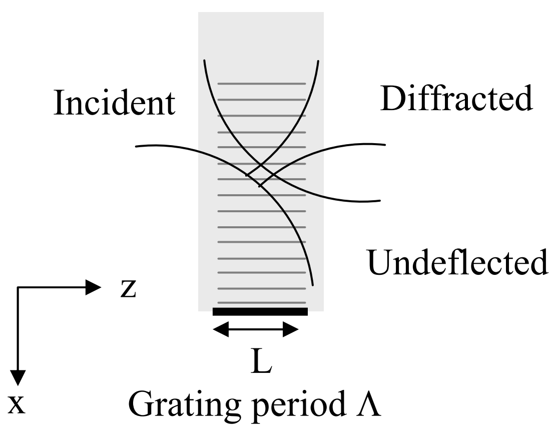
# AO modulators

## Separation of beams

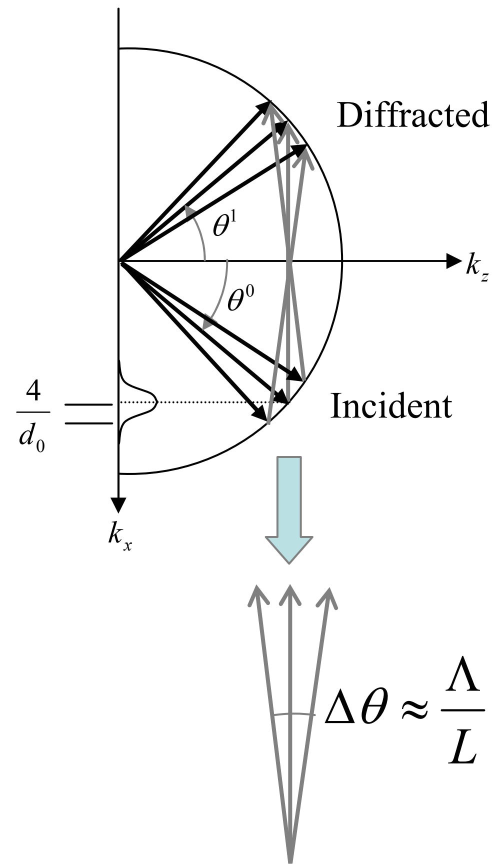
We clearly wish to minimize the transit time $\tau = d_0/V$
Thus we want small beam diameter *d0* and large velocity *V*.

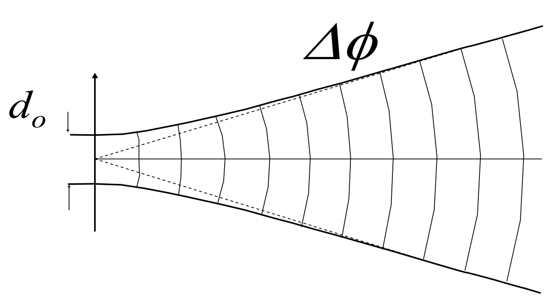
**Real space**

Gaussian beam of diameter $d_0$




**Fourier space**




Optic angular spectrum

Acoustic angular spectrum

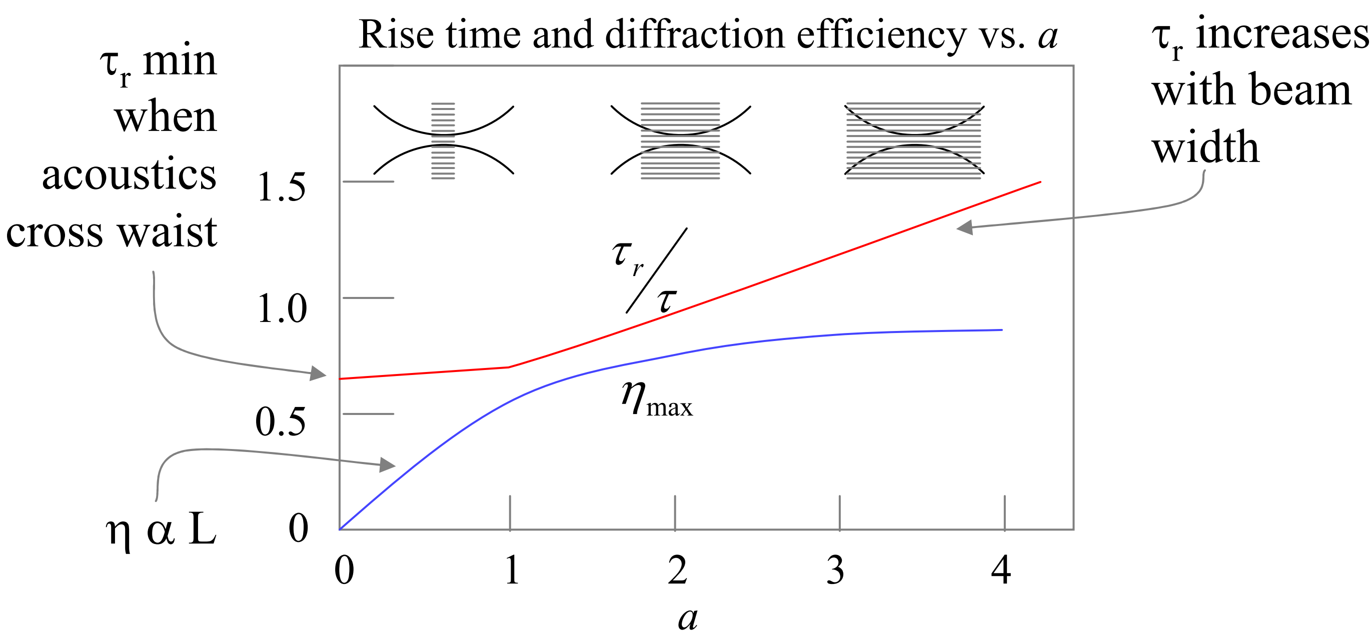
To separate undeflected from diffracted by *N* times the $1/e^2$ angle:

$$K \geq N\frac{8}{d_0} \quad \text{or} \quad d_0 \geq N\frac{4}{\pi}\Lambda \approx N\Lambda$$

# AO modulators

## Balance of design constraints

What is the optimal value of $a \equiv \dfrac{\Delta\phi}{\Delta\theta}$ ?




For $a \ll 1$, $\tau_r \approx 0.65\dfrac{d_0}{V}$ and $\eta_{max} \ll 1$ or fast but inefficient.

For $a = 1.5$, $\tau_r \approx 0.85\dfrac{d_0}{V}$ and $\eta_{max} \approx \dfrac{1}{2}$ which is a good balance.

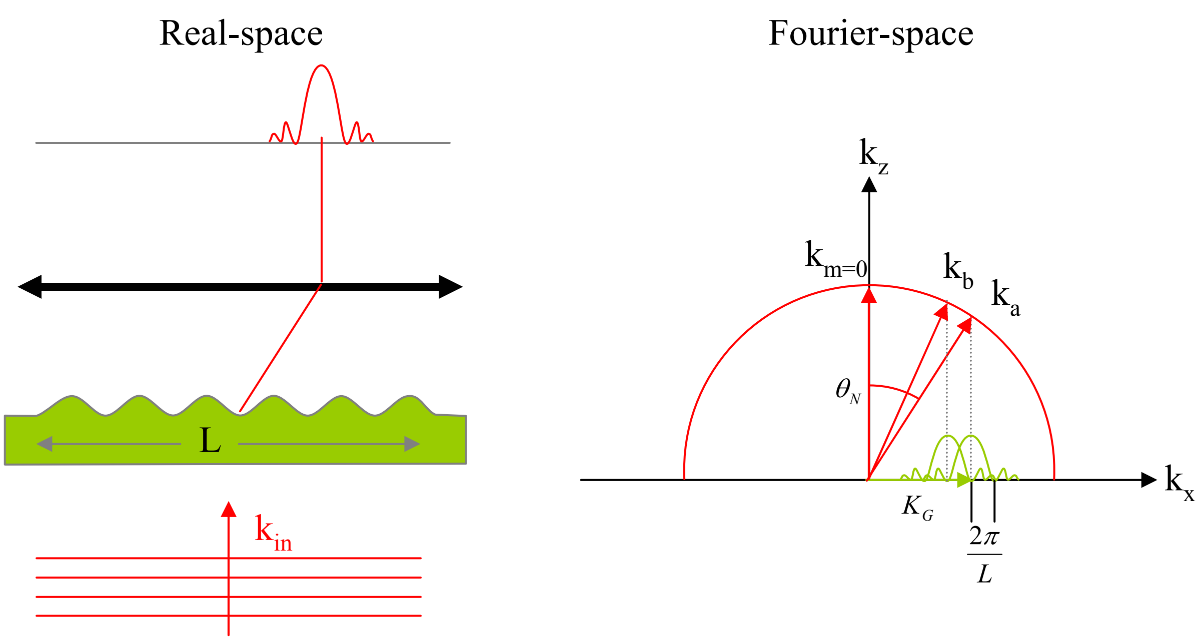
For $a\backslash >> 1$, $\tau_r$ increases and $\eta_{max} \to 1$ or slow but efficient.

Thus, to balance fast rise time and high efficiency, modulators typically operate near $a$=1.5

Xu and Stroud, Acousto-Optic Devices, Wiley,1992

# AO deflectors

## Primary specification = # spots




$$\theta_{a-null} = \theta_{b-peak}$$

Rayleigh resolvability criterion

$$\sin^{-1}\frac{2\pi/\Lambda_a - 2\pi/L}{k} = \sin^{-1}\frac{2\pi/\Lambda_b}{k}$$

$$\frac{L}{\Lambda_a} - 1 = \frac{L}{\Lambda_b}$$

$$N_b = N_a + 1$$

Diffracted spot moves resolvable angle when # grating lines +=1

$$N = \frac{L}{\Lambda_{\min}} - \frac{L}{\Lambda_{\max}} = L\left(f_{s-\max} - f_{s-\min}\right) = LB_s$$

Space bandwidth prod

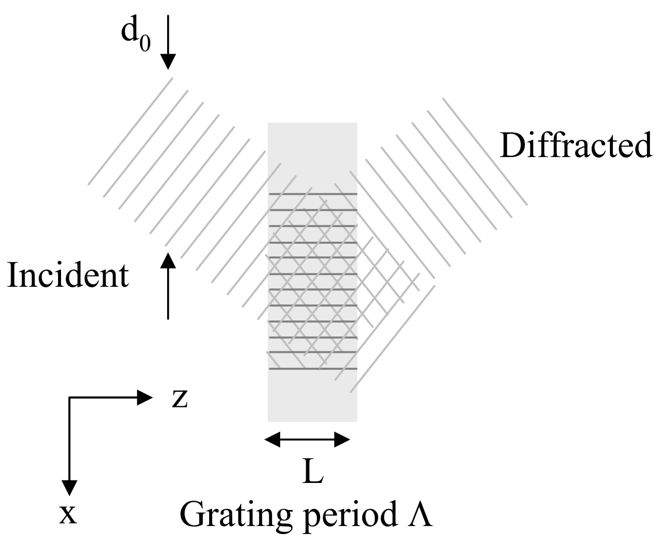
$$N = \frac{L}{\Lambda_{\min}} - \frac{L}{\Lambda_{\max}} = \frac{L}{V}\left(f_{t-\max} - f_{t-\min}\right) = \tau B_t$$

Time bandwidth prod
$B_t$ <= octave, typ.

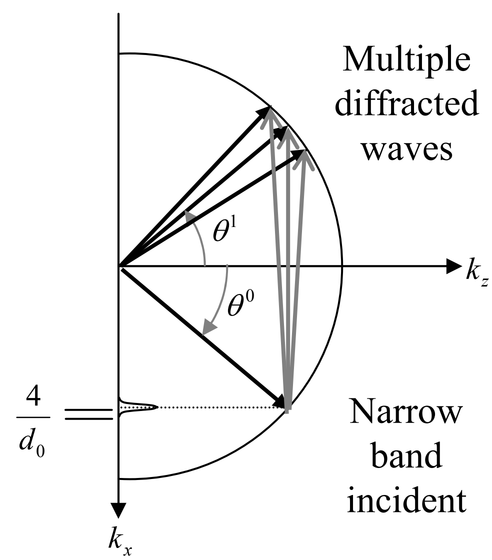
# AO deflectors
## Achieving many resolvable spots

- To achieve large space bandwidth product we want:
  - Largest possible incident beam diameter, $d_o$ and
  - smallest acoustic wavelength $\Lambda$ (=slow wave).
  - Complete opposite of modulator requirements!
- $d_0$ is limited by acoustic loss, $\Lambda$ by available modes of material

**Real space**



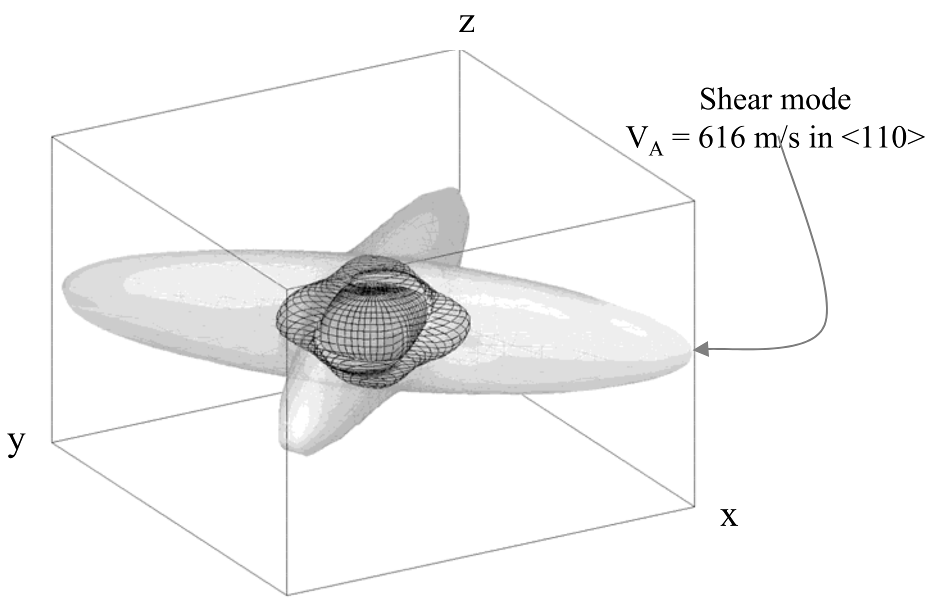

**Fourier space**



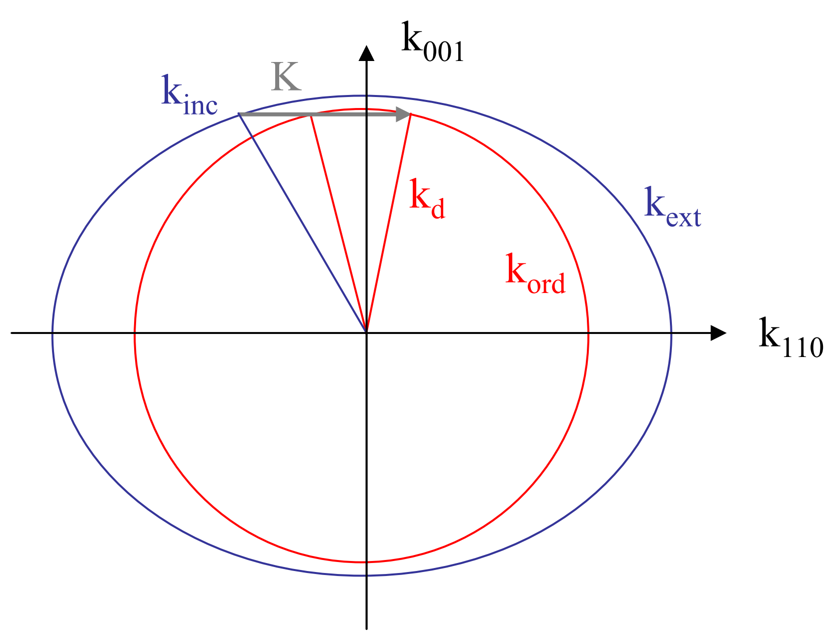

- These conditions are equivalent to $a$<<1 and thus the efficiency will be very low (most of the acoustic energy is wasted for any one deflection angle).
- So how can we increase $L$ and yet Bragg match over a large set of diffracted angles?

# Birefringent AODs

Acoustics propagated in <110> to take advantage of anomalously slow shear mode:

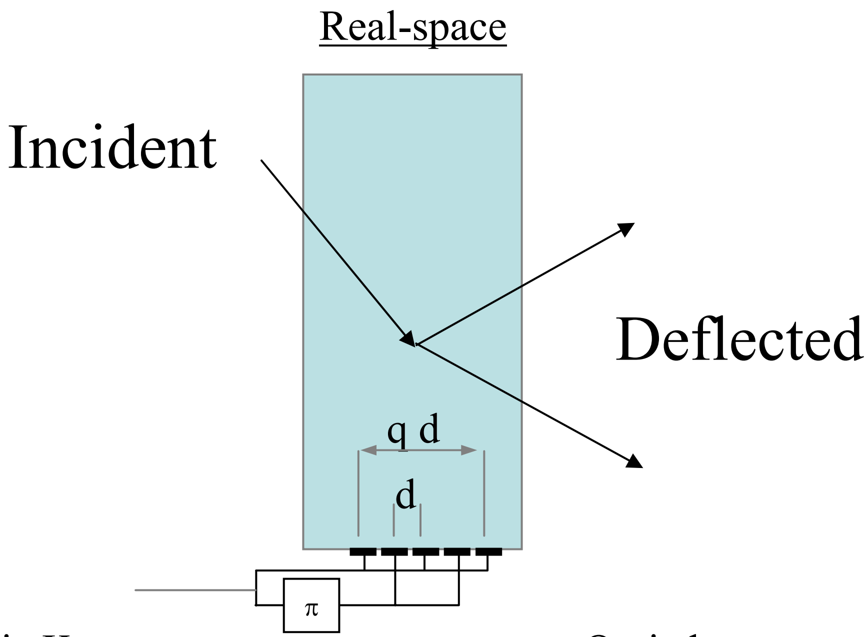


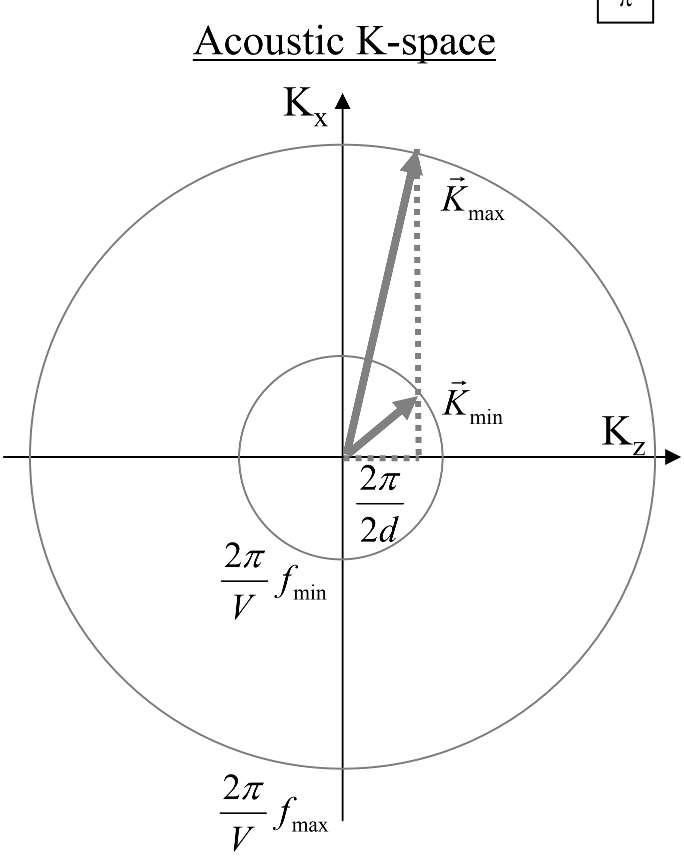

Optical activity of $TeO_2$ splits surfaces at optical axis.
This enables *tangential matching* of K to diffracted optical k surface:

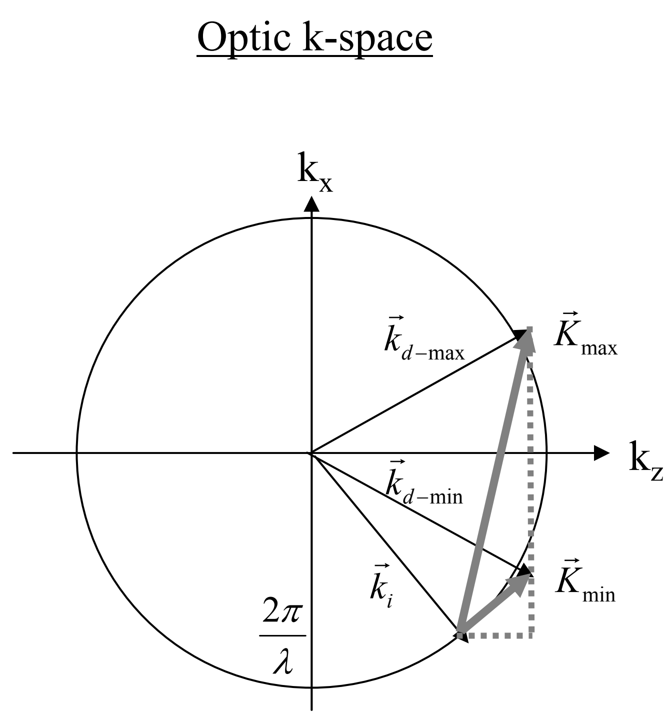


# Phased-array deflectors

## Tangential matching in isotropic material



# Bandshape of phased-array AOD