1. The following questions are variations from an exam. For each sequence, compute the limit if it exists. Also indicate which (if any) of the following convergences apply:

- sure
- almost sure
- in probability
- mean square
- in distribution

(a) \( X[n] \sim N(3, \frac{1}{n}) \).
(b) \( X[n] \) is uniformly distributed in \([-\frac{1}{n}, \frac{1}{n}]\).
(c) \( X[n] = \frac{\sum_{i=1}^{n} Y_i}{n} \), where the \( Y_i \) are i.i.d. with mean \( \mu_y \) and variance \( \sigma_y^2 \).
(d) \( X[n] = \frac{\sum_{i=1}^{n} (Y_i - \mu_Y)}{\sigma_Y \sqrt{n}} \), where the \( Y_i \) are i.i.d. with mean \( \mu_Y \) and variance \( \sigma_Y^2 \).
(e) \( X[n] = n!n^{-\alpha n^2} \), where \( \alpha \) is uniform in \([0, 1]\).

To show the relationship between these examples, sketch the set diagram relationship between different types of convergence, and indicate where (a), (b), etc. falls in the diagram.

2. This problem analyzes a modified version of the queueing for the coffee shop in the engineering lobby. In this version the engineers are impatient and are less likely to join the line the longer the line. When there are no customers, the average time until the next customer arrives is 1 minutes. When there are \( n \) customers, the average time until the next customer arrives is \( n + 1 \) minutes (or equivalently the arrival rate is \( \frac{1}{n+1} \) per minute). The average time to serve a customer is exponentially distributed with mean 2 minutes. A picture may be helpful.

(a) Compute the probability that there are \( n \) customers at the coffee shop.
(b) Is this system stable? If so, what is the average number of customers in the system. (Hint: part (a) may resemble a standard distribution)

3. In this problem you will test the hypothesis that data traffic is well modeled by a Poisson process. You will look at ethernet data traces over one hour on a busy ethernet segment. These traces were recorded at Bellcore (now known as Telcordia) in 1989 and are known as the August89 Busy data trace. The data can be downloaded from the class web page The file consists of two columns. The first is the time the packet was measured at the recording device (in seconds), and the second column is the number of bytes in the packet including headers. We will only use the timing information here.
(a) Plot the average packets per second when you average over 0.01 sec, 1 sec, and 1 minute intervals. Measure in consecutive jumping windows and plot the results for each time scale. Do not plot more than 200 averages in a plot (i.e. the 0.01 sec plot will only contain 2sec worth of data).

(b) Now, fit an exponential distribution to the packet interarrival times. What is the average arrival rate and interarrival time? Generate one hour worth of packets using this distribution. Repeat the plots from part (a). For each time scale, plot the part (a) and part (b) plots on the same graph.

(c) What can you say qualitatively about the real traffic versus the simulated traffic? Which is more bursty? Which tends to average out as we increase the averaging period?

Extra Credit: In the last problem, make two Q-Q plots for the trace data. One vs. the exponential distribution and a second vs. the Pareto distribution. Note there is a lot of data so you need to think how to manage the data.