1. Consider \( Z(t) = X(t) + N(t) \) where \( X(t) \) and \( N(t) \) are WSS and mutually uncorrelated with power spectral densities \( S_{XX}(\omega) \) and \( S_{NN}(\omega) \) and zero means. Let \( Y(t) = L\{Z(t)\} \) where \( L \) is a LTI system with impulse response \( h(t) \).
   
   (a) Compute the psd of the output \( Y(t) \).
   
   (b) Compute the cross-power spectral density of \( X \) and \( Y \), i.e. find \( S_{XY}(\omega) \) and \( S_{YX}(\omega) \).
   
   (c) Define the error \( \eta(t) \overset{\text{def}}{=} Y(t) - X(t) \) and compute the psd of \( \eta(t) \).
   
   (d) Define \( h(t) = a\delta(t) \) and compute the value of \( a \) which minimizes \( E[\eta^2(t)] = R_{\eta\eta}(0) \).

2. To estimate the size \( N \) of a fish population in a lake, \( k \) fish are caught, tagged, and released. Suggest estimators of \( N \) for each of the below methods. Indicate your estimators unbiasedness, and efficiency.
   
   (a) Catch \( n \) fish of which \( X \) are tagged.
   
   (b) Catch fish repeatedly with replacement until you catch \( X \) tagged fish (which happens in catch number \( n \)).

   Note \( X \) is the random variable in (a) and \( n \) is the random variable in (b). What assumptions are you making in each method?

3. Below are seven measurements of the ozone level (in ppm) taken at an environmental measuring station. Suppose that these have a normal distribution.

\[
0.06 
0.07 
0.08 
0.11 
0.12 
0.14 
0.21
\]

   (a) Find a 95% symmetric confidence interval for the mean, \( \mu \).
   
   (b) Find a 95% symmetric confidence interval for the variance, \( \sigma^2 \).

4. Let \( X[n] \) be a sequence of i.i.d. Gaussian variables with known mean \( \mu = 0 \) and unknown variance \( \sigma^2 \). Estimate the variance as \( \sigma^2[n] = \frac{1}{n} \sum (X[n])^2 \). Since the mean is known this is an unbiased estimate of the variance. We now use this estimate in the Gaussian density function \( f_X(x) \sim N(0, \sigma^2[n]) \). Because \( \sigma^2[n] \) is a random variable, so is the function \( f_X(x) \). Estimate the distribution of \( f_X(x) \) for large \( n \). For which \( x \) is this variation more significant, near the mean or in the tails? (Hint: this is confusing because we are asking for the variation in a density function. Unfortunately, this is a common case. We estimate distribution parameters and then use these in the distribution or density function. The estimator is a r.v. so the density which is a function of the estimator is a r.v. If it helps think of \( f_X \) as a function and the question is asking what is the distribution of the function of an estimator.)