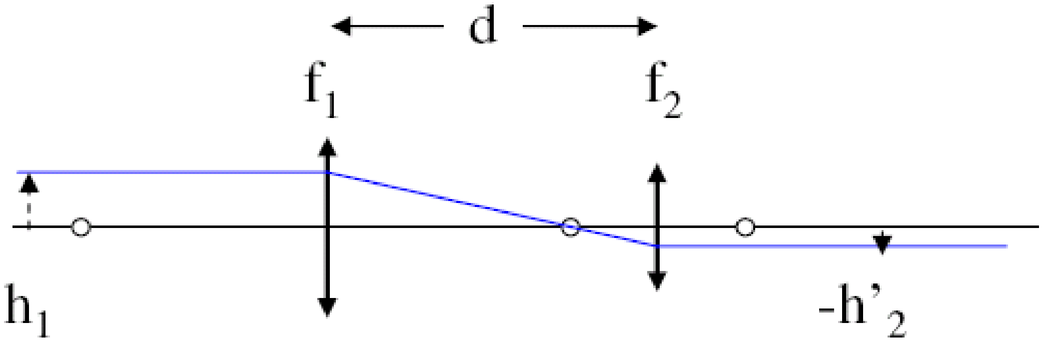
Instruments and the Optical Invariant

- Telescopes
- $M=1/M_\theta$
- Microscopes and eye pieces
- Camera
- Fiber Optics
- Spectrometers
- Optical Disk
    - media
    - laser isolation
    - optical head
    - optical system with servo signal
- Optical Invariant

Example: The Telescope

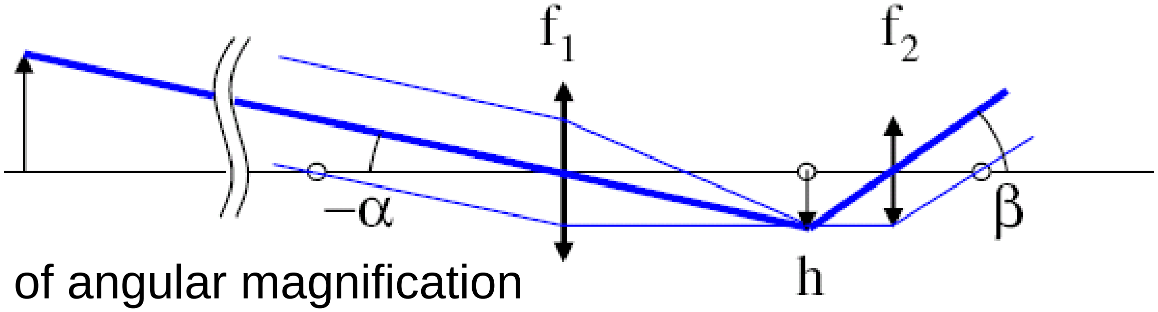
Keplerian

Shown in the *afocal* geometry (d=f1+f2). Relaxed eye focuses at ~1m, thus telescope are usually not afocal. Analysis simpler, however.



Afocal: system has no power: ray ǁ to OA does not intersect OA in image space

$$M \equiv \frac{h'_2}{h_1} = -\frac{f_2}{f_1}$$



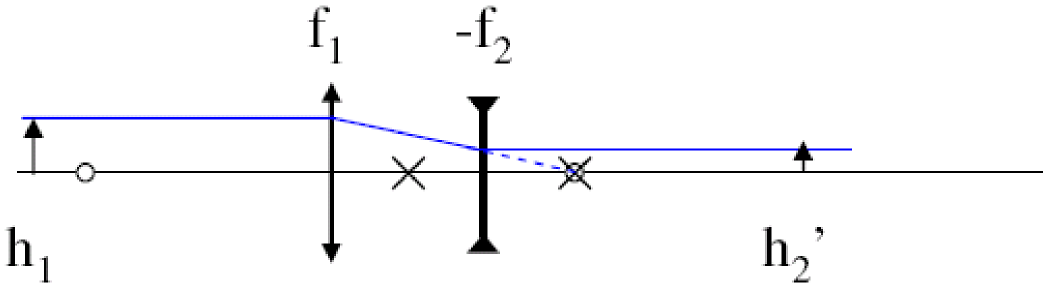
$$M_\theta \equiv \frac{\beta}{\alpha}$$ Definition of angular magnification

$$= -\frac{h/f_2}{h/f_1} = -\frac{f_1}{f_2}$$ Via similar triangles

$$= 1/M$$ This is both important and fundamental.

Example: The Telescope

Galilean



More compact, upright image. Same afocal condition: $d=f_1+f_2$

y	h_1	h_1	$h_1 - h_1(f_1+f_2)/f_1$	$h_1 - h_1(f_1+f_2))/f_1 = -h_2$
u	0	$-h_1/f_1$	$-h_1/f_1 – (h_1-h_1(f_1+f_2)/f_1)/f_2=0$	$-h_1/f_1 – (h_1-h_1(f_1+f_2)/f_1)/f_2$

$() =h_1f_2/f_1$

$-h2 = h_1f_2/f_1$

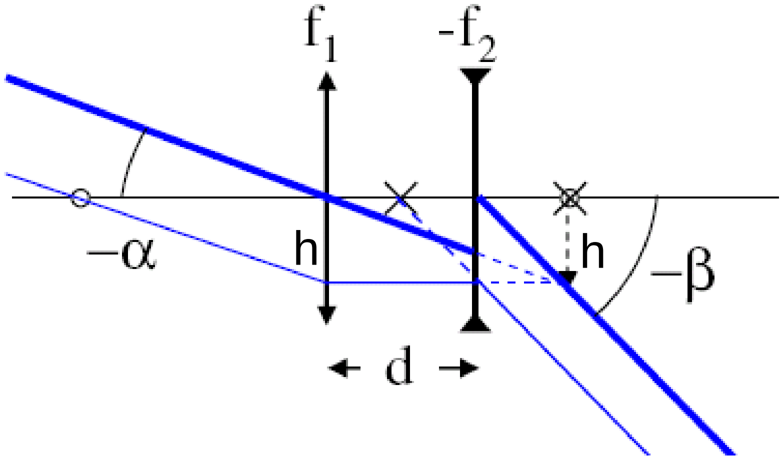
$$M = h_2/h_1 = -f_2/f_1$$

But f_2 is negative so $M>0$

Example: The Telescope

Galilean

$$M \equiv \frac{h_2'}{h_1} = -\frac{f_2}{f_1}$$



$$M_\theta \equiv \frac{\beta}{\alpha}$$

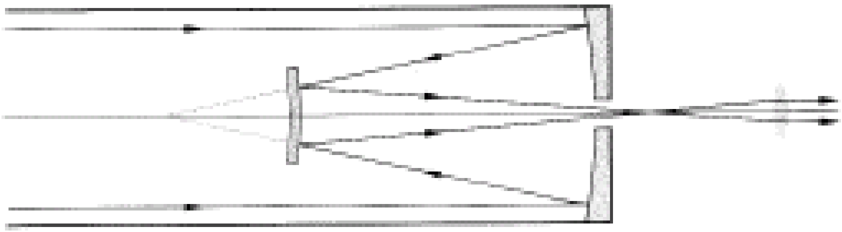
$$= \frac{h/f_2}{h/(-f_1)} = -\frac{f_1}{f_2}$$

$$= 1/M$$

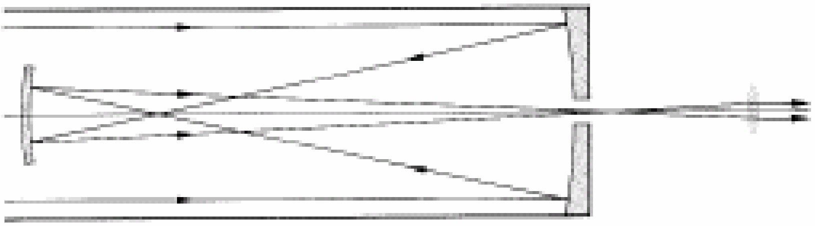
Note that formula is identical to Keplerian.
This is the advantage of the sign convention.

Reflective Telescopes

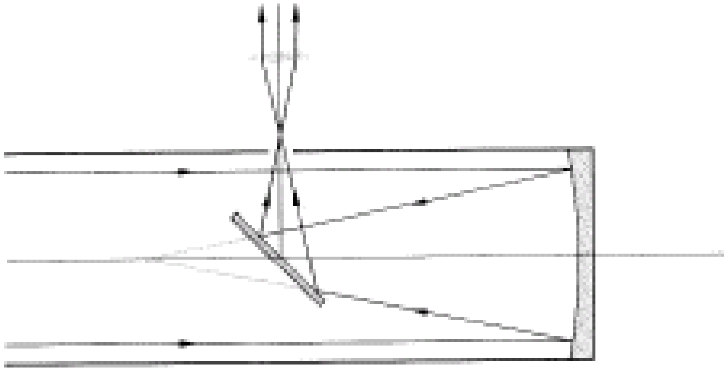
Magnification same as Keplerian / Galilean



Cassegrainian (d)



Gregorian (c)



Newtonian (b)

Chromatic aberration is very small with mirrors, transmission can be very high, light weight

Magnifier - through angles

Useful for infinite conjugates

$$M_v \equiv \frac{\beta}{\alpha}$$

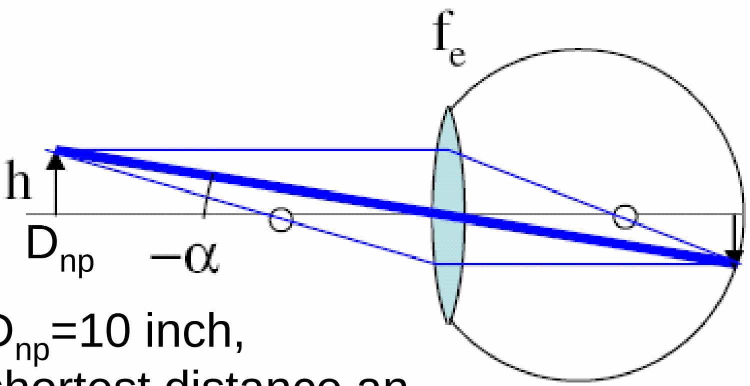
For a equal focal lengths, fe, visual magnification should be proportional to ratio of angles

$$= \frac{D_{np}}{-t_M}$$

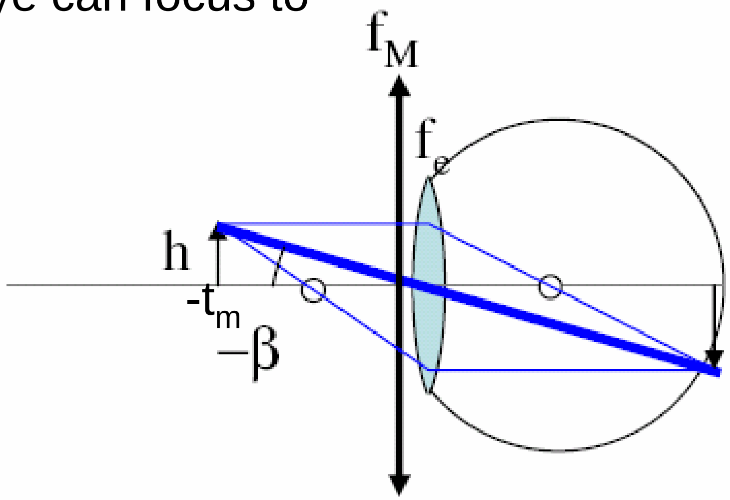
Via similar triangles

$$\boxed{= 1 + \frac{D_{np}}{f_M}}$$

via lens power equation

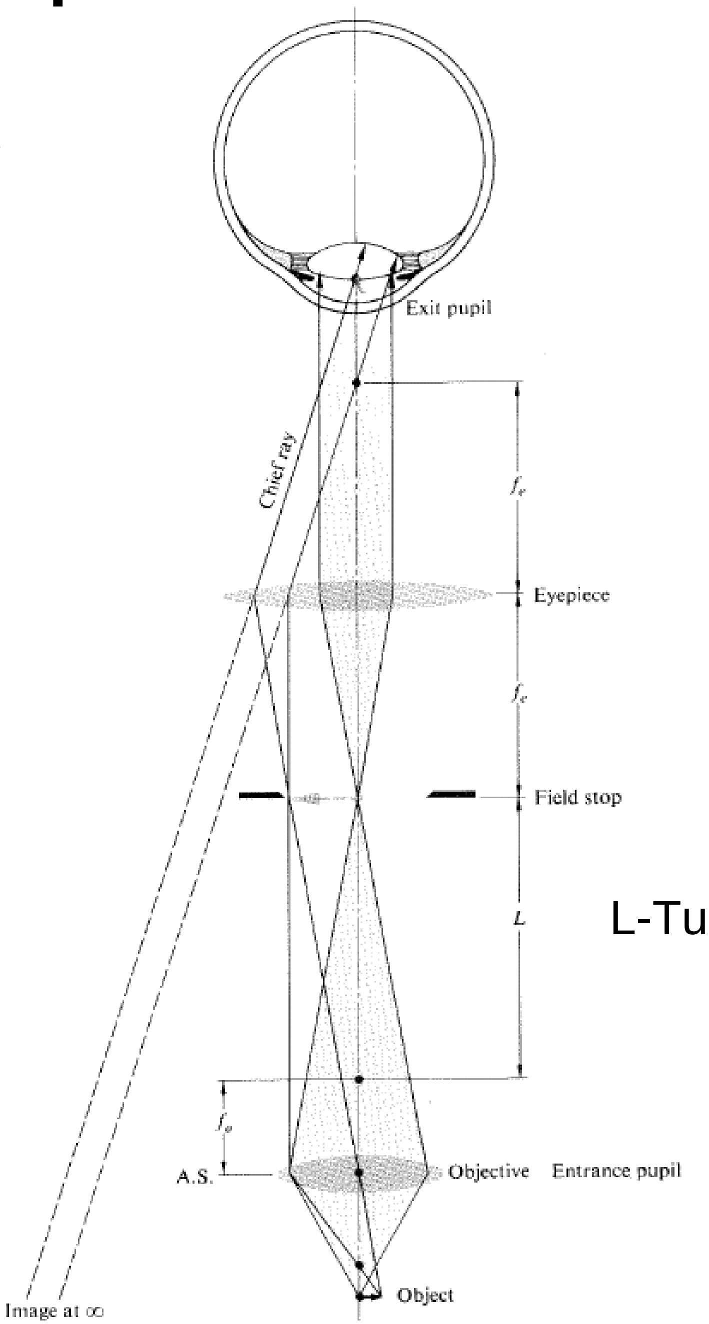


D_{np}=10 inch, shortest distance an eye can focus to



For an object at infinity, this becomes $M_v \equiv \frac{D_{np}}{f_M}$

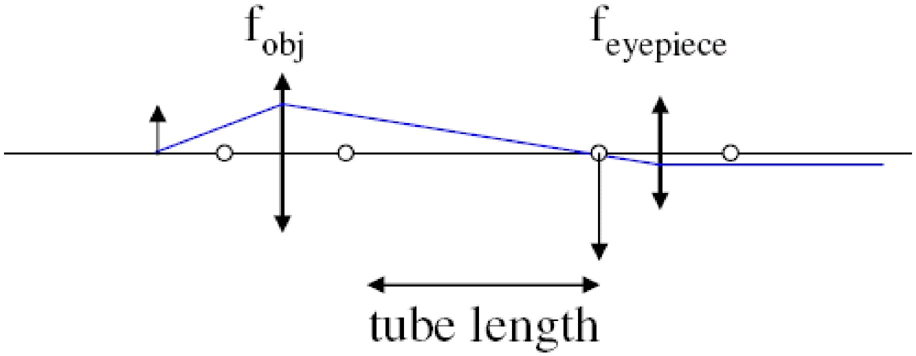
Compound Microscope



Focus by moving object relative to both lenses and stop

L-Tube length

Microscope



Focal system. Form image at infinity for simplicity of analysis.

Standard tube length is 160 mm.

Standard Near Point is 10 inches (254mm)

Visual magnification is product of linear mag of objective and mag of eyepiece:

$$M_{v-microscope} = M_{obj} M_{v-eyepiece}$$

$$= \left(-\frac{l_{tube}}{f_{obj}} \right) \left(\frac{D_{np}}{f_{eyepiece}} \right)$$

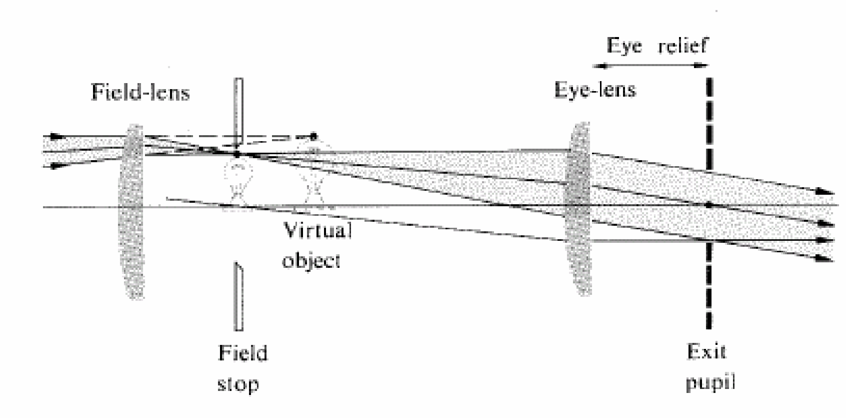
Remember $M=-x_i/F$ (thin lens)

Note eq.s are approximate $l_{tube} >> f_{obj}$, $D_{np} >> f_{eyepice}$

M_{obj}	f_{obj} [mm]	Typical NA
4	30	0.10
10	16	0.25
20	8	0.40
60	3	0.85
100	1.8	1.3

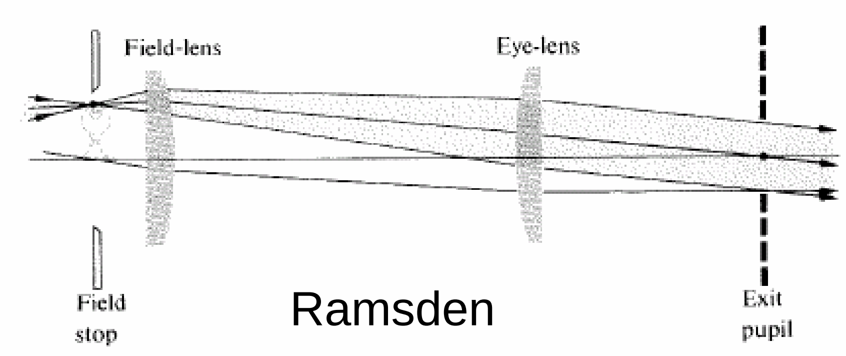
Eye Pieces

Used with microscope and telescopes



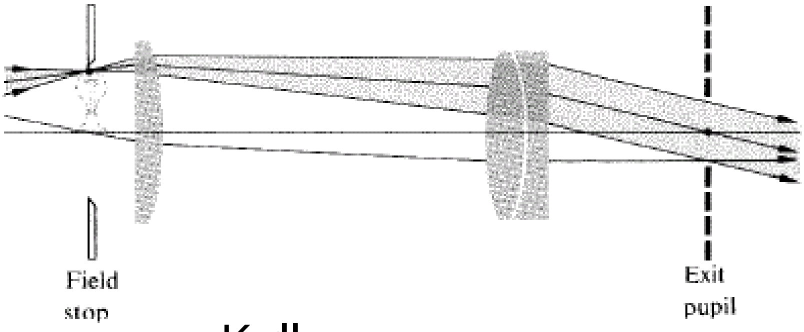
Huygens

Cheap but bad eye relief



Ramsden

Cheap but better eye relief (common)



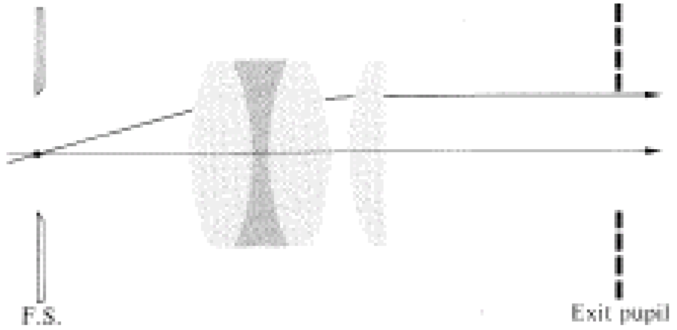
Kellner

Achromatic Ramsden, wide field

If standard NP=10 inches
a 10x eye piece would
have a F=1 inch

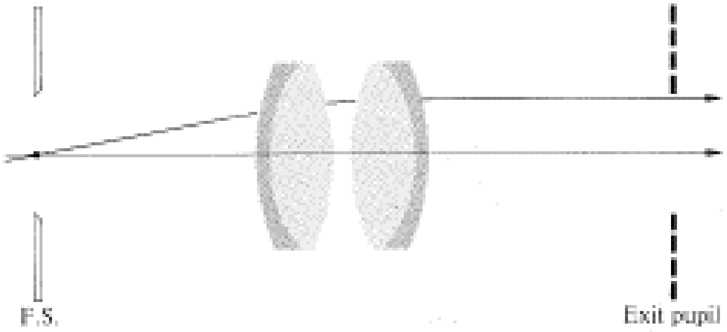
Can get Zemax examples

More Eye Pieces



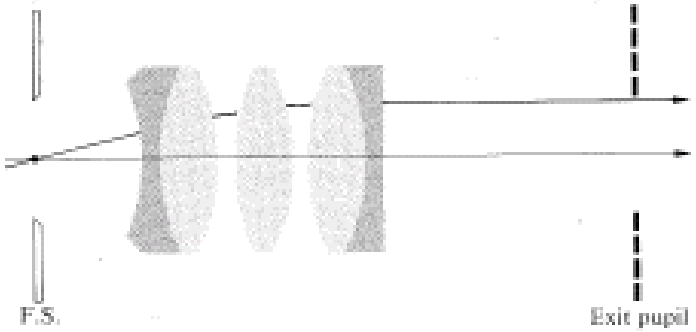
Orthoscopic

Better image quality



Plossl (symmetrical)

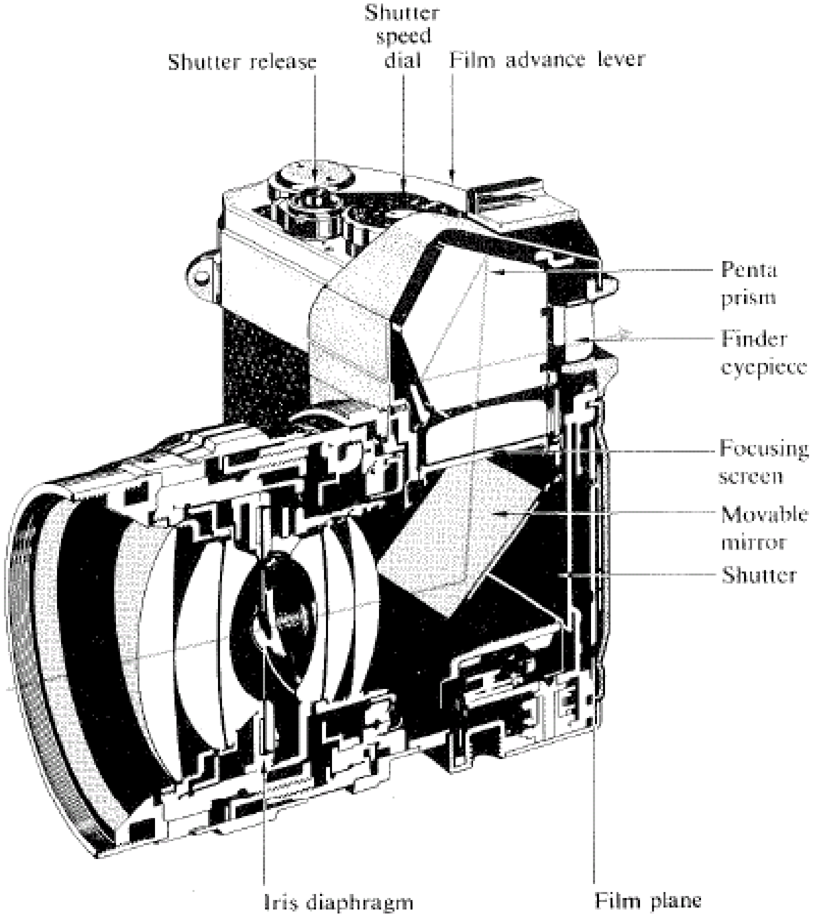
Better image quality over field



Erfle

Most common wide field EP

Camera



SLR – Single Lens Reflex Camera

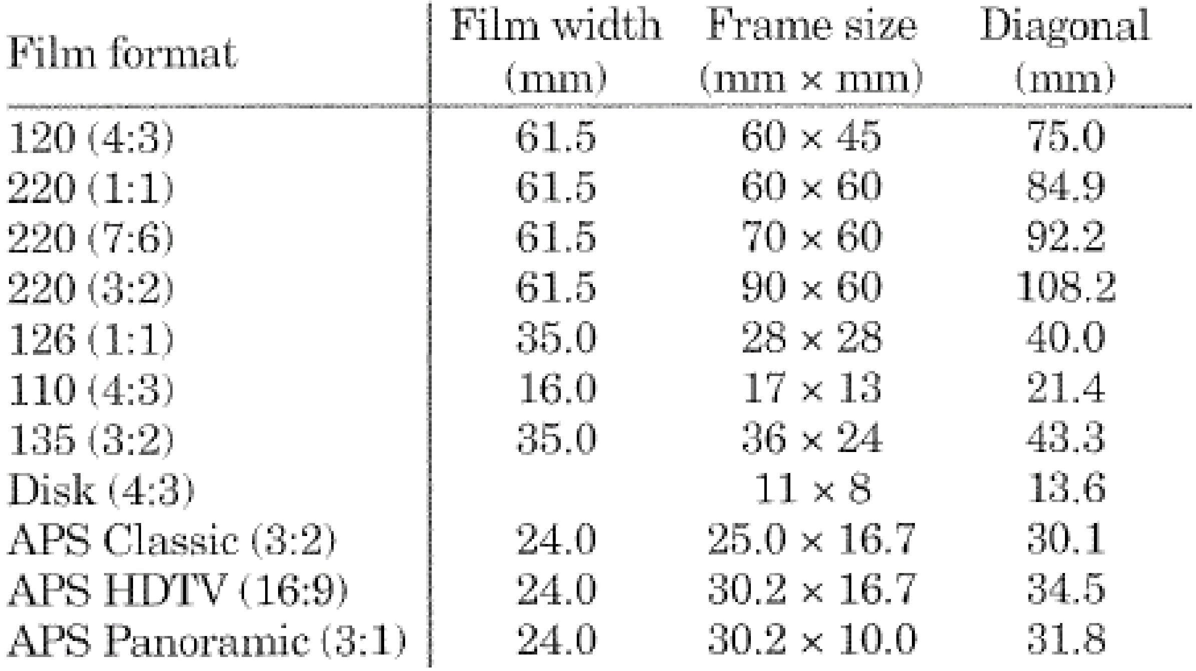
35mm Camera

- wide range of F available cheaply
- 46.5mm from mount to film plane
- Nikon, Canon major vendors
- Zeiss, Leica, Tamron, Tokina, Scheider, etc
- Image size: 24 mm×36 mm.

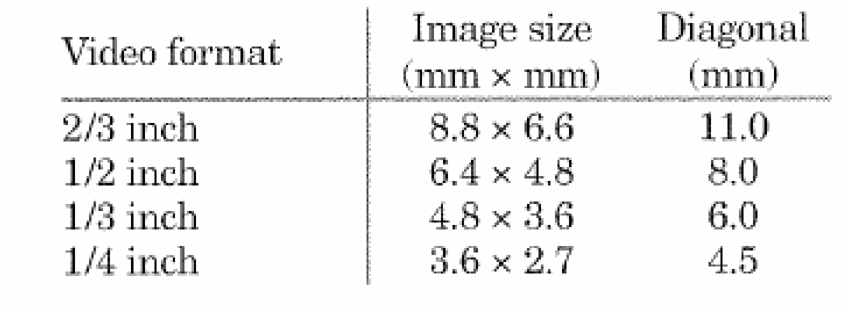
Medium and Large Format Camera

Hasselblad (Zeiss), Mammia
56.5 x 56.5mm film sizes plus
110mm F2, Hasselblad ~ 5-6k
BFL=74.9mm

Film (Field) sizes

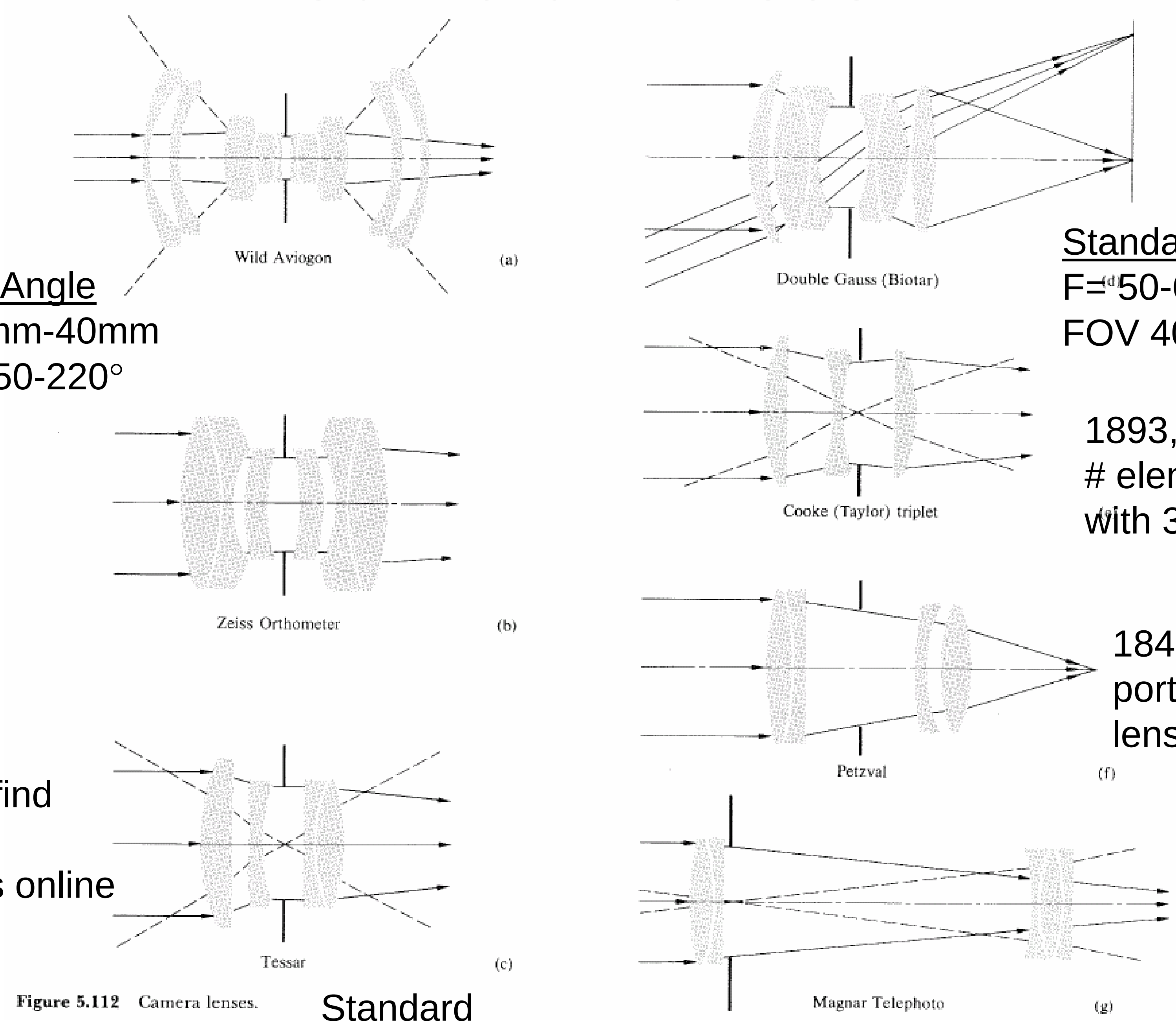


Film format	Film width (mm)	Frame size (mm × mm)	Diagonal (mm)
120 (4:3)	61.5	60 × 45	75.0
220 (1:1)	61.5	60 × 60	84.9
220 (7:6)	61.5	70 × 60	92.2
220 (3:2)	61.5	90 × 60	108.2
126 (1:1)	35.0	28 × 28	40.0
110 (4:3)	16.0	17 × 13	21.4
135 (3:2)	35.0	36 × 24	43.3
Disk (4:3)		11 × 8	13.6
APS Classic (3:2)	24.0	25.0 × 16.7	30.1
APS HDTV (16:9)	24.0	30.2 × 16.7	34.5
APS Panoramic (3:1)	24.0	30.2 × 10.0	31.8



Video format	Image size (mm × mm)	Diagonal (mm)
2/3 inch	8.8 × 6.6	11.0
1/2 inch	6.4 × 4.8	8.0
1/3 inch	4.8 × 3.6	6.0
1/4 inch	3.6 × 2.7	4.5

Camera Lenses



Wide Angle
F= 6mm-40mm
FOV 50-220°

Standard
F= 50-65mm
FOV 40-50°

1893, Fewest # elements with 3rd ab 0

1840, portrait lens

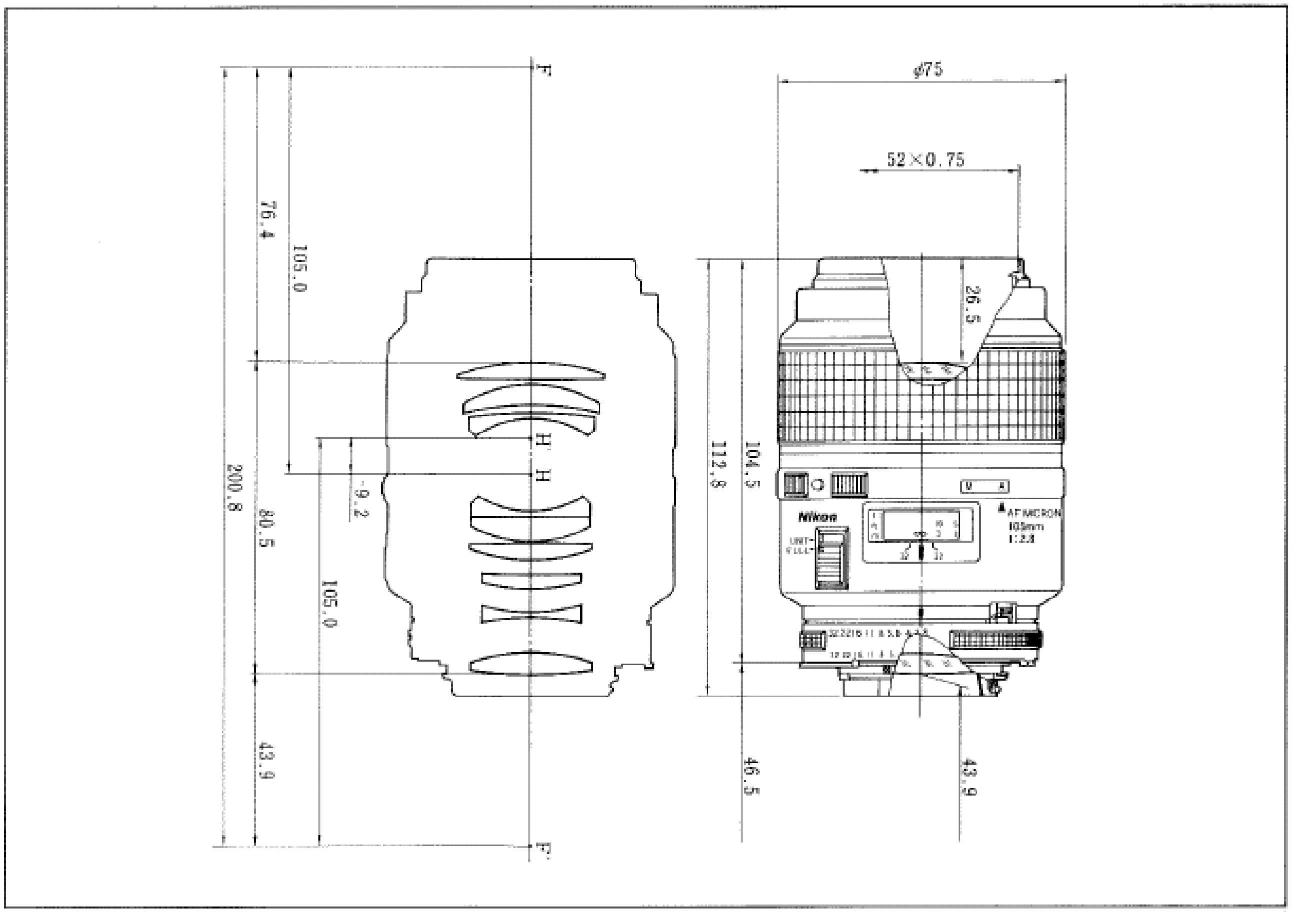
You can find Zemax examples online

Figure 5.112 Camera lenses.

Standard

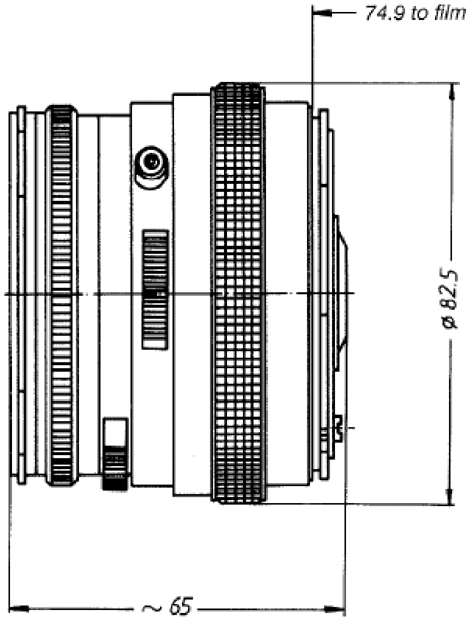
Nikon AF Micro-Nikkor

105mm f/2.8



unit: mm

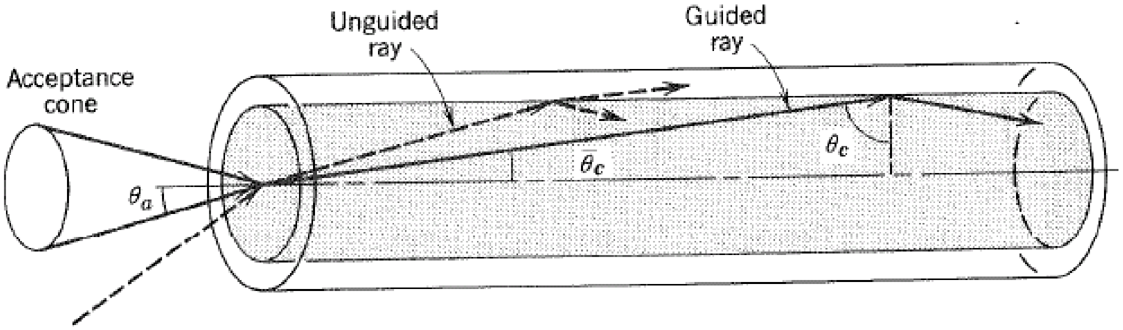
Hasselblad 80mm, f/2.8

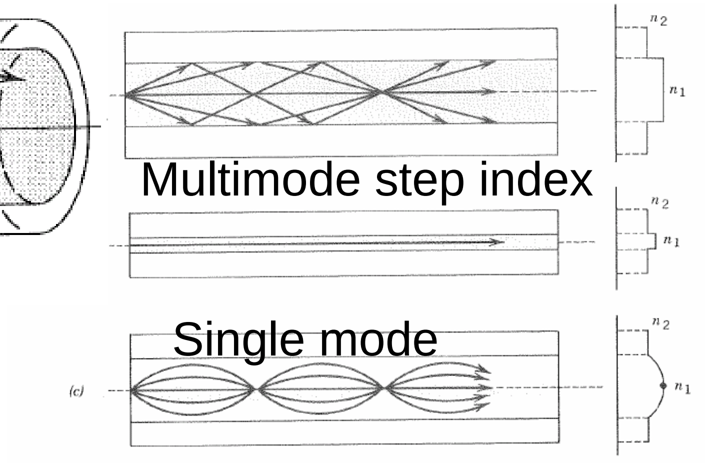


Number of lens elements:	7
Number of components:	5
f-number:	2.8
Focal length:	80.5 mm
Negative size:	56.5 x 56.5 mm
Angular field 2 w:	diagonal 52°, side 38°
Spectral range:	visible spectrum
f-stop scale:	2.8–4–5.6–8–11–16–22
Mount:	Prontor CF shutter
Filter mounting:	bayonet for Hasselblad series 60

Distance range:	∞ to 0.9 m
Position of entrance pupil:	26.6 mm behind the first lens vertex
Diameter of entrance pupil:	28.8 mm
Position of exit pupil:	25.7 mm in front of the last lens vertex
Diameter of exit pupil:	34.5 mm
Position of principal plane H:	39.0 mm behind the first lens vertex
Position of principal plane H′:	10.8 mm in front of the last lens vertex
Distance between first and last lens vertex:	46.4 mm

Fiber Optics





Multimode step index

Single mode

Multimode gradient index

What is the NA that the fiber can accept/ send out ? Core is n_1 and cladding is n_2

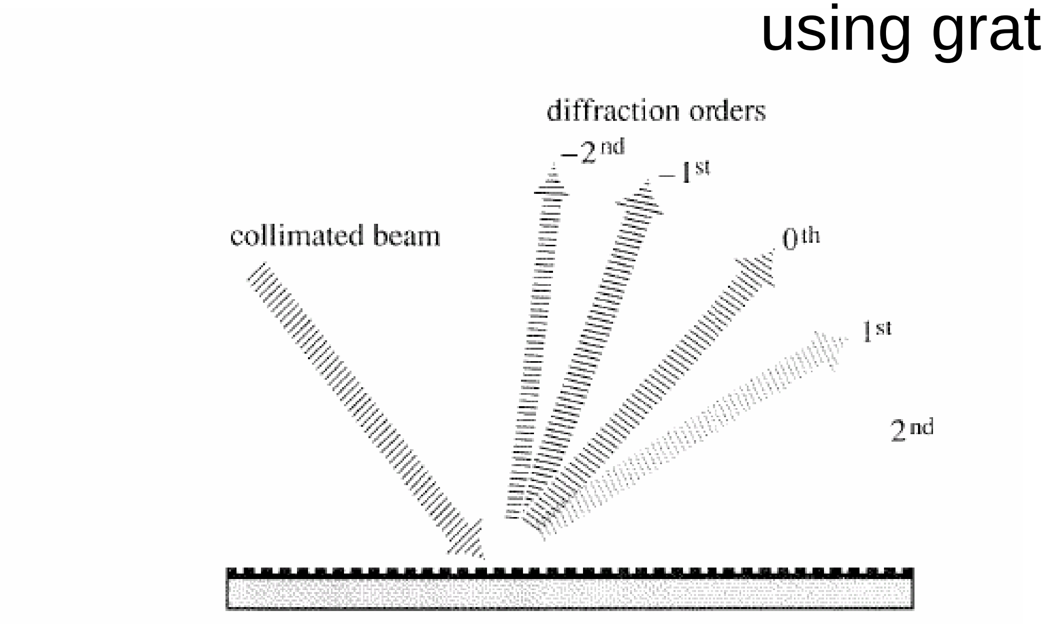
$$\sin\theta_a = NA = n_1 \sin\bar{\theta}_c = n_1\sqrt{1-\cos^2\bar{\theta}_c}$$

$$NA = n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \sqrt{n_1^2 - n_2^2}$$

Ex: $n_1=1.475$, $n_2=1.46$ then NA=.21 and $\theta_a=12$ degrees

Spectrometers

using gratings



Grating equation

$$\sin\alpha = \frac{m\lambda}{S} \pm \sin I$$

α is the angle diffracted
λ is the wavelength of light
m is the order number
I is the incident angle + sign is for transmission, - is for reflective grating
S – is the period of the grating (spacing of the grating lines)

Since angle depends on λ, can use to measure wavelength

Spectrometers

using gratings

The dispersion of the grating is $d\theta/d\lambda$ (differentiating the grating equation)

$$d\theta/d\lambda = m/(S \cos\theta)$$

The effective width of a line is equal to $\Delta\alpha=2\pi/N$, where N is the number of grating lines illuminated (assuming the Aperture Stop is the grating) $\Delta\alpha$ can be written as $(kS/2)(\sin\theta - \sin\theta_i)$ or $(kS/2)\cos\theta\,(\Delta\theta)$.

This can be written as

$d\theta_{min} = 2\lambda/(NS \cos\theta_m)$

This is the FULL angular width of a line due to instrument broadening

Plug this into above equation and solve for $d\lambda$ results in

$\lambda/d\lambda_{min} = mN = R$(resolving power) $= NS(\sin\alpha \pm \sin I)/\lambda$

Or $d\lambda_{min} = \lambda/mN$

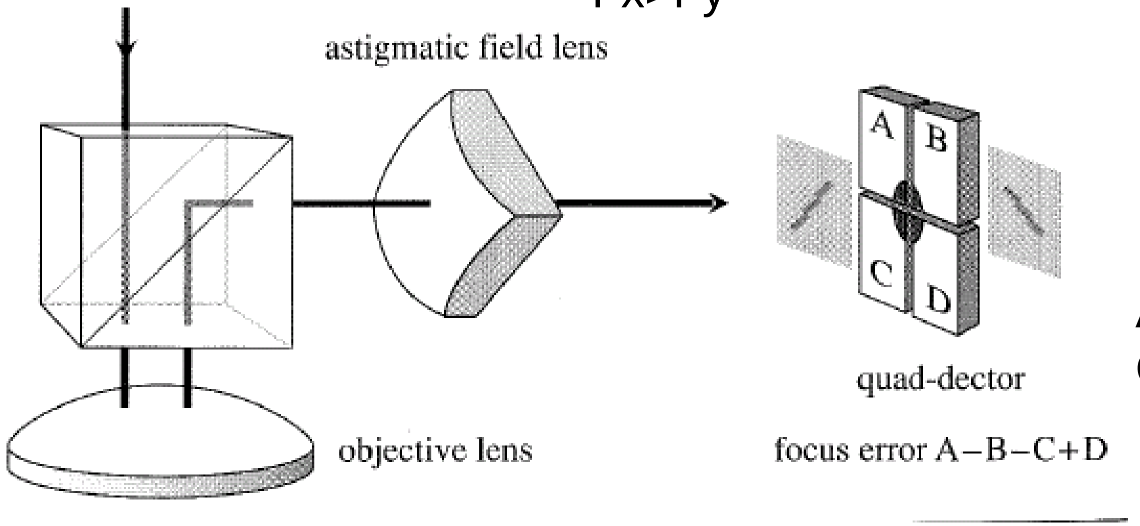
6in wide grating at 15,000lines/in in 2nd order will resolve 180,000 lines, at 540nm $d\lambda = 0.003$nm

Optical disks

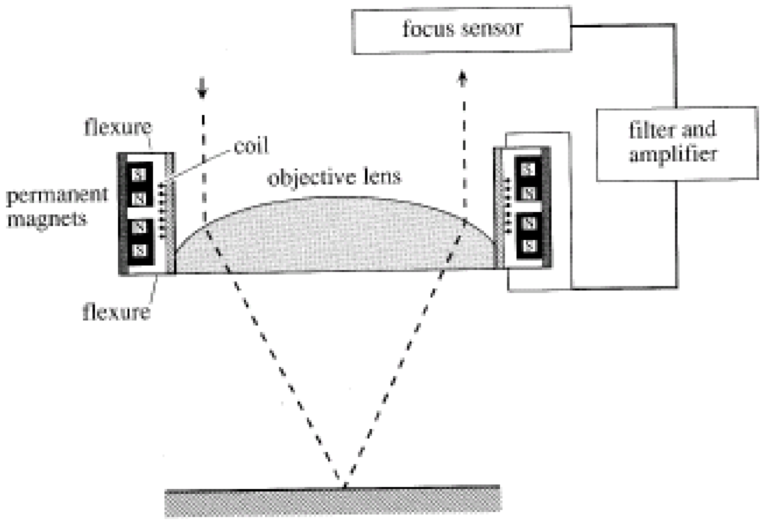
- Probably the largest volume optical system ever – CD,DVD, BD
  - 10’s of million devices per year
    - First video disk in early 1970 – killed (12 inch)
    - CD mid 1970’s
      - First CD-R from Sony cost $15,000
      - Current OEM price for CDROM is ~8 dollars
  - ~10 Billion disks per year
    - CD price are <20 cents a disk with 10 cents being IP royality
    - DVD’s maybe 50 cents
  - Record and read by focusing beam to diffraction limited spot and changing reflectivity or polarization state of media

Focus Servo

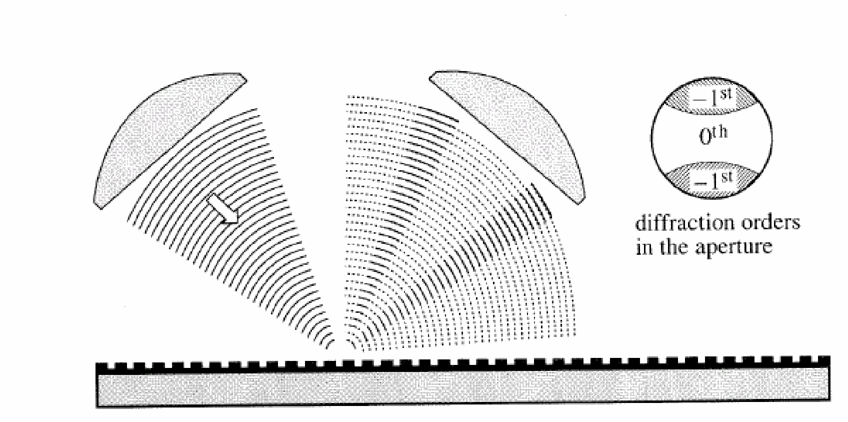
Fx>Fy

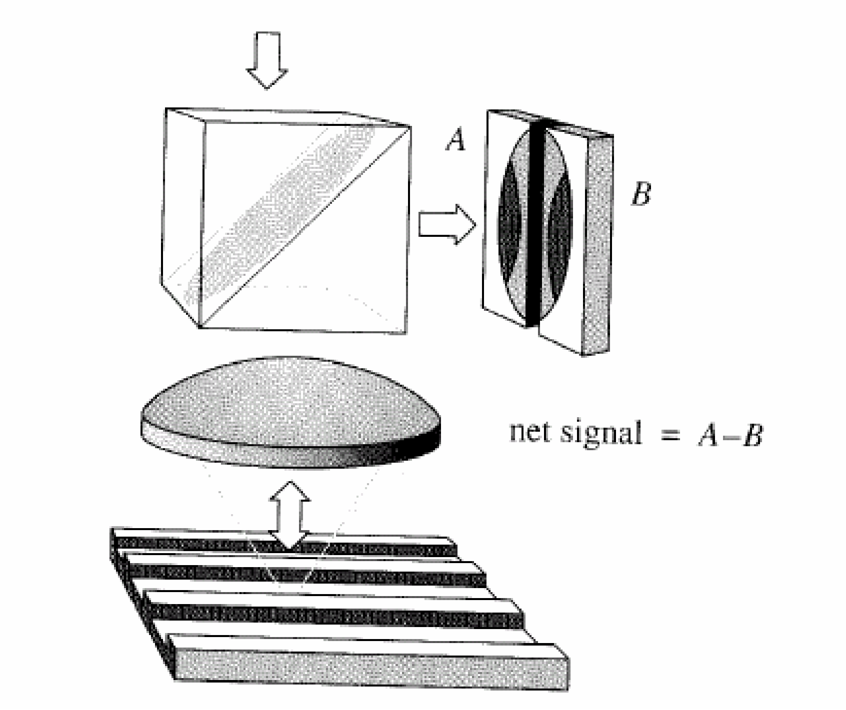


A+D-(B+C)
Or (AD-BC)



Tracking Servo



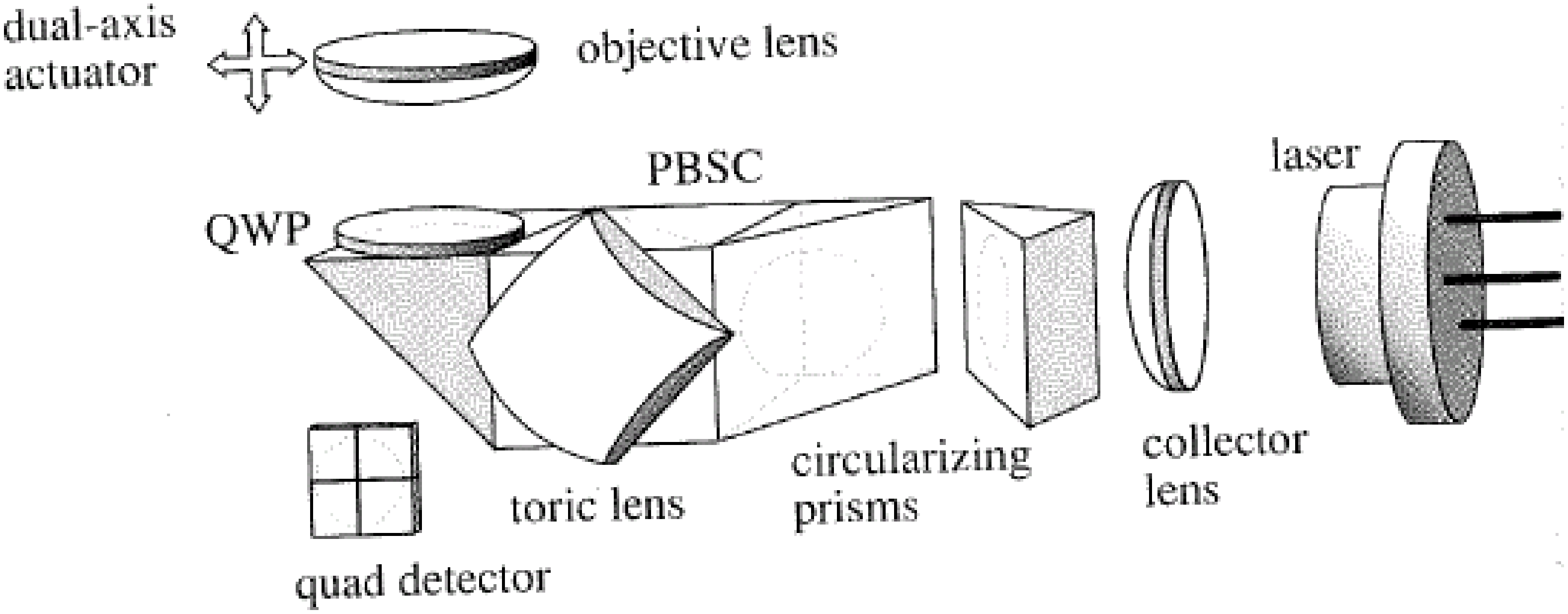


If NA of lens is > 0.5λ/p
orders will overlap

Where p is grating period
(track width)

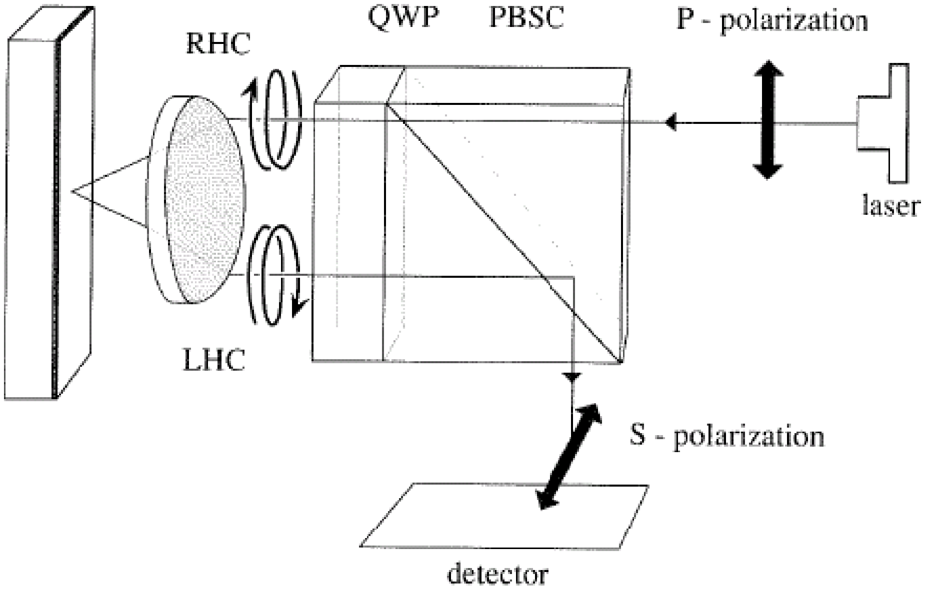
Optical Head

Write Once Drive



- MO has polarizer between PBS and toric lens
- Data signal is transitions for bright state to dark state. RLL codes are used to make sure timing stays sync'ed

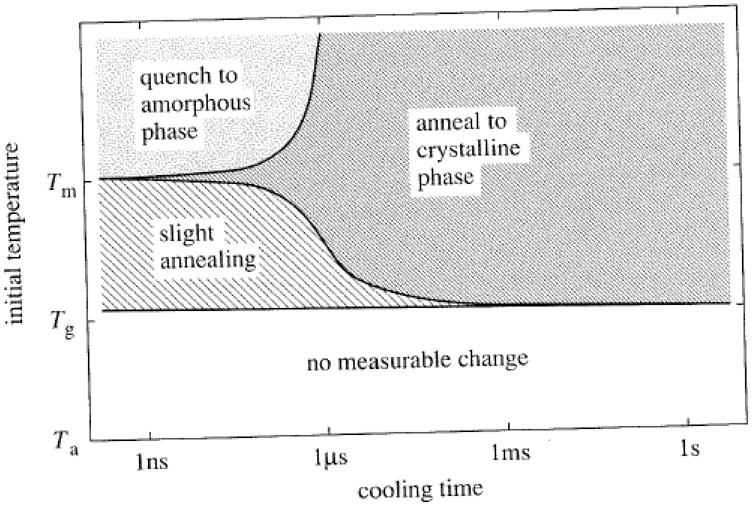
Laser Feedback Elimination

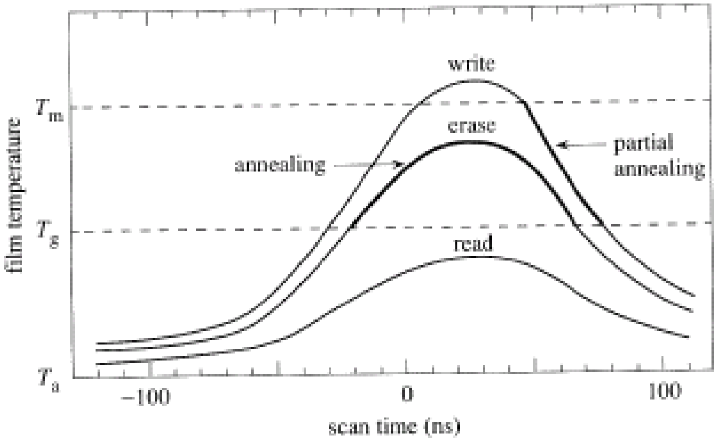


“Optical recording”, Alan Marchant, Addison Wesley

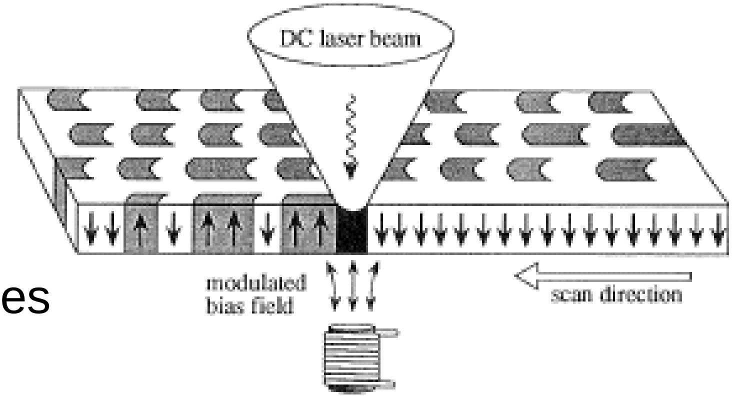
Recordable Media

Phase Change: Sole survivor





Examples include: InSeTe and GeSeTe and many others flavors

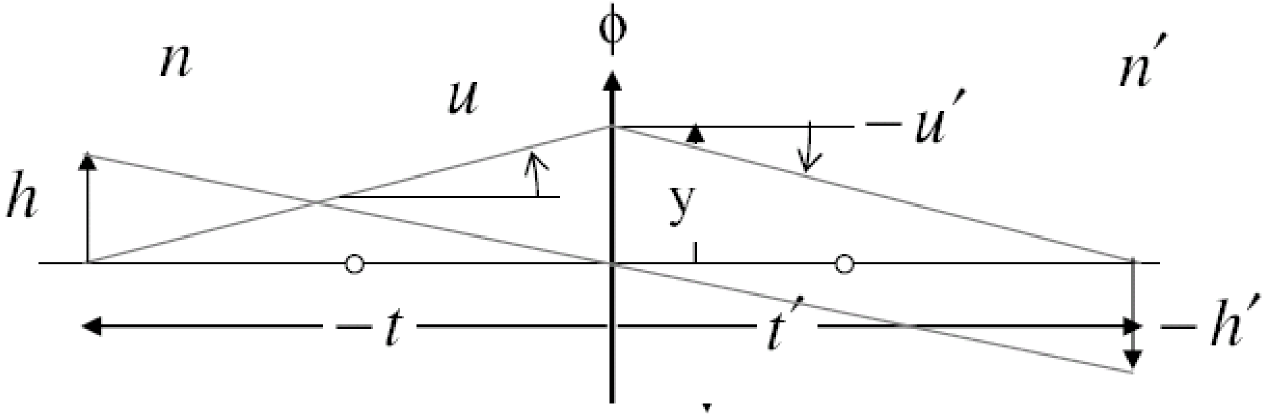


Magneto-optical materials

Being considered for next gen hard drives

Optical Invariant

At image/object plane (special case)



$$n\frac{h}{-t} = n'\frac{-h'}{t'}$$ Paraxial Snell's Law

$$u = \frac{y}{-t}$$ Triangles

$$-u' = \frac{y}{t'}$$

$$M \equiv \frac{h'}{h} = \frac{nt'}{n't} = \frac{n\,{}^{y}\!/_{-u'}}{n'\,{}^{-y}\!/_{u}} = \frac{nu}{n'u'}$$ Substitute into M

Optical Invariant

At image/object plane (special case)

In a cascaded system

$$M \equiv \frac{h'_k}{h_0} = \frac{n_0\,u_0}{n'_0 u'_0}\,\frac{n_1\,u_1}{n'_1 u'_1}\cdots\frac{n_k\,u_k}{n'_k u'_k} = \frac{n_0\,u_0}{n'_k u'_k}$$

$$\boxed{H = n'_k u'_k h'_k = n_0\,u_0 h_0}$$

A conserved quantity

Invariant – this expression has the same value everywhere in the optical system.

At an object or image plane the invariant is equal to the index times the object/image height times the half convergence/divergence angle of the axial beam

Optical invariant

aka Lagrange or Helmholtz invariant

At a general surface anywhere in the optical system the invariant is expressed as

$$H = n\left(u\,\bar{y} - \bar{u}\,y\right)$$

is conserved everywhere

Write the paraxial refraction equations for the marginal ray (PMR) and chief or pupil ray (PPR):

$$n'u' - nu = -\phi y$$

$$n'\bar{u}' - n\bar{u} = -\phi \bar{y}$$

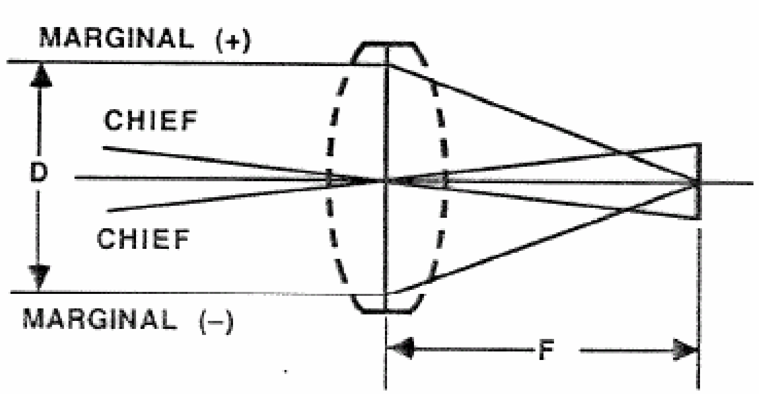
With a bit of algebra:

$$n'\left(u'\bar{y} - \bar{u}'y\right) = n\left(u\,\bar{y} - \bar{u}\,y\right)$$

The 3D version for throughput is that the product of the object/image area times the solid angle of collection is invariant

Basic Definitions

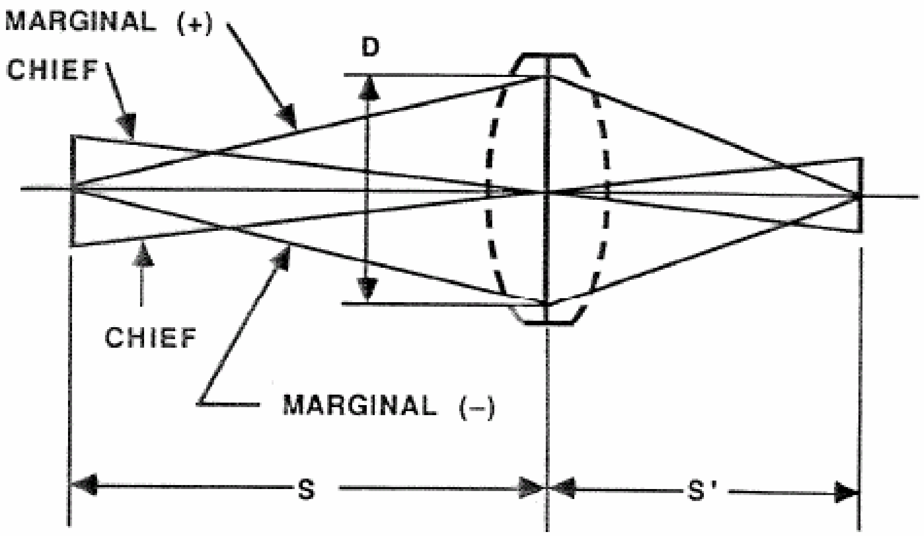
"INFINITE CONJUGATE"



OBJECT OR IMAGE AT INFINITY

$F/D = (F/\#)$

"FINITE CONJUGATE"



OBJECT AND IMAGE AT FINITE DISTANCES $S'/D = (F/\#)_{EFFECTIVE}$

Examples

- Given object and image slopes and object height can find height of image.
  - uo = .333, ui=-0.04755, h=20mm
  - M=h'/h => **h'= (20)(0.333)/(-0.04755)= -14.0187mm**

- Image height for lens with object at infinity
  - Y for axial ray is 0, slope of u is zero, u_p is half FOV

$$INV = h'n'u' = -y_1 n u_p$$

$$h' = -u_p y_1/u' \text{ for } n=n'$$

$F= -y_1/u'$ so

$h' = u_p F$ or F $\tan u_p$ for non paraxial case

Optical invariant

aka Lagrange or Helmholtz invariant

Using the invariant, at the object (or image) of limited field diameter L: $y = 0$, $\bar{y}$ = edge of field, u = maximum ray angle

$$H_{obj} = nu\bar{y} = NA\frac{L}{2} = .6\frac{\lambda}{r_0}\frac{L}{2} \approx \frac{\lambda}{2}N_{spots}$$

Rayleigh Resolution
($NA = 0.6\lambda/\Delta r$)

Thus we have found the information capacity of the optical system, aka the space-bandwidth product:

$$\boxed{N_{spots} = \frac{2}{\lambda}H_{obj}}$$

Question

If an object that is 1 cm^2 with 1 sr of solid angle is images to 2cm^2 area,

What is the solid angle of this image ?

Homework #2

Available at the website under homework

http://ecee.colorado.edu/~ecen4616
http://ecee.colorado.edu/~ecen5616

Due in 2 weeks