Apertures and Stops

• Chief ray, marginal ray, axial ray
• Stops and Pupils
• NA
• Effective FL, F/# and depth of field
• Vignetting
• Example
Optics of Finite Size

Introduction

• Up to now, all optics have been infinite in transverse extent. Now we’ll change that.
• Types of apertures: edges of lenses, intermediate apertures ("stops").
• Two primary questions to answer:
  • What is the angular extent (NA) of the light that can get through the system. “Aperture” = largest possible angle for object of zero height. Depends on where the object is located.
  • What is the largest object that can get through the system = “Field”. At the edge of the field, the angular transmittance is one half of the on axis value.
• Will find each of these with two particular rays, one for each of above.
• Will find two specific stops, one of which limits aperture and one which limits field.
• The conjugates to these stops in object and image space are important and get their own names.
• Finally, this will allow us to understand the total power efficiency of the system.
The aperture stop and the paraxial marginal ray

Launch an axial ray, the *paraxial marginal ray*, from the object.
- Increase the ray angle until it just hits some aperture.
- This aperture is the *aperture stop*.
- The sin of $\alpha$ is the *numerical aperture*.
- The aperture stop determines the system resolution, light transmission efficiency and the depth of field/focus.
Pupils

The images of the aperture stop

• Axial rays at the object (image) appear to enter (exit) the system entrance (exit) pupil.
• When you look into a camera lens, it is the *pupil* you see (the image of the stop).
• Both pupils and the aperture stop are conjugates.
Windows
Images of the field stop

- Launch an axial ray, the *chief ray*, from the aperture stop.
- Increase the ray angle until it just hits some aperture.
- This aperture is the *field stop*.
- The angle of the chief ray in object space is the *angular field of view*.
- The height of the chief ray at the object is the *field height*.
- The chief ray determines the spatial extent of the object. When that object is very far away, it is convenient to use an angular field of view.
- When the field stop is not conjugate to the object, *vignetting* occurs, cutting off ~half the light at the edge of the field.
Field stops and windows

Definitions

The entrance (exit) window is the image of the field stop in object (image) space.
Numerical aperture
The measure of angular bandwidth

\[ \text{NA} \equiv n \sin \alpha \]
Definition of numerical aperture.  Note inclusion of \( n \) in this expression.

\[ = \frac{D/2}{F} \]
Paraxial approximation

\[ F \# \equiv \frac{1}{2 \text{NA}} \]
Definition of F-number

\[ = \frac{F}{D} \]
Paraxial approximation

\[ r_0 = 0.6 \frac{\lambda}{\text{NA}} \approx \lambda \text{F\#} \]
Radius of Airy disk, resolution of system

NA is the conserved quantity in Snell’s Law because it represents the transverse periodicity of the wave:

\[ k_x = 2\pi f_x = \frac{2\pi}{\lambda_0} n \sin \alpha = \frac{2\pi}{\lambda_0} \text{NA} \]

Therefore \( \text{NA}/\lambda_0 \) equals the largest spatial frequency that can be transmitted by the system. Note that NA is a property of cones of light, not lenses.
Effective F#

Lens speed depends on its use

\[ F\# = \frac{1}{2NA} = \frac{f}{D} \]

Infinite conjugate condition

Finite conjugate condition

\[ F^i\# = \frac{t'}{D} = \frac{f(1-M)}{D} = (1-M)F\# \]

\[ F^o\# = \frac{t}{D} = \frac{f(1-\frac{1}{M})}{D} = (1-\frac{1}{M})F\# \]

or

\[ NA^i = NA/(1-M) \]
\[ NA^o = NA/(1-\frac{1}{M}) \]
**Depth of focus**

**Dependence on F#**

Your detection system has a finite resolution of interest (e.g. a digital pixel size) = $\rho$

\[ \delta_z = 2\rho F^\# = \frac{\rho}{NA^*} \]

\[ = 4\lambda(F^\#)^2 = 1.2 \frac{\lambda}{(NA^*)^2} \]

If $\rho = 2r_0$ = Airy disk diameter

First eq. is “detector limited”, second is “diffraction limited” Thus as we increase the power of the system (NA increases) the depth of field decreases linearly for a fixed resolvable spot size $\rho$ or quadratically for the diffraction limit $r_0$. 

The object distance which is perfectly in focus on the detector defines the nominal system focus plane. When this plane is positioned such that the far point goes to infinity, then the system is in the “hyperfocal” condition and the nominal focus plane is called the “hyperfocal distance.”

This is very handy for fixed focus cameras – you can take a portrait shot (if the person is beyond the near point) out to a landscape and not notice the defocus.
Vignetting
Losing light apertures or stops

Take a cross-section through the optical system at the plane of any aperture. Launch a bundle of rays off-axis and see how they get through this aperture:

Unvignetted
\[ a \geq |y| + |\bar{y}| \]

Fully vignetted
\[ a \leq |\bar{y}| - |y| \]
Pupil matching
Off-axis imaging & vignetting

Exit pupil of telescope (image of A.S. in image space) matches entrance pupil of following instrument (eye).

Note that the entrance pupil in the system above will appear to be ellipsoidal for off-axis points. Thus even in this well-designed, aberration-free case, off-axis points will not be identical on axis.

Vignetting: Further, we might find extreme rays at off-axis points are terminated. This loses light (bad) but also loses rays at extreme angles, which might limit aberrations (good).
In a *telecentric system* either the EP or the XP is located at infinity. The system shown above is *doubly telecentric* since both the EP and the XP are at infinity. All doubly telecentric system are afocal.

When the stop is at the front focal plane (just lens f2 above) the XP is at infinity and slight motions of the image plane will not change the image height.

When the stop is at the back focal plane (just lens f1 above) the EP is at infinity and small changes in the distance to the object will not change the height at the image plane.

Telecentricity is used in many metrology systems.
Analyzing an Optical System

1. Setup the system array (distances, component locations, focal length, and clear apertures

2. Run an marginal ray using a y-u trace

3. Find the aperture stop (calculate the ratio of clear aperture to ray height (smallest r/yk is aperture stop)

4. Run a chief ray trace

5. Calculate as in step 3 above to find field stop

6. If few components, use thin lens equation to find location and size of the pupils and windows.

7. If there are many components, use y-u trace forward and backwards to find pupils and windows.

8. Sizes of pupils can be determined by taking the slope angle of the chief ray at the aperture stop and dividing by the image and object space slope angles of that ray to get the magnification of the aperture stop in image and object space, respectively. Multiplying the aperture stop size by the magnification gives the exit and entrance pupil sizes. (Optical invariant is the basis).

9. A full report of a optical system contains: image location and magnification, the location of stops, pupils, and windows, the effective focal length of the system, F/#, and its angular field of view.
Detailed example
Holographic data storage

We're going to design the imaging path
1 to 1 imaging
Aperture in Fourier plane

SLM

Lens

Aperture

Material

CCD

\[ D_{\text{SLM}} = 10 \text{ mm} \]
\[ p = 10 \mu \text{m} \]

\[ D_{\text{Lens}} = 10 \text{ mm} \]
\[ f = 20 \text{ mm} \]

\[ D_{\text{Ap}} = 2 \text{ mm} \]

\[ D_{\text{CCD}} = 10 \text{ mm} \]
1 to 1 imaging
Aperture in Fourier plane

Fourier diffraction calculation for a random pixel pattern.
Finding the aperture stop
Trace the paraxial marginal ray (PMR)

\[
\begin{bmatrix}
 y_k \\
 u'_k
\end{bmatrix} =
\begin{bmatrix}
 1 & 0 \\
 -\phi_k & 1
\end{bmatrix}
\begin{bmatrix}
 y_k \\
 u_k
\end{bmatrix}
\]

\[
\begin{bmatrix}
 y_{k+1} \\
 u_{k+1}
\end{bmatrix} =
\begin{bmatrix}
 1 & t'_k \\
 0 & 1
\end{bmatrix}
\begin{bmatrix}
 y_k \\
 u'_k
\end{bmatrix}
\]

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What does the aperture stop do?

Limits the NA of the system

What is the lens effective NA?

\[ NA^* = \frac{D / 2}{f (1 - M)} = \frac{5}{20(-2)} = \frac{1}{8} \]

However we have stopped the system down to \( NA = \frac{2}{40} = \frac{1}{20} \)

Pixel size = 10 \( \mu m \) \( \implies \) \( NA_{pix} = \frac{\lambda}{p} = \frac{.5}{10} = \frac{1}{20} \)

The aperture stop thus:
1. Determines the system field of view
2. Controls the radiometric efficiency
3. Limits the depth of focus
4. Impacts the aberrations
5. Sets the diffraction-limited resolution
Find the entrance pupil
(image of aperture stop in object space)

Find the entrance pupil
(image of aperture stop in object space)

\[-\frac{1}{t_0} + \frac{1}{t_1} = \frac{1}{f}\]

\[\Rightarrow \frac{1}{20} + \frac{1}{t_1} = \frac{1}{20}\]

\[\Rightarrow t_1 = \infty\]

What’s *that* mean? If the entrance pupil is at infinity, then every point on the object radiates into the same cone:
Find the field stop

Trace the chief ray (PPR)

Entrance and exit windows at field stop (since it is lens).
Required lens diameter
And why windows should be at images

Object vignette at edges

Note that the extreme marginal ray makes the same angle with the chief ray (PPR) as the paraxial marginal ray (PMR) makes with the axis. Thus to fit the extreme marginal ray through an aperture, we require:

\[ D \geq 2\left(|y| + |\bar{y}|\right) = 2(2 + 5) = 12 \]

Now the camera is the field stop and the entrance window is the SLM.
- The optical system now captures the same light from each SLM pixel and it can be adjusted for all pixels via the aperture stop.
- Independently, we can adjust the field size.
Thicken the lenses

\[ D_{SLM} = 10 \text{ mm} \]
\[ p = 10 \mu\text{m} \]
\[ D_{Lens} = 12 \text{ mm} \]
\[ F = 20 \text{ mm} \]
\[ D_{Ap} = 2 \text{ mm} \]
\[ D_{CCD} = 10 \text{ mm} \]
Optical invariant
aka Lagrange or Helmholtz invariant

Using the invariant, at the object (or image) of limited field diameter
\( L: y = 0, \bar{y} = \text{edge of field, } u = \text{maximum ray angle} \)

\[
H_{\text{obj}} = nu \bar{y} = NA \frac{L}{2} = 0.6 \frac{\lambda}{r_0} \frac{L}{2} \approx \frac{\lambda}{2} N_{\text{spots}}
\]

Rayleigh Resolution
\( (NA = 0.6\lambda/\Delta r) \)

Thus we have found the information capacity of the optical
system, aka the space-bandwidth product:

\[
N_{\text{spots}} = \frac{2}{\lambda} H_{\text{obj}}
\]
Optical invariant
for the holographic storage example

\[ N_{\text{spots}} \approx \frac{2H}{\lambda_0} \]

Definition of optical invariant

\[ = \frac{2\gamma_0 NA_0}{\lambda_0} \]

\[ = \frac{D_{SLM}}{\lambda_0} \]

Stopped-down NA

\[ = \frac{D_{SLM}}{\lambda_0/p} \]

So we have correctly designed this system to transmit the proper number of resolvable spots.
Summary of Rays and NA

"INFINITE CONJUGATE"

OBJECT OR IMAGE AT INFINITY
\[ F/D = (F/#) \]

"FINITE CONJUGATE"

OBJECT AND IMAGE AT FINITE DISTANCES
\[ S'/D = (F/#)_{\text{EFFECTIVE}} \]
Summary of Stops and Pupils
Reading

W. Smith “Modern Optical Engineering”

Chapter 9 and
Chapter 5
\[
t_A(\xi, \eta) = \left( \left[ \text{rect}\left(\frac{\xi}{X}\right)\text{rect}\left(\frac{\eta}{Y}\right) \right] \otimes \left[ \frac{1}{\Delta} \text{comb}\left(\frac{\eta}{\Delta}\right)\delta(\xi) \right] \right) \text{rect}\left(\frac{\eta}{N\Delta}\right)
\]
\[ t_A(\xi, \eta) = \left( \left[ \text{rect}\left(\frac{\xi}{X}\right)\text{rect}\left(\frac{\eta}{Y}\right) \right] \otimes \left[ \frac{1}{\Delta} \text{comb}\left(\frac{\eta}{\Delta}\right)\delta(\xi) \right] \right) \text{rect}\left(\frac{\eta}{N\Delta}\right) \]

To facilitate the use of convolution theorem, break up equation into individual contributors,

\[ t_A(\xi, \eta) = (s(\xi, \eta) \otimes t(\xi, \eta)) \times u(\xi, \eta) \]

Where,

\[ s(\xi, \eta) = \text{rect}\left(\frac{\xi}{X}\right)\text{rect}\left(\frac{\eta}{Y}\right) \]

\[ t(\xi, \eta) = \frac{1}{\Delta} \text{comb}\left(\frac{\eta}{\Delta}\right)\delta(\xi) \]

\[ u(\xi, \eta) = \text{rect}\left(\frac{\eta}{N\Delta}\right) \]

Now apply convolution theorem to \( s, t \) and \( u \),

\[ \mathcal{F}(s) = XY \text{sinc}(Xu) \otimes \text{sinc}(Yv) \]

\[ \mathcal{F}(t) = \text{comb}(\Delta v) \]

\[ \mathcal{F}(u) = N\Delta \text{sinc}(N\Delta u) \]

Where \( u = \frac{x}{Xf} \) and \( v = \frac{y}{Xf} \).

Now apply convolution theorem to \( t_A \),

\[ \mathcal{F}(t_A) = (\mathcal{F}(s) \times \mathcal{F}(t)) \otimes \mathcal{F}(u) \]

\[ = ([XY \text{sinc}(Xu) \otimes \text{sinc}(Yv)] \times \text{comb}(\Delta v)) \otimes N\Delta \text{sinc}(N\Delta u) \]

\[ \propto ([\text{sinc}(Xu) \otimes \text{sinc}(Yv)]) \times (\text{comb}(\Delta v) \otimes \text{sinc}(N\Delta u)) \]
To find the intensity pattern from an aperture illuminated by a plane wave,

\[ I(\frac{x}{\lambda f}, \frac{y}{\lambda f}) \propto \mathcal{F}(t_A) \times \mathcal{F}(t_A)^* \]

Note that if \( \Delta = 2Y \) then the even diffraction orders of the comb function will be zero.