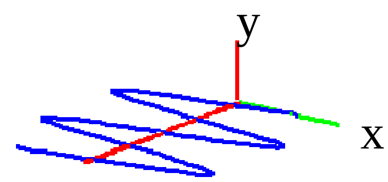
Polarization of Light

- Polarization states
- Stokes Vectors
- Mueller Matrices
- Jones Matrices
- Components
- Examples

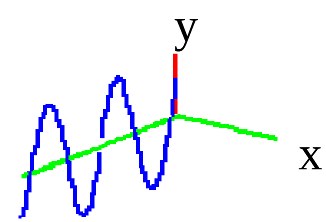
Polarization states

Linear states of polarization

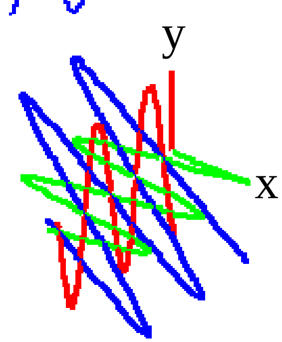
Linear x polarized $|E_x| \neq 0, \; |E_y| = 0$



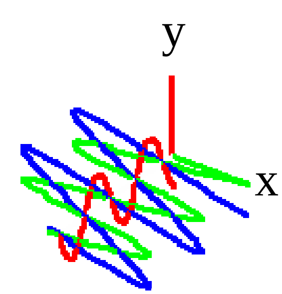
Linear y polarized $|E_x| = 0, \; |E_y| \neq 0$



Linear -45° polarized $|E_x| = |E_y|, \; \phi_y - \phi_x = \pi$



Linear θ polarized $|E_x| \neq |E_y|, \; \phi_y - \phi_x = 0, \pi$

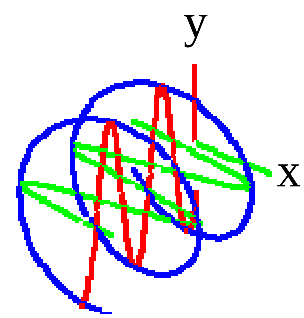


O'Shea 1.6, Saleh & Teich 6

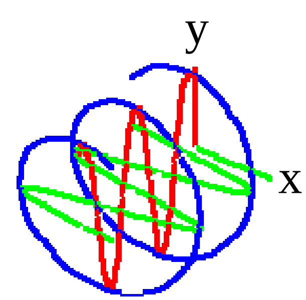
Polarization states

Elliptical states of polarization

Right-hand circular $|E_x| = |E_y|,\ \phi_y - \phi_x = \pi/2$



Left-hand circular $|E_x| = |E_y|,\ \phi_y - \phi_x = -\pi/2$

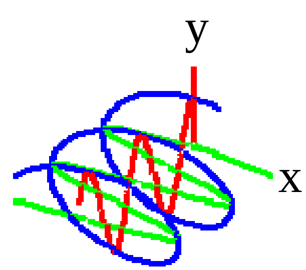


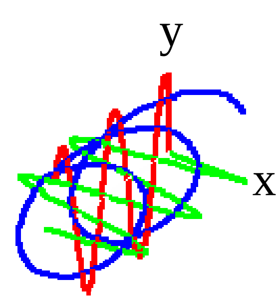
Elliptical

$|E_x| \neq |E_y|,$

and/or

$\phi_y - \phi_x \neq \pm \pi/2$





Stokes vectors

Complete description of polarization state

Perform 6 irradiance measurements with ideal polarizers:

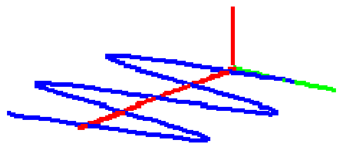
E_x	Horizontal linear
E_y	Vertical linear
E_{45}	45^o linear
E_{135}	135^o linear
E_R	Right circular
E_L	Left circular

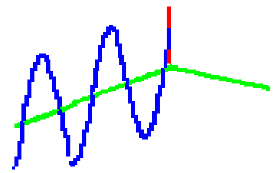
$$\vec{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} E_H + E_V \\ E_H - E_V \\ E_{45} - E_{135} \\ E_R - E_L \end{bmatrix}$$

Stokes vectors

Complete description of polarization state

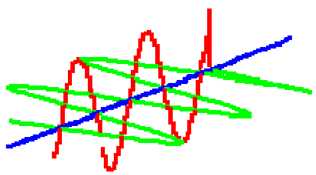
$$H = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

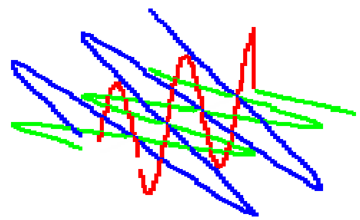




$$V = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

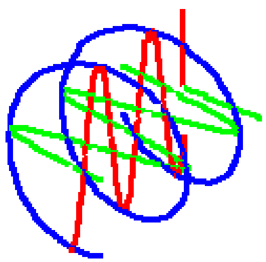
$$45 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

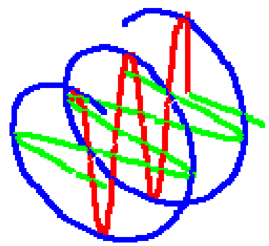




$$135 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

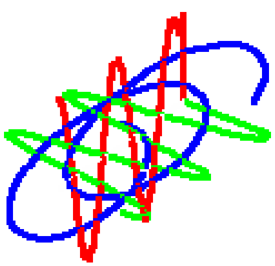
$$R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

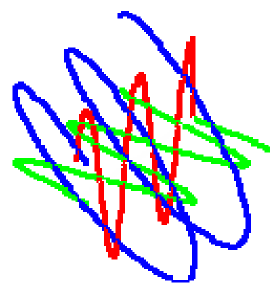




$$L = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ .8 \\ .6 \end{bmatrix}$$

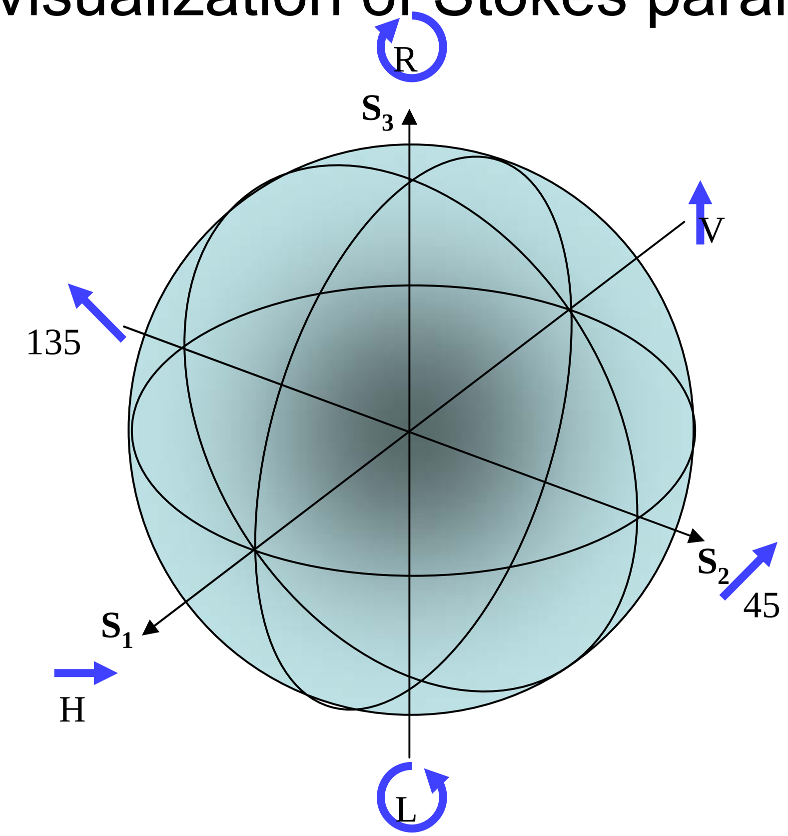




$$\begin{bmatrix} 1 \\ 0 \\ -.8 \\ -.6 \end{bmatrix}$$

Poincarè sphere

Useful visualization of Stokes parameters



$$S_i \cdot S_j = S_{i1} S_{j1} + S_{i2} S_{j2} + S_{i3} S_{j3}$$

- Surface is polarized, center is unpolarized
- Equator is linear polarized
- North hemisphere is right elliptical, south is left elliptical
- Orthogonal polarizations are on opposite points of sphere

Stokes vectors

Degree of polarization

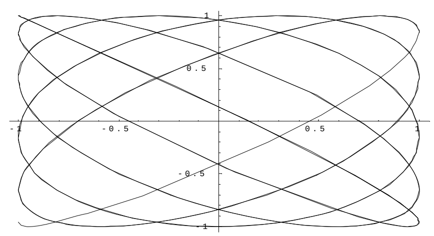
$$DoCP = \frac{S_3}{S_0}$$ Degree of circular polarization

$$DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}$$ Degree of linear polarization

$$DoP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$ Degree of polarization

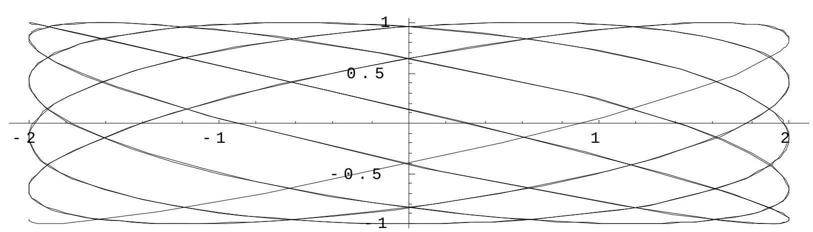
Stokes vectors are polychromatic on addition:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



H + V at different frequencies give Lissagous figure with no <u>average</u> polarization state

$$\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



2H + V = H polarized DoP = 1/3

Mueller Matrics

Complete polarization modeling

4x4 matrix of real values describing transformation of polarizations.
Can describe de-polarizing elements.
Eigenvectors are eigenpolarization of system (unchanged).

E.g. Linear polarizer

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & & \\ \frac{1}{2} & \frac{1}{2} & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & & \\ \frac{1}{2} & \frac{1}{2} & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Moral: We are going to use Jones vectors/matrices to do designs, but you should know their limitations. Jones vectors deal only with systems with perfect temporal and spatial coherence. **Systems with finite coherence can be partially polarized and then you must use Mueller matrices and Stokes parameters**.

Jones vectors

Simplified for fully polarized systems

$$\vec{J} \equiv \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

where A_x and A_y are the <u>complex</u> amplitudes of the x and y **polarized electric <u>fields</u>**.

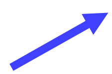
$$J_1 \cdot J_2 \equiv A_{1x} A^*_{2x} + A_{1y} A^*_{2y}$$

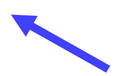
Inner product

$$H \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$V \equiv \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

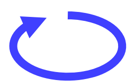
$$45 \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

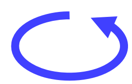




$$135 \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$R \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$





$$L \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

Table of Jones and Mueller matrices

Linear optical element		Jones matrix	Mueller matrix
Horizontal linear polarizer	↔	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Vertical linear polarizer	↕	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at +45°	⤢	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at −45°	⤡	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Quarter-wave plate, fast axis vertical		$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Quarter-wave plate, fast axis horizontal		$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$
Homogeneous circular polarizer right	↻	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
Homogeneous circular polarizer left	↺	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

Rotation matrices

Components not aligned with x or y

$$\vec{J}\,' = R(\theta)\vec{J}$$

Transform for Jones Vectors

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Coordinate Transform Matrix

$$T\,' = R(\theta)\,T\,R(-\theta)$$

Transform for Jones Matrices

Three types of physics

1. Diattenuation (polarization dependent loss)
   - Transmission is polarization dependent
   - “Polarizers”
   - A.k.a.: polarization dependent loss (PDL), dichroism
2. Retardance
   - Optical path length is polarization dependent.
   - “Wave plates”, optical activity, electro-optic
   - A.k.a.: polarization mode dispersion (PMD)
   - Poincarè sphere geometry
3. Depolarization
   - The degree of polarization may decrease depending on input polarization
   - A.k.a.: polarization scrambling

Polarizers

$$P_x \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Jones matrix of linear polarizer passing H

Power transmission of analyzer and arbitrary linear polarization:

$$J_{Out} = P_x L(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$$

$$T = J_{Out} \cdot J_{Out} = \cos^2\theta$$

Malus' Law

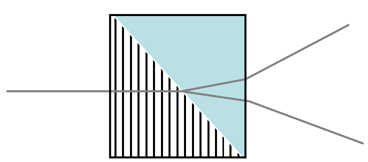
Dichroism: polarization dependent absorption

$$P_x \equiv \begin{bmatrix} e^{-\alpha z} & 0 \\ 0 & e^{-\beta z} \end{bmatrix}$$

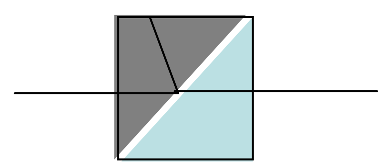
E.g.: Sheet polarizer, polarizing sunglasses, Polarcor™, wire-grid

Reflective polarizers: Brewster's angle, thin-film coatings

Crystal polarizers



E.g.: Wollaston, Rochon, Sénarmont

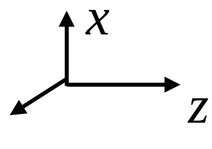


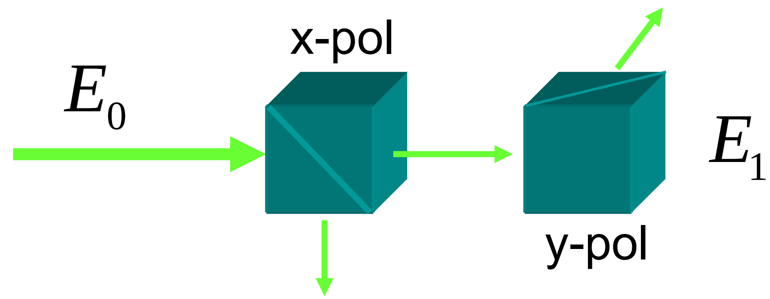
E.g.: Glan-Focault

Jones matrix example

Cascaded polarizers

Crossed polarizers:



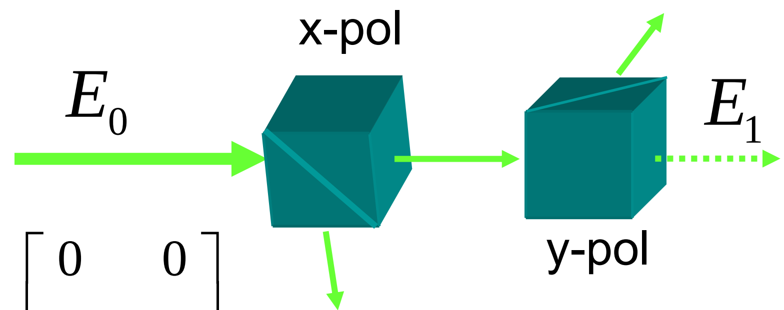


$$E_1 = \mathbf{A}_y \mathbf{A}_x E_0$$

$$\mathbf{A}_\mathbf{y} \mathbf{A}_\mathbf{x} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

so no light leaks through.

Rotation tolerance



$$\mathbf{A}_\mathbf{y} \mathbf{A}_\mathbf{x}(\varepsilon) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \varepsilon & 0 \end{bmatrix}$$

$$\mathbf{A}_\mathbf{y} \mathbf{A}_\mathbf{x}(\varepsilon) \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \varepsilon & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 \\ \varepsilon E_x \end{bmatrix}$$

So $I_{out} \approx \varepsilon^2 I_{in,x}$

Retarders

$$R \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

Jones matrix of retarder with fast axis in x

$$\Gamma = \frac{2\pi}{4} = \frac{\pi}{2}$$

Quarter-wave plate.
Converts linear into circular.

$$\Gamma = \frac{2\pi}{2} = \pi$$

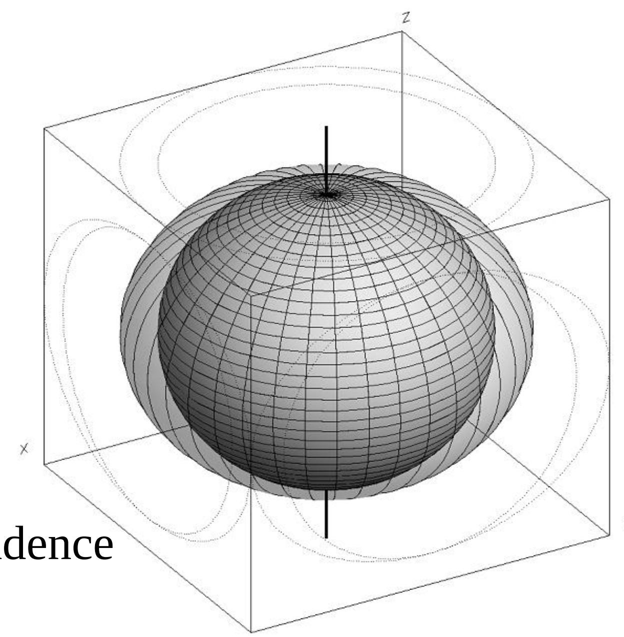
Half-wave plate.
Converts linear into linear.

$$\frac{1}{n^2(\theta)} = \left\{ \frac{1}{n_o^2}, \frac{\mathrm{C}os^2\theta}{n_o^2} + \frac{\mathrm{Sin}^2\theta}{n_e^2} \right\}$$

E.g. Half-wave plate.

$$\Gamma = \frac{2\pi}{\lambda_o}(n_e - n_o)L = \pi$$

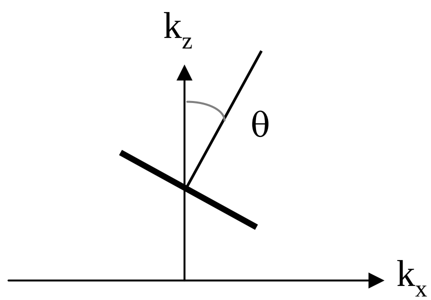
$$L = \frac{\lambda_o}{2\Delta n}$$

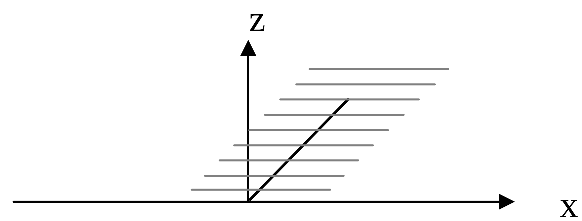


Quartz QWP is only 48 microns thick at 1550 nm.
Crossing plates of differing dispersion can reduce λ, θ dependence

Power walk-off

Consider a Fourier propagation problem in which the k-surface is a tilted line:



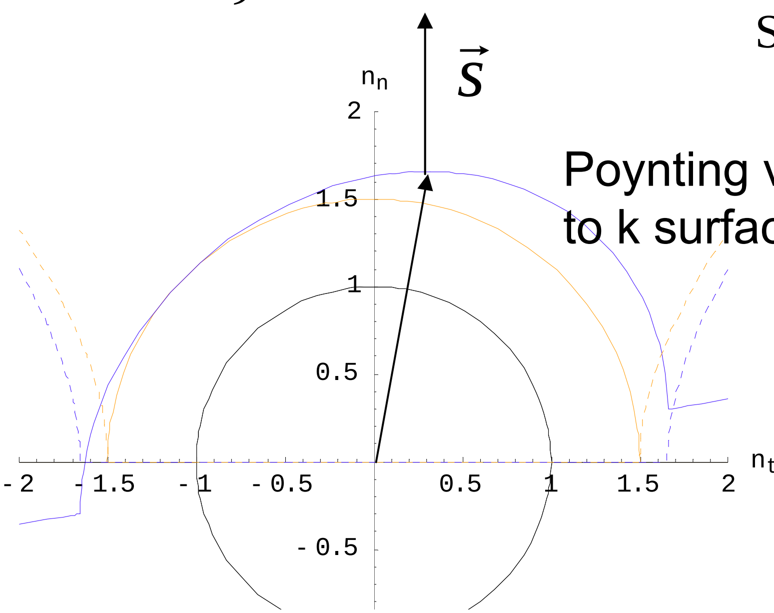


$$E(x,z) = F_x^{-1}\left\{ F_x^{1}\{E(x,0)\} e^{-j\frac{dk_z}{dk_x}k_x z} \right\}$$

Fourier propagator

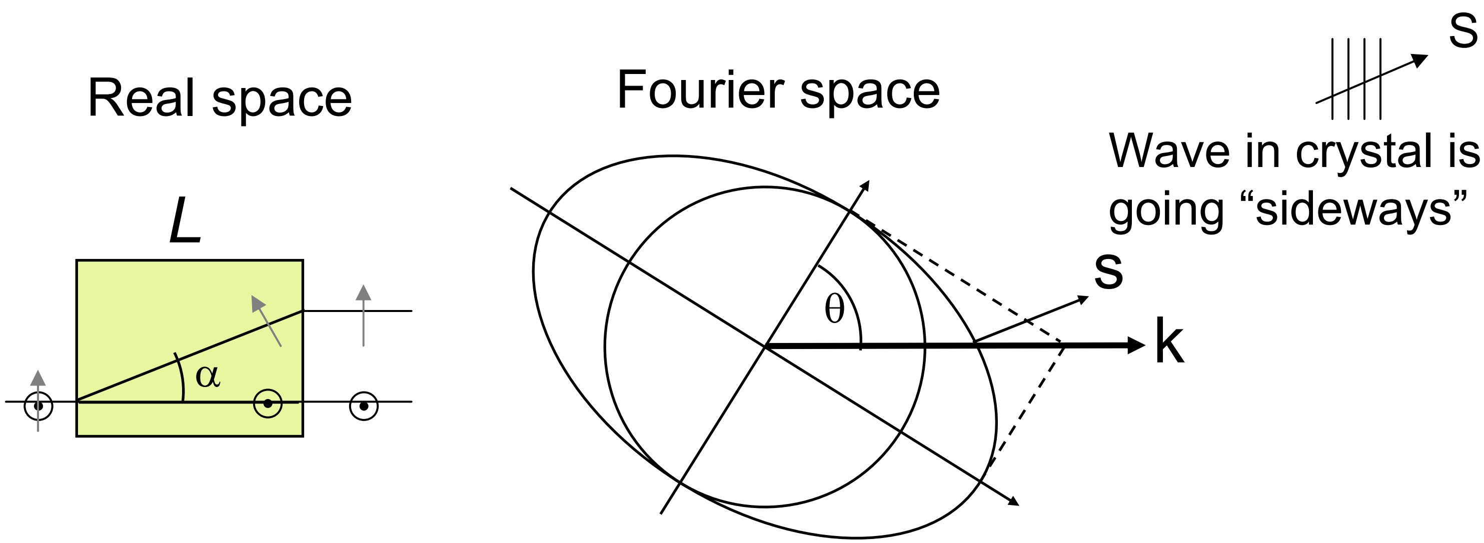
$$= E\left(x - \frac{dk_z}{dk_x} z, 0 \right)$$

Shift theorem



Poynting vector is normal to k surface

Beam displacing polarizer



Separates polarizations with very high isolation (limited only by crystal scatter). Emerging polarizations are parallel to a very high degree since the crystal can be flat to an arc-second.

Beam displacing polarizer

$$\alpha = \arctan\left[\frac{n_o^2}{n_e^2}\tan\theta\right] - \theta$$

Walk-off angle vs. crystal cut angle

$$\theta_{maxBD} = \arctan\left(\frac{n_e}{n_o}\right)$$

Crystal orientation for maximum walk-off

$$n_e(\theta_{maxBD}) = \left[\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}\right]^{-1/2} = \sqrt{\frac{n_o^2 + n_e^2}{2}}$$

Phase velocity at this propagation angle

$$\Delta n = \left(\sqrt{\frac{n_o^2 + n_e^2}{2}} - n_o\right)$$

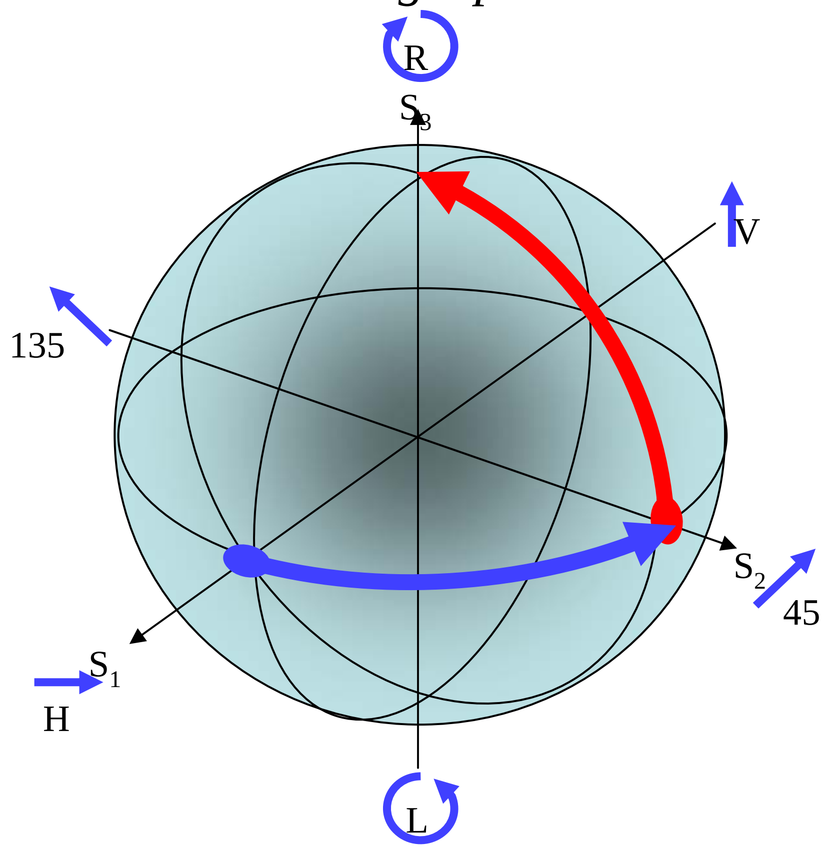
Differential optical path length

$$= \left(\sqrt{\frac{1.9447^2 + 2.1486^2}{2}} - 1.9447\right) = 0.1045$$

YVO_4 at 1.55 μm

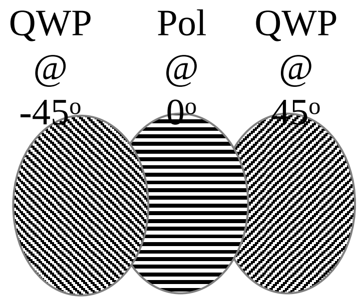
Poincarè and retarders

- Retarders rotate the polarization state on the Poincarè sphere.
- Axis of rotation connects *eigenpolarizations* of the retarder.

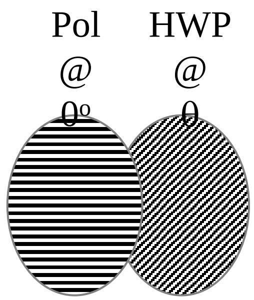


- **Example1: QWP with horizontal axis converts 45º linear into RHC**
- **Example2: Optically active (or Faraday) rotator converts H to 45º**

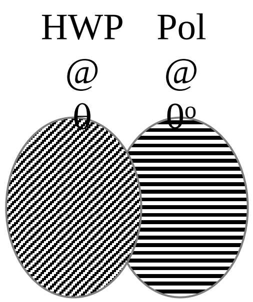
Useful combinations



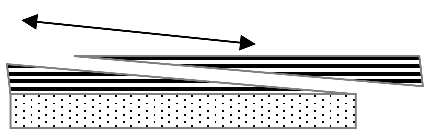
Circular polarizer



Adjustable linear polarizer



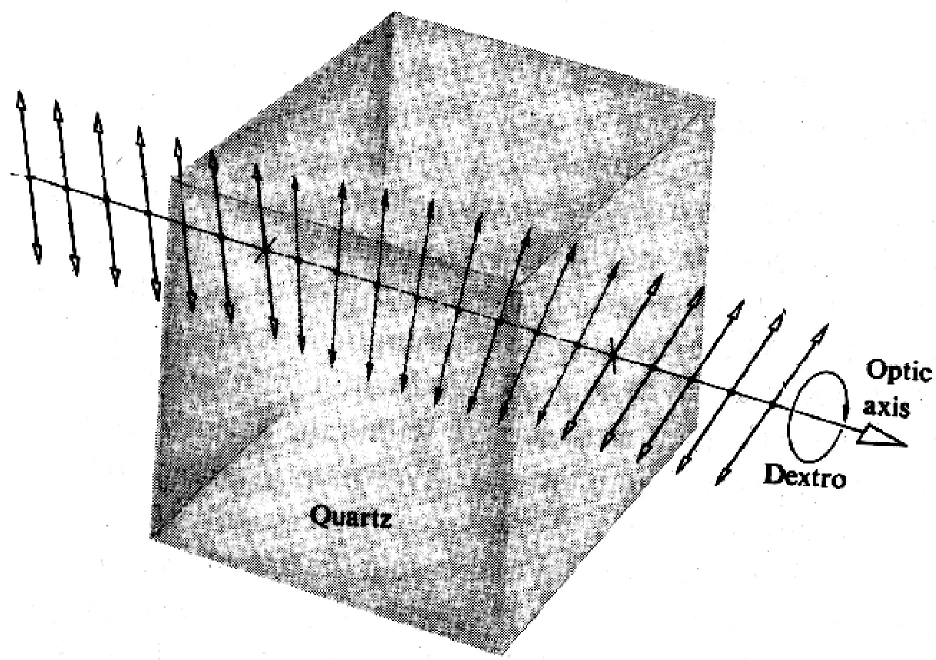
Adjustable transmission



Babinet-Soleil adjustable retarder

Optical activity

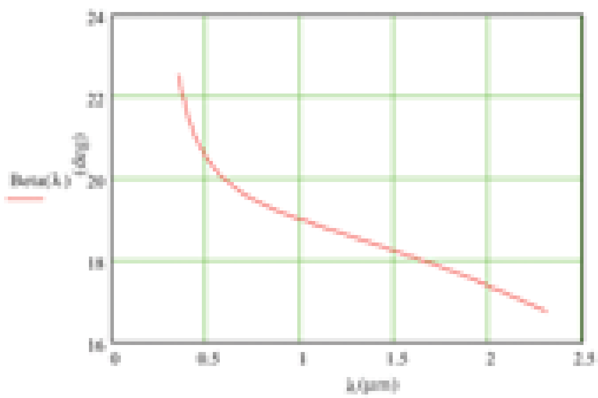
Retarders with circular eigenstates

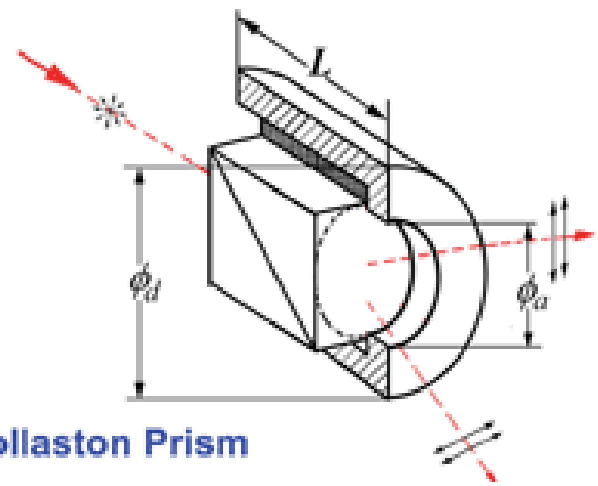


$$OA \equiv \begin{bmatrix} \cos \delta/2 & \sin \delta/2 \\ -\sin \delta/2 & \cos \delta/2 \end{bmatrix}$$

Wollaston

- Wollaston polarizer is made of two birefringent material prisms that are cemented together. The deviations of the ordinary and extraordinary beams are nearly symmetrical about the input beam axis. The separation angle exhibits chromatic dispersion, as shown in the blow. Any separation angle can be designed upon the requirement. Typical separation angle vs wavelength is shown in the plot below for Calcite.





Wollaston Prism

Extinsion Ratio: 5x10 $^{-6}$

Beam Separation Angle:16.7-22.5 at 980nm

Faraday Rotator

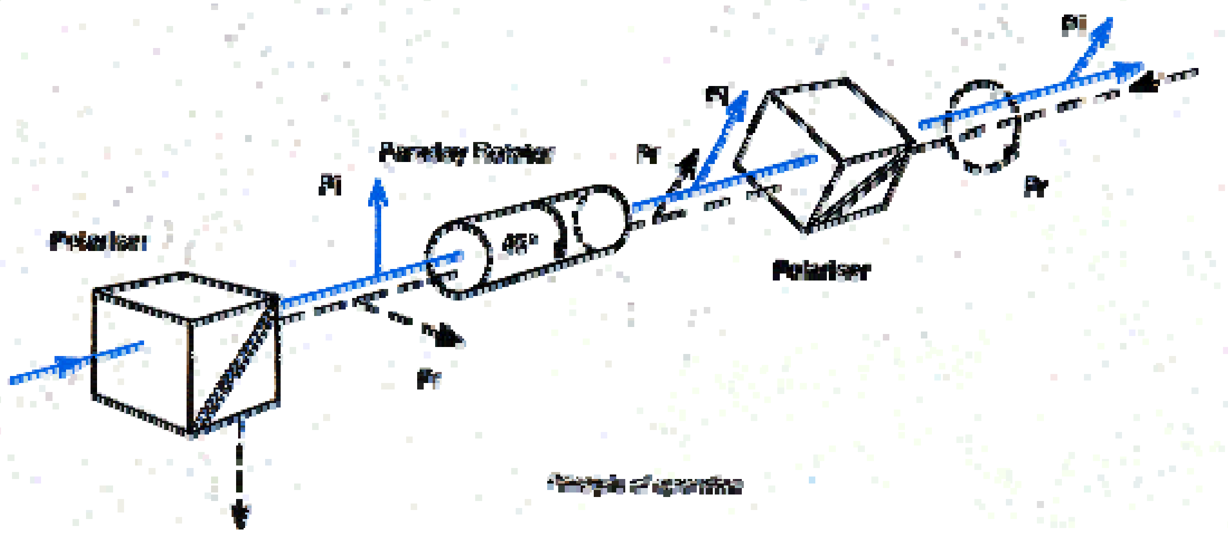
- A **Faraday rotator** is an optical device that rotates the polarization of light due to the Faraday effect, which in turn is based on a magneto-optic effect.
- The Faraday rotator works because one polarization of the input light is in ferromagnetic resonance with the material which causes its phase velocity to be higher than the other.
- The plane of linearly polarized light is rotated when a magnetic field is applied parallel to the propagation direction (permanent magnets). The empirical angle of rotation is given by:

$$\beta = VBd$$

- Where β is the angle of rotation (in radians).B is the magnetic flux density in the direction of propagation (in teslas). d is the length of the path (in metres) where the light and magnetic field interact.Then *V* is the Verdet constant for the material. This empirical proportionality constant (in units of radians per tesla per metre, rad/(T·m)) varies with wavelength and temperature and is tabulated for various materials.
- Note that direction of rotation given by sign of *B* Rotation is <u>**independent of propagation**</u> direction (NOT like waveplate)
- Faraday rotators are used in optical isolators to prevent undesired back propagation of light from disrupting or damaging an optical system.

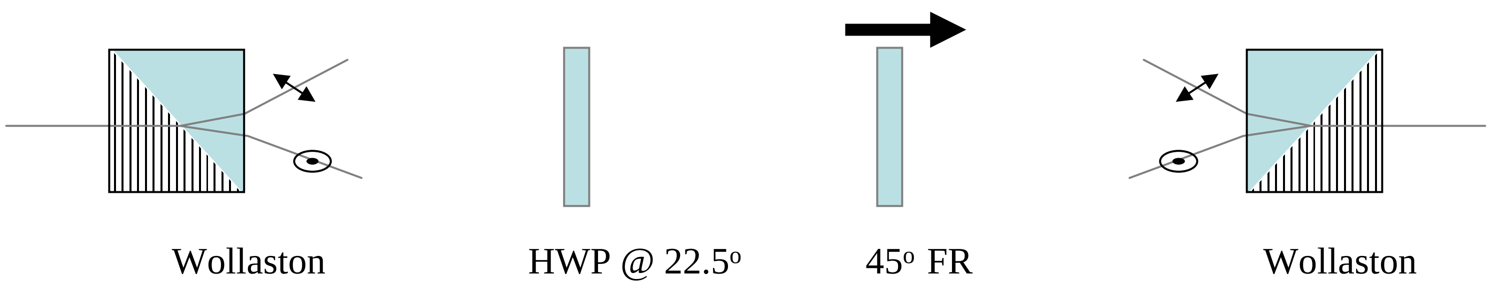
Polarization Dependent Isolator

- The polarization dependent isolator, or Faraday isolator, is made of three parts, an input polarizer (polarized vertically), a Faraday rotator, and an output polarizer, called an analyzer (polarized at 45 degrees)
- Light traveling in the forward direction becomes polarized vertically by the input polarizer. The Faraday rotator will rotate the polarization by 45 degrees. The analyser then enables the light to be transmitted through the isolator.
- Light traveling in the backward direction becomes polarized at 45 degrees by the analyzer. The Faraday rotator will again rotate the polarization by 45 degrees. This means the light is polarized horizontally (the rotation is insensitive to direction of propagation). Since the polarizer is vertically aligned, the light will be extinguished

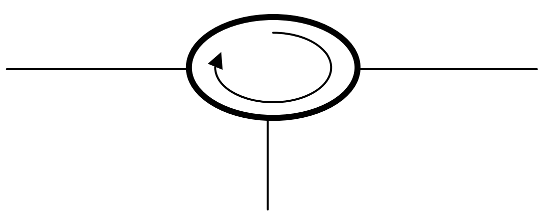


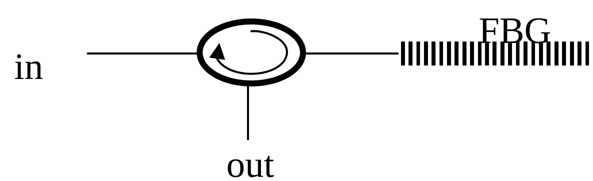
Nonreciprocal optics
aka Faraday rotators

Example: Polarization **independent isolator**



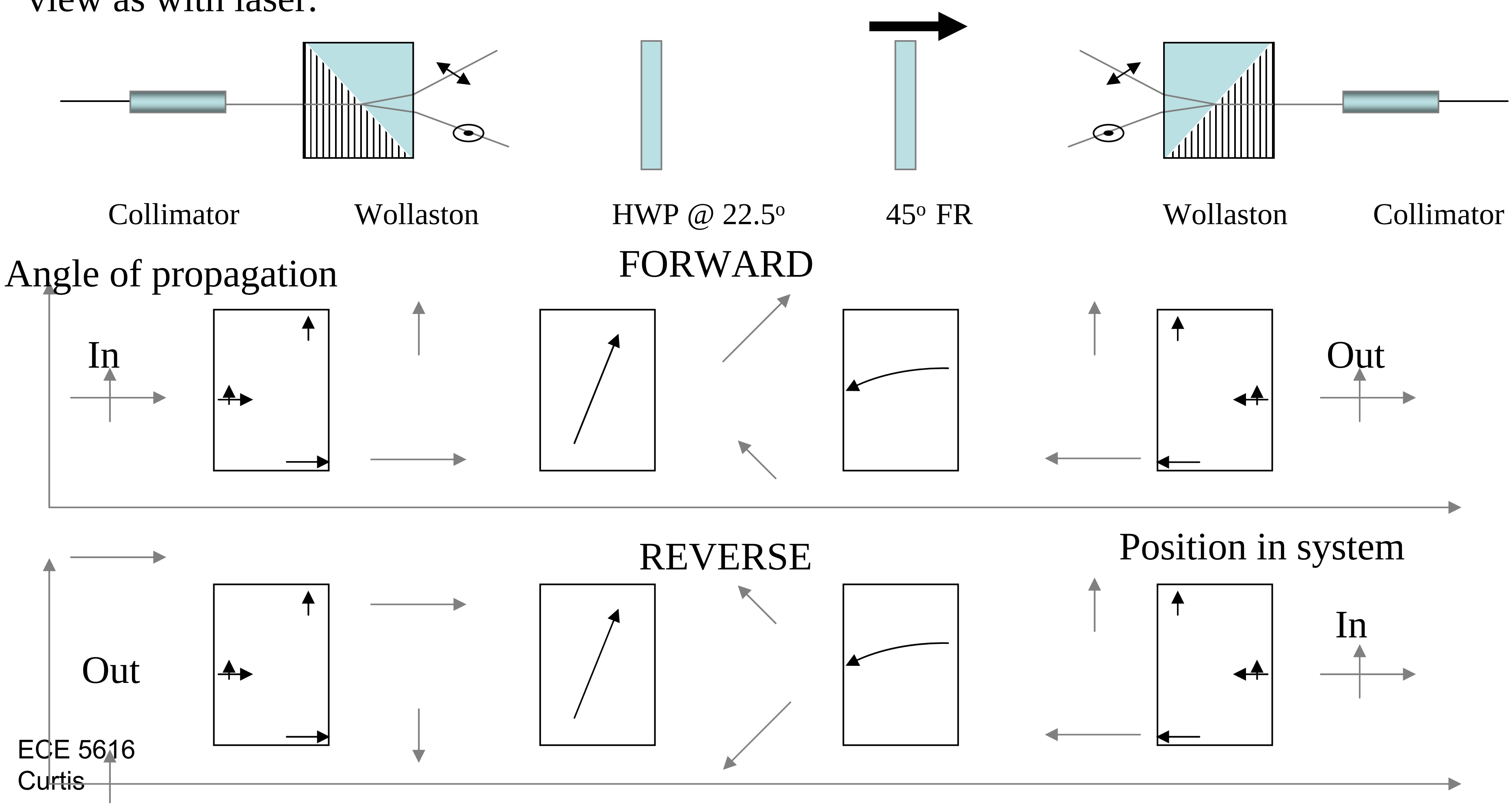
Similar optics can form a pseudo-circulator:



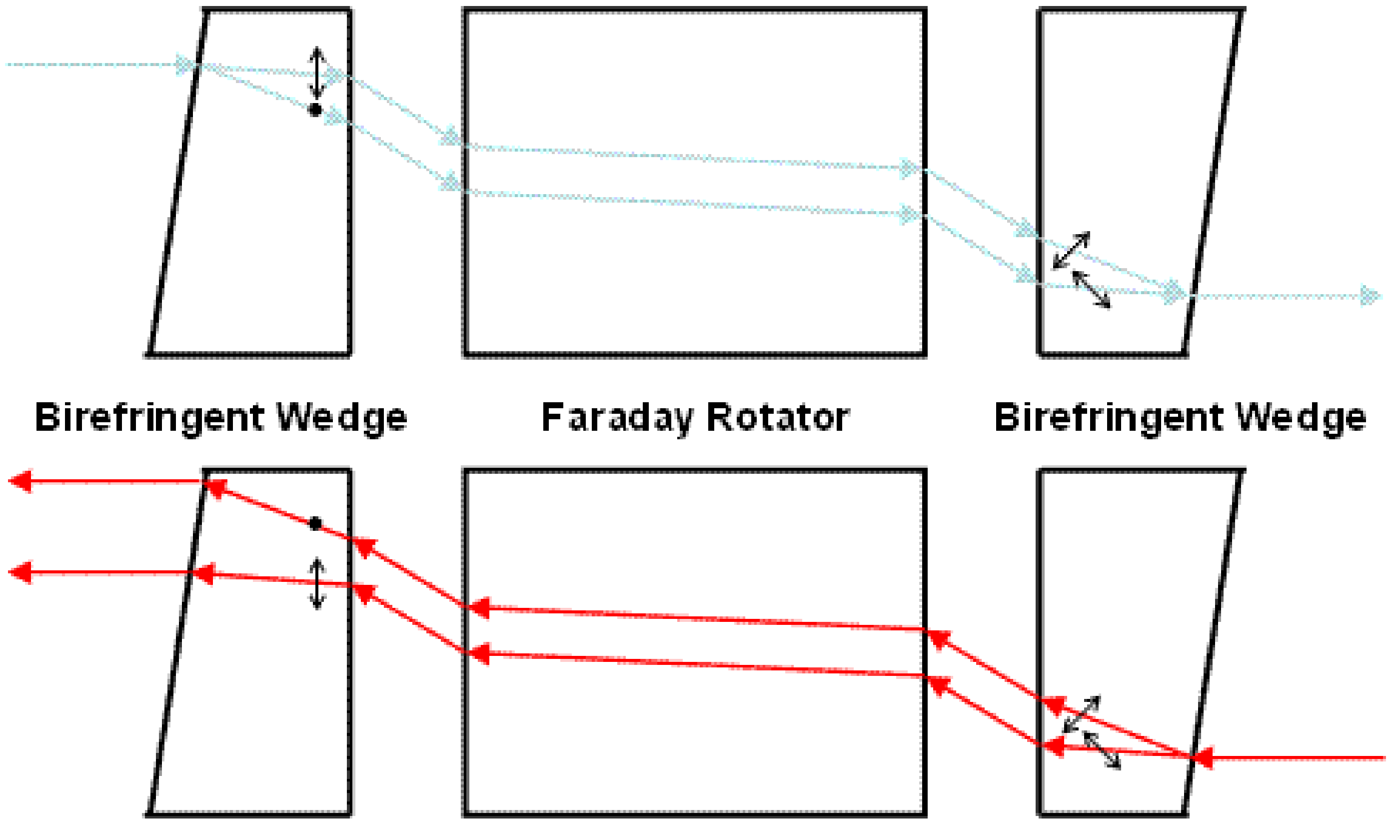


Design notation for polarization manipulation

View of polarization as if looking into laser. This is the EE convention and yields RHC with thumb pointing into viewers eyes, along with laser. To use the Physics convention, consider view as with laser.

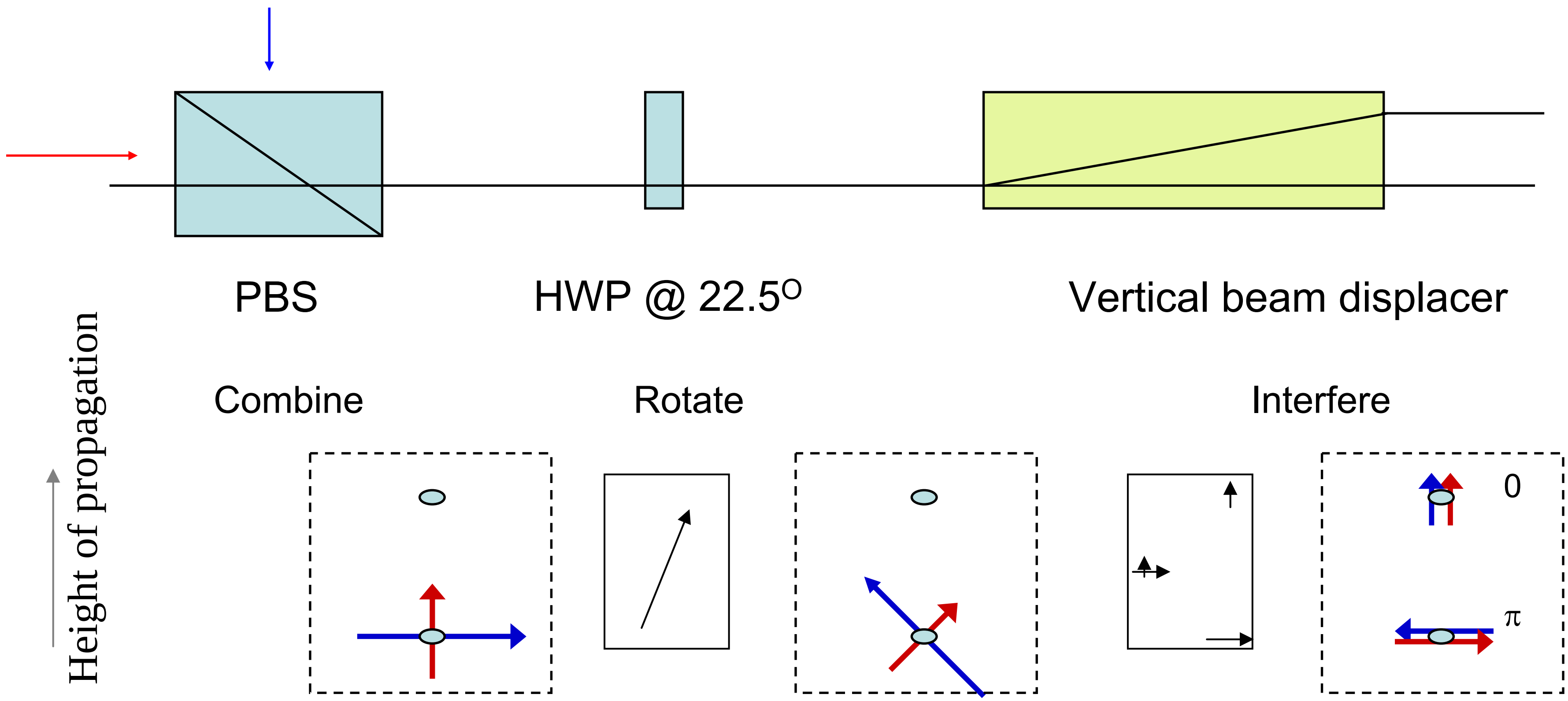


Polarization Independent Isolator



Another example

Spatial phase-shifting interferometer



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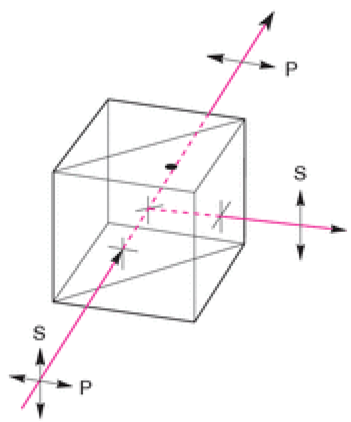
Many thanks to Bill Chang for teaching me this notation.

PBS Example



NOTE: To avoid damage, beam must enter prism on the side marked with a dot.

Material	SF 2, NSSK grade, precision annealed optical glass
Wavefront Distortion	≤λ/4 at 632.8 nm over the clear aperture
Clear Aperture	Central diameter, >80% of dimension
Surface Quality	20-10 scratch-dig
Efficiency	T_p >80%, >90% average, R_s >99.5% average
Extinction Ratio	T_p/T_s >500:1, 1000:1 average
Transmitted Beam Deviation	≤5 arc min
Reflected Beam Deviation	90° ±5 arc min
Angle of Incidence	0° ±5°
Dimensions Tolerance	±0.25 mm
Antireflection Coating	Broadband, multilayer coating, R_{avg} <1.0% per surface
Temperature Range	-50 °C–90 °C
Durability	MIL-C-675C
Cleaning	Non-abrasive method, acetone or isopropyl alcohol on lens tissue recommended (see Care and Cleaning of Optics) Cemented optic, do not immerse in a solvent
Damage Threshold	2 kW/cm² CW, 1 J/cm² with 10 nsec pulses, typical

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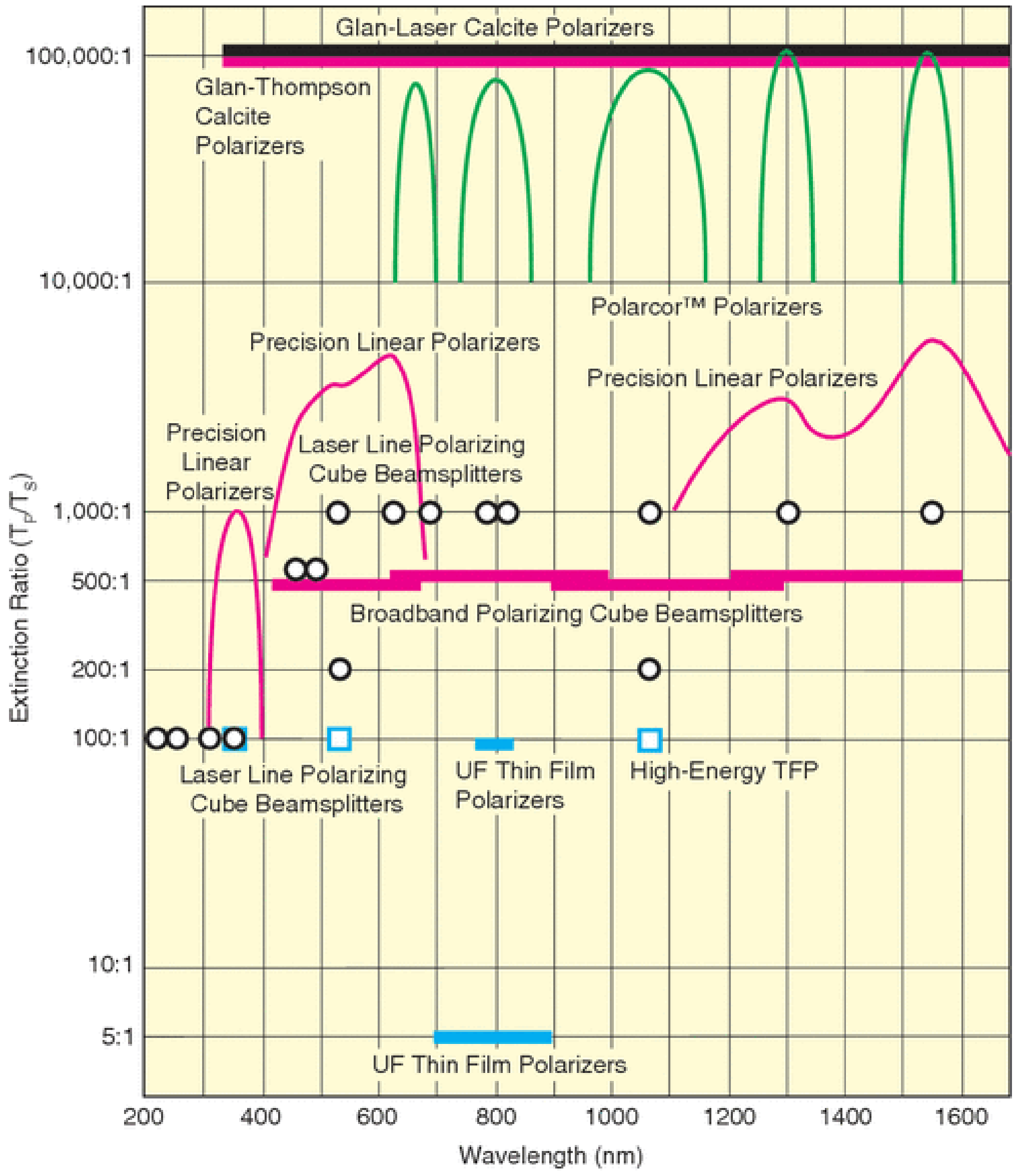
Polarizers



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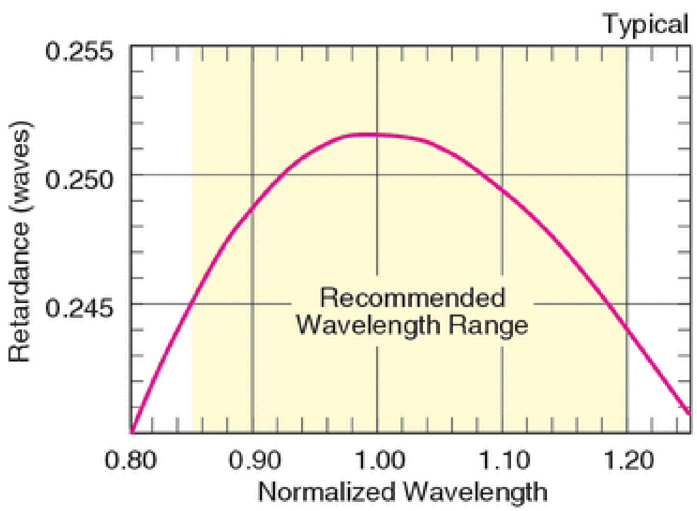
Zero Order Waveplates



For . $\Gamma = \lambda/2$ the plate is called a „0" order half-wave plate. Thickness of the plate is:

$$d = \frac{\lambda}{2(n_e - n_o)}$$

Retarder Material	Birefringent polymer film stack
Substrate Material	BK 7, grade A, fine annealed optical glass
Retardation	λ/4 or λ/2
Retardation Accuracy	±λ/100
Wavefront Distortion	≤λ/4 at 632.8 nm over the full aperture
Clear Aperture	10.2 mm
Surface Quality	40-20 scratch-dig
Transmitted Beam Deviation	≤1 arc min
Acceptance Angle	± 7°
Thickness	3.56 mm
Housing Diameter	25.4 ±0.13 mm
Housing Thickness	6.2 mm
Temperature Range	-20 °C to 50 °C
Antireflection Coating	Broadband, multilayer coating, R_{avg} <0.5%
Cleaning	Non-abrasive method, acetone or isopropyl alcohol
Damage Threshold	500 W/cm^2 CW, 0.3 J/cm^2 with 10 nsec pulses, visible
Housing	Black anodized aluminum

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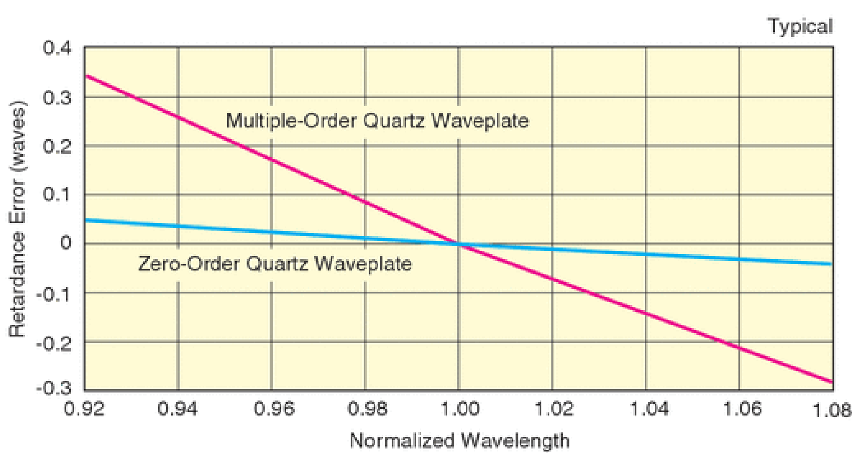
Multiple order Waveplates

Must order for laser line
Incident angles must be close to normal
Multiple nulls with polarizer



Material	Quartz, schlieren grade
Construction	Single plate
Retardation	λ/4 or λ/2
Retardation Accuracy	±λ/300 at 20°C ±1°C
Wavefront Distortion	≤λ/10 at 632.8 nm over the full aperture
Clear Aperture	≥central 90% of diameter
Surface Quality	10-5 scratch-dig
Wedge	<0.5 arc sec
Diameter Tolerance	+0/-0.25 mm
Reflectivity per Surface	Single wavelength: R <0.5% total, or 0.25% per surface Dual wavelength: R_{avg} <1.5%
Thickness	Single wavelength: 1 mm, nominal Dual wavelength: 0.5–4 mm, nominal
Cleaning	Non-abrasive method, acetone or isopropyl alcohol on lens tissue recommended, caution: fragile, thin optic
Damage Threshold	2 MW/cm² CW, 2 J/cm² with 10 nsec pulses

Newport Catalogue