1. The Drude model expresses the wavelength dependence of the relative dielectric constant $\epsilon_r(\lambda) = \epsilon_r(\lambda) - j\epsilon_r(\lambda)$ of materials with highly mobile electrons by the expression

$$\epsilon(\lambda) = 1 - \frac{\lambda^2 \lambda_r}{\lambda_p^2 (\lambda_r - j\lambda)}$$

where $\lambda_p$ is the plasma wavelength and $\lambda_r$ is the relaxation wavelength. Assuming $\lambda_p = 0.1 \mu m$ and $\lambda_r = 100 \mu m$,

(a) Sketch the variation of $\epsilon_r(\lambda)$ as a function of $\log_{10}(\lambda)$ for $-7 \leq \log_{10}(\lambda) \leq 1$ (that is from $0.1 \mu m \leq \lambda \leq 10 m$),

(b) Sketch the variation of $\epsilon_r(\lambda)$ as a function of $\log_{10}(\lambda)$ for $-7 \leq \log_{10}(\lambda) \leq 1$,

(c) estimate the value of complex index of refraction $N = n - jK$ at $\lambda = 1 \mu m$ and at $\lambda = 10 m$,

(d) calculate the reflectivity $R$ of a slab of this material (silver-like) at at $\lambda = 1 \mu m$ and at $\lambda = 10 m$.

2. A lifeguard at a mountain lake becomes aware of an emergency situation. A child is going down for the third time 10 meters out into the lake. The shorefront runs from north to south. The lifeguard is 10 meters west of the shore and 50 meter north of the child (making her 20 meters west of the child) when she becomes aware of the dire situation. She knows that she can run at 15 m/s on sand (watch out Usain Bolt) and can swim at 1.5m/s in the lake water (likewise for Michael Phelps). She rapidly needs to plot a path to the victim that minimizes her time of arrival at the child’s location in the water. Assuming she will run along one straight line path in the sand and swim along another straight line path in the water,

(a) What is the ratio of the sin of the angle (normal to the lakeshore), sin $\theta_i$, she will run along on the beach to the angle she will swim at in the water sin $\theta_t$.

(b) Calculate what her initial angle (relative to the normal to the lakeshore) should be to minimize her time of arrival.

(c) Although the lifeguard is working on her PhD in engineering, she cannot immediately calculate angles to milli-radians in her head. What is the best strategy to saving the life?

(d) Calculate the true minimum time to arrival versus the time to arrival in a simpler approximate solution. Will the lifeguard be sued?

3. The spatial coherence length of an incoherent source with aperture $d$ scales linearly with the observation distance, inversely with aperture opening with a proportionality of $\lambda$.

(a) Calculate the coherence of sunlight ($\bar{\lambda}$) at the surface of the earth given that the radius of the sun is $7 \times 10^8 m$ at a distance of $1.6 \times 10^{11} m$. 

(b) Grimaldi needs to carry out a diffraction experiment in his castle cloister using a 1 cm aperture. His window is a meter from his optical table. Calculate the aperture size he will need to use in the window in order to sufficiently increase his coherence length to the aperture size.

(c) Grimaldi decides it would be better to use starlight rather than sunlight for his experiment. Calculate the distance at which a star of solar radius would need to be at in order to produce a 1 cm coherence length on the surface of the earth. Is there a problem?

(d) For a last ditch attempt, Grimaldi decides to use a candle as a light source. Calculate the size of the pinhole he would have to place next to the candle in order to have a 1 cm coherence length at a 1 m distance from the candle. Describe how this solution compares with the original attempt with sunlight.

(e) Grimaldi decides to call his friend Galileo to borrow two lenses. Describe briefly how he should use these lenses. Estimate how much sunlight could be captured by 10 cm lenses given the initial coherence length and size of the pinhole.