ECEN 5645
Introduction to Optoelectronics
Class Meeting 1

Plane and Diverging Waves
Today’s Topics

• Course Overview
  – Instructor/grader, website and grading
• Course Objectives
• Course Outline
• Introduction to Optoelectronics
  – Wave Optics
    • Plane and Diverging Waves
Instructor, Grader, office hours

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    • M 1:00 – 1:50 p. m.
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• Grader – Keyon Janani
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On the Website
ecee.colorado.edu/~ecen5645/

- [http://eccee.colorado.edu/~ecen5645/](http://eccee.colorado.edu/~ecen5645/)
- Syllabus/Schedule
- Homeworks are posted
- Powerpoint lecture slides will be posted as pdf’s after class
- Other useful material will be posted such as class announcements
Grading

• Grading
  • 11 homeworks (40%)
    • A major portion of the grade will come from assigned problems presented in-class
  • 3 midterms (45%)
    • 9/24/15, 10/29/15 and 12/03/15
  • Project (15%)
    • Project proposal
    • Progress report with completion schedule
    • In-class presentation
    • Final Report
Course Objectives

• **Overall:** Preparation for analysis, use and design of information bearing optoelectronic systems
  
  – Employ wave concepts in description of energy flow in guided wave systems
  
  – Conceptualize information flow in optical communication system
  
  – Determine salient characteristics of electrical signals extracted from optical carriers including optical, electrical and conversion noise
Course Outline and Schedule

• The Wave Nature of Light (5 weeks)
• Dielectric Waveguides and Optical Fibers (5 Weeks)
• Photodetection (4 Weeks)
• Project Presentations (1 Week)
Today

Plane and Diverging Waves

• Waves
• Phase velocity
• Wavefronts
• The wave equation
• Gaussian beams

Note: The following slides are primarily from S. O. Kasap
Chapter 1  Wave Nature of Light

Physicists use the wave theory on Mondays, Wednesdays and Fridays and the particle theory on Tuesdays, Thursdays and Saturdays.

—Sir William Henry Bragg

Augustin Jean Fresnel (1788–1827) was a French physicist and a civil engineer for the French government who was one of the principal proponents of the wave theory of light. He made a number of distinct contributions to optics including the well-known Fresnel lens that was used in lighthouses in the nineteenth century. He fell out with Napoleon in 1815 and was subsequently put under house arrest until the end of Napoleon’s reign. During his enforced leisure time he formulated his wave ideas of light into a mathematical theory.  （© INTERFOTO/Alamy.）

If you cannot saw with a file or file with a saw, then you will be no good as an experimentalist.

—Attributed to Augustin Fresnel
Light is an electromagnetic wave

An electromagnetic wave is a traveling wave that has time-varying electric and magnetic fields that are perpendicular to each other and the direction of propagation $z$. 
\[ E_x = E_o \cos(\omega t - kz + \phi_o) \]

\(E_x\) = Electric field along \(x\) at position \(z\) at time \(t\)
\(k\) = Propagation constant = \(2\pi/\lambda\)
\(\lambda\) = Wavelength
\(\omega\) = Angular frequency = \(2\pi \nu\) (\(\nu\) = frequency)
\(E_o\) = Amplitude of the wave
\(\phi_o\) = Phase constant; at \(t = 0\) and \(z = 0\), \(E_x\) may or may not necessarily be zero depending on the choice of origin.

\((\omega t - kz + \phi_o) = \phi = \text{Phase of the wave}\)

This is a monochromatic plane wave of infinite extent traveling in the positive \(z\) direction.
Wavefront

A surface over which the phase of a wave is constant is referred to as a **wavefront**

A wavefront of a plane wave is a plane **perpendicular** to the direction of propagation

The interaction of a light wave with a nonconducting medium (conductivity = 0) uses the electric field component $E_x$ rather than $B_y$.

**Optical field** refers to the electric field $E_x$. 
A plane EM wave traveling along \( z \), has the same \( E_x \) (or \( B_y \)) at any point in a given \( xy \) plane. All electric field vectors in a given \( xy \) plane are therefore in phase. The \( xy \) planes are of infinite extent in the \( x \) and \( y \) directions.
Phase Velocity

The time and space evolution of a given phase $\phi$, for example that corresponding to a maximum field is described by

$$\phi = \omega t - kz + \phi_o = \text{constant}$$

During a time interval $\delta t$, this constant phase (and hence the maximum field) moves a distance $\delta z$. The phase velocity of this wave is therefore $\delta z / \delta t$. The phase velocity $v$ is

$$v = \frac{\delta z}{\delta t} = \frac{\omega}{k} = v \lambda$$
Phase change over a distance $\Delta z$

$$\phi = \omega t - kz + \phi_o$$

$$\Delta \phi = k\Delta z$$

The phase difference between two points separated by $\Delta z$ is simply $k\Delta z$

since $\omega t$ is the same for each point

If this phase difference is 0 or multiples of $2\pi$ then the two points are in phase. Thus, the phase difference $\Delta \phi$ can be expressed as $k\Delta z$ or $2\pi \Delta z/\lambda$
Recall that
\[ \cos \phi = \text{Re}[\exp(j \phi)] \]

where \( \text{Re} \) refers to the real part. We then need to take the real part of any complex result at the end of calculations. Thus,

\[ E_x(z,t) = \text{Re}[E_o \exp(j \phi_o) \exp(j(\omega t - kz))] \]

or

\[ E_x(z,t) = \text{Re}[E_c \exp j(\omega t - kz)] \]

where \( E_c = E_o \exp(j \phi_o) \) is a complex number that represents the amplitude of the wave and includes the constant phase information \( \phi_o \).
Wave Vector or Propagation Vector

Direction of propagation is indicated with a vector \( \mathbf{k} \), called the wave vector, whose magnitude is the propagation constant, \( k = \frac{2\pi}{\lambda} \). \( \mathbf{k} \) is perpendicular to constant phase planes.

When the electromagnetic (EM) wave is propagating along some arbitrary direction \( \mathbf{k} \), then the electric field \( E(\mathbf{r},t) \) at a point \( \mathbf{r} \) on a plane perpendicular to \( \mathbf{k} \) is

\[
E(\mathbf{r},t) = E_o \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_o)
\]

If propagation is along \( z \), \( \mathbf{k} \cdot \mathbf{r} \) becomes \( kz \). In general, if \( \mathbf{k} \) has components \( k_x, k_y \) and \( k_z \) along \( x, y \) and \( z \), then from the definition of the dot product, \( \mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z \).
A traveling plane EM wave along a direction $\mathbf{k}$

$$E(\mathbf{r},t) = E_o \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_o)$$
Maxwell’s Wave Equation

\[
\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \varepsilon_o \varepsilon_r \mu_o \frac{\partial^2 E}{\partial t^2} = 0
\]

A plane wave is a solution of Maxwell’s wave equation

\[
E_x = E_o \cos(\omega t - kz + \phi_o)
\]

Substitute into Maxwell’s Equation to show that this is a solution.
Spherical Wave

$$E = \frac{A}{r} \cos(\omega t - kr)$$

Wavefronts
Examples of possible EM waves

Optical divergence refers to the angular separation of wave vectors on a given wavefront.
The radiation emitted from a laser can be approximated by a Gaussian beam. Gaussian beam approximations are widely used in photonics.
Gaussian Beam

The intensity across the beam follows a Gaussian distribution

\[ I(r,z) = \frac{2P}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) \]

\[ \theta = \frac{w}{z} = \frac{\lambda}{\pi w_0} \]

\[ 2\theta = \text{Far field divergence} \]
The Gaussian Intensity Distribution is Not Unusual

The Gaussian intensity distribution is also used in fiber optics. The fundamental mode in single mode fibers can be approximated with a Gaussian intensity distribution across the fiber core.

\[
I(r) = I(0) \exp\left(-\frac{2r^2}{w^2}\right)
\]
\[
2\theta = \text{Far field divergence}
\]

\[
2^{1/2}w_o
\]

\[
\theta = \text{Rayleigh range}
\]

\[
z_o = \frac{\pi w_o^2}{\lambda}
\]
Gaussian Beam

Rayleigh range

\[ Z_o = \frac{\pi W_o^2}{\lambda} \]

\[ 2w = 2w_o \left[ 1 + \left( \frac{z}{z_o} \right)^2 \right]^{1/2} \]

\[ 2w = 2w_o \left[ 1 + \left( \frac{z\lambda}{\pi W_o^2} \right)^2 \right]^{1/2} \]
Real and Ideal Gaussian Beams

Real beam $M^2 > 1$
Gaussian beam $M^2 = 1$

Definition of $M^2$

\[
M^2 = \frac{w_{or} \theta_r}{w_o \theta} = \frac{w_{or} \theta_r}{(\lambda / \pi)}
\]

\[2w_r = 2w_{or} \left[ 1 + \left( \frac{z \lambda M^2}{\pi w_{or}^2} \right)^2 \right]^{1/2}
\]
Real Gaussian Beam

Real beam $M^2 > 1$

Gaussian beam $M^2 = 1$

$w_{or} = w_o$

$2w_r = 2w_{or} \left[ 1 + \left( \frac{z \lambda M^2}{\pi w^2_{or}} \right)^2 \right]^{1/2}$

Correction note: Page 10 in textbook, Equation (1.11.1), $w$ should be $w_r$ as above and $w_{or}$ should be squared in the parantheses.
Two spherical mirrors reflect waves to and from each other. The optical cavity contains a Gaussian beam. This particular optical cavity is symmetric and confocal; the two focal points coincide at $F$. 
EXAMPLE 1.1.1 A diverging laser beam

Consider a He-Ne laser beam at 633 nm with a spot size of 1 mm. Assuming a Gaussian beam, what is the divergence of the beam? What are the Rayleigh range and the beam width at 25 m?

Solution

Using Eq. (1.1.7), we find

$$2\theta = \frac{4\lambda}{\pi(2w_o)} = \frac{4(633 \times 10^{-9} \text{ m})}{\pi(1 \times 10^{-3} \text{ m})} = 8.06 \times 10^{-4} \text{ rad} = 0.046^\circ$$

The Rayleigh range is

$$z_o = \frac{\pi w_o^2}{\lambda} = \frac{\pi[(1 \times 10^{-3} \text{ m})/2]^2}{(633 \times 10^{-9} \text{ m})} = 1.24 \text{ m}$$

The beam width at a distance of 25 m is

$$2w = 2w_o \left[1 + \left(\frac{z}{z_o}\right)^2\right]^{1/2} = (1 \times 10^{-3} \text{ m}) \left[1 + \left(\frac{25 \text{ m}}{1.24 \text{ m}}\right)^2\right]^{1/2} = 0.0202 \text{ m} \approx 20 \text{ mm}.$$