ECEN 5645
Introduction to Optoelectronics
Class Meeting 14
Step Index Fiber II
Today’s Topics

• Types of step index fiber
  – Weakly guiding glass – single and multimode
  – Specialty fiber
  – Silicon wire
  – Plastic fiber

• Simplified picture of fiber propagation
  – Parameters and dimensionless propagation constant
  – Phase and group delay
  – Mode field diameter

• Examples of weakly guiding fibers
The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. The coordinates $r$, $\phi$, $z$ are used to represent any point $P$ in the fiber. Cladding is normally much thicker than shown.
Important Fiber Parameters

\[ M = \frac{V^2}{2} \quad V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} NA \]

\[ \frac{\Delta \tau}{L} = \frac{n_1 - n_2}{c} = \frac{NA^2}{(n_1 + n_2 + 2)^2c} \]

\[ a_{SM} \leq \frac{2.4\lambda}{2\pi NA} \]

- We consider the number of modes, multimode dispersion per unit length and single mode radius
Numerical Aperture \( NA \)

Maximum acceptance angle \( \alpha_{\text{max}} \) is that which just gives total internal reflection at the core-cladding interface, i.e. when \( \alpha = \alpha_{\text{max}} \) then \( \theta = \theta_c \). Rays with \( \alpha > \alpha_{\text{max}} \) (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

\[
NA = \left( n_1^2 - n_2^2 \right)^{1/2} \\
\sin \alpha_{\text{max}} = \frac{\left( n_1^2 - n_2^2 \right)^{1/2}}{n_o} = \frac{NA}{n_o} \\
V = \frac{2\pi a}{\lambda} NA
\]

\( 2\alpha_{\text{max}} = \) total acceptance angle

\( NA \) is an important factor in light launching designs into the optical fiber.
Different Types of Step Index Fiber

- Weakly guiding multimode
  - Historical telecom
  - Data communication
  - Two mode
- Weakly guiding single mode
- Specialty fiber – holey fiber
- Plastic fiber
- Silicon nanowire
Weakly Guiding SI Telecom Fiber

\[ a = 25 \mu m \]
\[ NA = 0.2 \]
\[ @0.85 \mu m \rightarrow V \approx 37 \]
\[ M \approx 685 \]

\[
\Delta \tau \frac{L}{L} = \left(0.2\right)^2 \times 3.3 \frac{\mu s}{km} = 37 \frac{ns}{km}
\]

\[
a_{SM} \leq \frac{2.4 \times 0.85}{1.26} \approx 1.62 \mu m
\]

- Step index MM telecom was not used
Weakly Guiding SI Datacom Fiber

\[ a = 50 \mu m \]
\[ \text{NA} = 0.3 \]
\[ \text{@} 0.85 \mu m \rightarrow V \approx 110 \]
\[ M \approx 11500 \]

\[ \frac{\Delta T}{L} = \frac{(0.3)^2}{3.} \times 3.3 \frac{\mu s}{\text{km}} = 100 \frac{\text{ns}}{\text{km}} \]

\[ a_{SM} \leq \frac{2.4 \times 0.85}{1.6} \approx 1.10 \mu m \]

- Step index MM telecom was not used
Graded Index (GRIN) Fiber

(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.

(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.
Graded versus Step

- As far as glass (germanosilicate) fiber is concerned, there are equivalent to fabricate
- The dispersion of graded index fiber is two orders of magnitude smaller than step
- Step index glass fiber has found no application
- Multimode fiber is easier to align/splice/etc
- Data communications is dominated by graded index germanosilicate fiber
Weakly Guiding SM Telecom Fiber

\[ a = 5 \mu m \]
\[ NA = 0.1 \]
\[ @1.3 \mu m \rightarrow V \approx 1.89 \]
\[ M \approx 2 \]
\[ a_{SM} \leq \frac{2.4 \times 1.55}{0.63} \approx 6.0 \mu m \]

- When the fiber is single mode (two polarization) there is no MM dispersion
- Dispersion in intermodal (material + WG)
Weakly Guiding Few M Pump Fiber

- Mode control is not possible for large $M$
- With only LP01 and LP11, there is little mode coupling yet modes overlap significantly

$a = 8\mu m$
$NA = 0.1$
$\lambda = 1.55 \mu m\rightarrow V \approx 3.5$
$M \approx 6$

$$a_{SM} \leq \frac{2.4 \times 1.55}{0.63} \approx 6.0 \mu m$$
Specialty Fiber: Holey Fiber

- Most common use is comb generation
- Multicore is hard to align
Plastic Fiber

\[ a = 6000 \mu m \]

\[ \text{NA} = 1. \]

\[ @0.6\mu m \quad \rightarrow \quad V \approx 10^5 \]

\[ M \approx 5 \times 10^9 \]

\[ \frac{\Delta T}{L} = \frac{(1)^2}{3} \times 3.3 \frac{\mu s}{km} = 1.1 \frac{\mu s}{km} \]

\[ a_{SM} \leq \frac{2.4 \times 0.6}{12} \approx 0.12 \mu m \]

- 6 mm core PMMA fiber is MM op in the visible
Silicon Nanowire

\[
\begin{align*}
\text{a} &= 100\text{nm} \\
\text{NA} &= 3.5 \\
\text{at } 1.3\mu\text{m} \\&\implies V \approx 1 \\
\text{M} &\approx 0.5
\end{align*}
\]

\[
a_{SM} \leq \frac{2.4 \times 1.3}{12 \times 3.5} \approx 0.12\mu\text{m}
\]

- Nanowire, nanotubes, etc. are easily to produce and considered for light bundles
Simplified picture of fiber propagation

- Parameters and dimensionless propagation constant
- Phase and group delay
- Mode field diameter
Optical Fiber Parameters

\[ n = \frac{(n_1 + n_2)}{2} = \text{average refractive index} \]

\[ \Delta = \text{normalized index difference} \]

\[ \Delta = \frac{(n_1 - n_2)}{n_1} \approx \frac{(n_1^2 - n_2^2)}{2} \]

V-number

\[ V = \frac{2\pi a}{\lambda} (n_1^2 - n_2^2)^{1/2} = \frac{2\pi a}{\lambda} (2n_1 n\Delta)^{1/2} \]

\( V < 2.405 \) only 1 mode exists. Fundamental mode

\( V < 2.405 \) or \( \lambda > \lambda_c \) Single mode fiber

\( V > 2.405 \) Multimode fiber

Number of modes

\[ M \approx \frac{V^2}{2} \]
Modes in an Optical Fiber

Normalized propagation constant

\[ b = \frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2} \]

\[ k = \frac{2\pi}{\lambda} \]

Normalized propagation constant \( b \) vs. \( V \)-number for a step-index fiber for various LP modes

\[ b \approx \left( 1.1428 - \frac{0.996}{V} \right)^2 \quad (1.5 < V < 2.5) \]
Group Velocity and Group Delay

Consider a single mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 µm, operating at 1.5 µm. What are the group velocity and group delay at this wavelength?

\[
b \approx \left( 1.1428 - \frac{0.996}{V} \right)^2 \quad 1.5 < V < 2.5
\]

\[
b = \frac{(\beta / k) - n_2}{n_1 - n_2} \quad \rightarrow \quad \beta = n_2 k [1 + b \Delta]
\]

\[k = \frac{2\pi}{\lambda} = 4,188,790 \text{ m}^{-1} \quad \text{and} \quad \omega = \frac{2\pi c}{\lambda} = 1.255757 \times 10^{15} \text{ rad s}^{-1}\]

\[V = (2\pi a / \lambda)(n_1^2 - n_2^2)^{1/2} = 1.910088\]

\[b = 0.3860859, \quad \text{and} \quad \beta = 6.044796 \times 10^6 \text{ m}^{-1}.
\]

Increase wavelength by 0.1% and recalculate. Values in the table.
Group Velocity and Group Delay

<table>
<thead>
<tr>
<th>Calculation →</th>
<th>V</th>
<th>k (m⁻¹)</th>
<th>ω (rad s⁻¹)</th>
<th>b</th>
<th>β (m⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 1.500000 , \mu m$</td>
<td>1.910088</td>
<td>4188790</td>
<td>$1.255757 \times 10^{15}$</td>
<td>0.3860859</td>
<td>$6.044818 \times 10^6$</td>
</tr>
<tr>
<td>$\lambda' = 1.50150 , \mu m$</td>
<td>1.908180</td>
<td>4184606</td>
<td>$1.254503 \times 10^{15}$</td>
<td>0.3854382</td>
<td>$6.038757 \times 10^6$</td>
</tr>
</tbody>
</table>

$$V_g = \frac{d\omega}{d\beta} = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.254503 - 1.255757) \times 10^{15}}{(6.038757 - 6.044818) \times 10^6} \approx 2.07 \times 10^8 \, \text{m s}^{-1}$$

The group delay $\tau_g$ over 1 km is 4.83 ms
Mode Field Diameter (2w)

Electric field of the fundamental mode

Intensity in the fundamental mode LP_{01}

Note: Maximum set arbitrarily to 1

Intensity \( \propto v_g \times E(r)^2 \)

\[ E(r) = E(0) \exp\left[-\left(\frac{r}{w}\right)^2\right] \]

\[ E(r)^2 = E(0)^2 \exp[-2\left(\frac{r}{w}\right)^2] \]
Mode Field Diameter

\[ E(r)^2 = E(0)^2 \exp[-2(r/w)^2] \]

Note: Maximum set arbitrarily to 1

Intensity \( \propto v_g \times E(r)^2 \)

2\( w \) = Mode Field Diameter (MFD)

Marcuse MFD Equation

\[ 2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6}) \]

0.8 < V < 2.5

\[ 2w \approx (2a)(2.6V) \]

1.6 < V < 2.4
Mode Field Diameter ($2w$)

\[ E(r)^2 = E(0)^2 \exp[-2(r/w)^2] \]

and

\[ \text{Intensity } \propto v_g \times E(r)^2 \]

Area of a circular thin strip (annulus) with radius \( r \) is \( 2\pi rdr \). Power passing through this strip is proportional to \( E(r)^2(2\pi r)dr \)

\[
\text{Fraction of optical power within MFD} = \frac{\int_{0}^{w} E(r)^2 2\pi r dr}{\int_{0}^{\infty} E(r)^2 2\pi r dr} = 0.865
\]
\[ E(r)^2 = E(0)^2 \exp[-2(r/w)^2] \]

Power density

\[ E(r)^2 \]

86% of total power

Gaussian

\[ 1/e^2 \]

\[ w \]

\[ 0 \]

\[ -w \]

\[ 2a \]

\[ 2w \]

Mode Field Diameter (2w)

86% of total power

Fraction of optical power within MFD = 86%

This is the same as the fraction of optical power within a radius \( w \) from the axis of a Gaussian beam (See Chapter 1)
Examples of Weakly Guiding Fibers

- Typical core diameters are the order of 50 microns and NA’s range from 0.2 to 0.4
Example: A multimode fiber

Calculate the number of allowed modes in a multimode step index fiber which has a core of refractive index of 1.468 and diameter 100 µm, and a cladding of refractive index of 1.447 if the source wavelength is 850 nm.

Solution:

Substitute, $a = 50$ µm, $\lambda = 0.850$ µm, $n_1 = 1.468$, $n_2 = 1.447$ into the expression for the $V$-number,

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi \times 50}{0.850}(1.468^2 - 1.447^2)^{1/2} = 91.44.$$  

Since $V \gg 2.405$, the number of modes is

$$M \approx \frac{V^2}{2} = \frac{(91.44)^2}{2} = 4181$$

which is large.
Example: A single mode fiber

What should be the core radius of a single mode fiber which has a core of $n_1 = 1.4680$, cladding of $n_2 = 1.447$ and it is to be used with a source wavelength of 1.3 μm?

Solution:

For single mode propagation, $V \leq 2.405$. We need,

$$V = \left(\frac{2\pi a}{\lambda}\right)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

or

$$\left[\frac{2\pi a}{(1.3 \ \mu m)}\right]\left(1.468^2 - 1.447^2\right)^{1/2} \leq 2.405$$

which gives $a \leq 2.01 \ \mu m$.

Rather thin for easy coupling of the fiber to a light source or to another fiber; $a$ is comparable to $\lambda$ which means that the geometric ray picture, strictly, cannot be used to describe light propagation.
Example: Single mode cut-off wavelength

Calculate the cut-off wavelength for single mode operation for a fiber that has a core with diameter of 8.2 µm, a refractive index of 1.4532, and a cladding of refractive index of 1.4485. What is the $V$-number and the mode field diameter (MFD) for operation at $\lambda = 1.31$ µm?

Solution:

For single mode operation,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

Substituting for $a$, $n_1$ and $n_2$ and rearranging we get,

$$\lambda > [2\pi(4.1 \text{ µm})(1.4532^2 - 1.4485^2)^{1/2}]/2.405 = 1.251 \text{ µm}$$

Wavelengths shorter than 1.251 µm give multimode propagation.

At $\lambda = 1.31$ µm,

$$V = 2\pi[(4.1 \text{ µm})/(1.31 \text{ µm})](1.4532^2 - 1.4485^2)^{1/2} = 2.30$$

Mode field diameter MFD
Solution (continued)

Mode field diameter MFD from the Marcuse Equation is

\[
2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})
= 2(4.1)[0.65 + 1.62(2.30)^{-3/2} + 2.88(2.30)^{-6}]
\]

\[
2w = 9.30 \mu m \quad \text{86% of total power is within this diameter}
\]

\[
2w = (2a)(2.6/V) = 2(4.1)(2.6/2.30) = 9.28 \mu m
\]

\[
2w = 2a[(V+1)/V] = 11.8 \mu m \quad \text{This is for a planar waveguide, and the definition is different than that for an optical fiber}
\]
Numerical Aperture NA

Maximum acceptance angle $\alpha_{\text{max}}$ is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha = \alpha_{\text{max}}$ then $\theta = \theta_c$. Rays with $\alpha > \alpha_{\text{max}}$ (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

$$NA = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\sin \alpha_{\text{max}} = \frac{\left(n_1^2 - n_2^2\right)^{1/2}}{n_o} = \frac{\text{NA}}{n_o}$$

$$2\alpha_{\text{max}} = \text{total acceptance angle}$$

$V = \frac{2\pi a}{\lambda} \text{NA}$

$NA$ is an important factor in light launching designs into the optical fiber.
Example: A multimode fiber and total acceptance angle

A step index fiber has a core diameter of 100 µm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

Solution:

The numerical aperture is

\[ NA = (n_1^2 - n_2^2)^{1/2} = (1.480^2 - 1.460^2)^{1/2} = 0.2425 \text{ or } 25.3\% \]

From, \[ \sin\alpha_{max} = \frac{NA}{n_o} = \frac{0.2425}{1} \]
Acceptance angle \[ \alpha_{max} = 14^\circ \]

Total acceptance angle \[ 2\alpha_{max} = 28^\circ \]

\( V \)-number in terms of the numerical aperture can be written as,
\[ V = \frac{(2\pi a)}{\lambda}NA = \frac{(2\pi 50 \ \mu m)}{(0.85 \ \mu m)}(0.2425) = 89.62 \]

The number of modes, \( M \approx \frac{V^2}{2} = 4016 \)

Normalized refractive index
\[ \Delta = \frac{n_1 - n_2}{n_1} = 0.0135 \text{ or } 1.35\% \]
Example: A single mode fiber

A typical single mode optical fiber has a core of diameter 8 µm and a refractive index of 1.460. The normalized index difference is 0.3%. The cladding diameter is 125 µm. Calculate the numerical aperture and the total acceptance angle of the fiber. What is the single mode cut-off frequency $\lambda_c$ of the fiber?

Solution:

The numerical aperture

$$NA = (n_1^2 - n_2^2)^{1/2} = [(n_1 + n_2)(n_1 - n_2)]^{1/2}$$

Substituting $(n_1 - n_2) = n_1\Delta$ and $(n_1 + n_2) \approx 2n_1$, we get

$$NA \approx [(2n_1)(n_1\Delta)]^{1/2} = n_1(2\Delta)^{1/2} = 1.46(2\times0.003)^{1/2} = 0.113 \text{ or } 11.3\%$$

The acceptance angle is given by

$$\sin\alpha_{max} = NA/n_o = 0.113/1 \text{ or } \alpha_{max} = 6.5^\circ, \text{ and } 2\alpha_{max} = 13^\circ$$

The condition for single mode propagation is $V \leq 2.405$ which corresponds to a minimum wavelength $\lambda_c$ is given by

$$\lambda_c = [2\pi a NA]/2.405 = [(2\pi)(4 \text{ µm})(0.113)]/2.405 = 1.18 \text{ µm}$$

Wavelengths shorter than 1.18 µm will result in multimode operation.