Dispersion and Bit Error Rate
Today’s Topics

• An Optical Transmission System
• A Transimpedance Amplifier
• Problem 2.15
• An Eye Diagram
• Calculating BER
• Problem 2.16
• ISI and Bitrate
An optical transmitter $T$ is fed information that is modulated onto an optical carrier, transmitted over an optical channel $C$ and converted back to electrical information in a receiver $R$. 
• A receiver front end FE is fed modulated optical intensity. The FE plus filter F (generally a TIA) convert photons to filtered voltage and converts (in D) the analog voltage to a digital stream.
A Receiver

• Consists of a detector, a front end (pre-amplifier and possibly amplifier) and decoder
• The most important figure of merit (FoM) in any communication system is the SNR
• In a digital system, the bit error rate is sometimes the most measurable FoM
• Subsequent amplification can degrade the SNR, but never increase it
An Optical Receiver

\[ i(t) = R \int_{t-\tau_d}^{t} I_o A_d dt = R P_{opt}(t) \]

\[ P_e = i^2(t) R_L \]

- The electrical SNR is the square of the optical SNR
- Pulse smearing (ISI) is evident in the limit of the integral
Sources of Noise

- Shot noise – counting noise – proportional to the square root of the number of photons received during a bit period
- Thermal noise – dominates at high bit rates – is due to thermal fluctuation of dissipation
- Excess noise

\[ P_{th} = k_B T \Delta f \]

\[ \text{SNR}_e \approx \frac{P_e}{P_{th}} \]
• Signals are not simply levels – lines have finite thickness - due to thermal noise
• Threshold levels need to be set with noise in mind
Thresholding and thermal noise

- If noise on the two levels is different, do not want to set the threshold at the signal mean
Inter Symbol Interference

- Digital decoding assumes a bit frame
- If signal extends past the frame, there is ISI
• When the dispersion is large enough then ISI alone, even without thermal noise, causes errors.
Problem 2.15

- Solution by Jordan Stone
Kasap Problem 2.15

\[ n_1 = 1.448; \quad n_2 = 1.440; \quad a = 3 \text{ um}; \quad \lambda = 1.5 \text{ um} \]

\[ b = 0.3860859 \]

\[ \beta = n_2 k [1 + b \Delta] = 6.04472 \text{ um}^{-1} \]

\[ \beta' = 6.038664 \text{ um}^{-1} \]

\[ v_g = \frac{\omega' - \omega}{\beta' - \beta} = 2.07067 \times 10^8 \frac{m}{s} \]

\[ \tau_g = \frac{1 \text{ km}}{v_g} = 4.82935 \text{ \mu s} \]

These agree with the Example to within rounding. The approximation for delta (and thus b) is a good one.
• A receiver front end FE is fed modulated optical intensity. The FE plus filter F (generally a TIA) convert photons to filtered voltage and converts (in D) the analog voltage to a digital stream.
A Receiver Realization

- Does an impedance transformation
- Includes signal buffering
- No filtering or bandwidth control though
A Transimpedance Amplifier

- A detector is a current source – need to convert current to voltage for subsequent stages
- The circuit is AC – need back bias on detector
A More Realistic TIA with Xmitter

- TIA is a good fit to a non-loading link
- Includes both the transimpedance and filtering
The electrical SNR involves the load resistance as well as the bandwidth.
• Signals are not simply levels due to thermal noise
• Threshold levels need to be set with noise in mind
• If gaussian levels are assumed, one can optimize the threshold and obtain an error function expression for the bit error rate.
• Curves are determined by error function
• BER can be improved by coherent communication and advanced coding
Problem 2.16

• Solution by Imbert Wang
From my code for Problem 1.8, I found the following refractive indices:

\[ n(0) = 1.4473 \]
\[ n(13.5) = 1.4682 \]

We can find the refractive index of the core that satisfies the condition:

\[ NA = 0.10 \]

where

\[ NA = (n_1^2 - n_2^2)^{1/2} \]
\[ (n_1^2 - 1.4473^2)^{1/2} = 0.10 \]
\[ n_1^2 = 2.105 \]
Assuming linear increase from \( n(0) \) to \( n(13.5) \):

\[
n(x) = n(0) + \frac{n(13.5) - n(0)}{13.5} \times x
\]

\[
x = \frac{(1.451 - 1.4473) \times 13.5}{1.4683 - 1.4473}
\]

\[
= 2.39
\]

Core composition should be 97.61% \( \text{SiO}_2 \)–2.39% \( \text{GeO}_2 \)
Thresholding and ISI

- The broadening of the pulses fills in the zeros as well as pulls down the levels of the ones.
• Digital decoding assumes a bit frame
• If signal extends past the frame, there is ISI
• When the dispersion is large enough then ISI alone, even without thermal noise, causes errors.
Including ISI in Decision

• Even if the lines have zero thickness, the ISI can make decisions complicated
Optical Bandwidth

An optical fiber link for transmitting analog signals and the effect of dispersion in the fiber on the bandwidth, $f_{op}$. 
Dispersion and Maximum Bit Rate

Maximum Bit Rate

\[ B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta \tau_{1/2}} \]

Dispersion

\[ \frac{\Delta \tau_{1/2}}{L} = D_{ch} \Delta \lambda_{1/2} \]

Bit Rate \times Distance is

inversely proportional to dispersion

inversely proportional to line width

of laser
Example: Bit rate and dispersion

Consider an optical fiber with a chromatic dispersion coefficient $8 \text{ ps km}^{-1} \text{ nm}^{-1}$ at an operating wavelength of $1.5 \mu m$. Calculate the bit rate distance product ($BL$), and the optical and electrical bandwidths for a 10 km fiber if a laser diode source with a FWHP linewidth $\Delta \lambda_{1/2}$ of 2 nm is used.

Solution

For FWHP dispersion,

$$\frac{\Delta \tau_{1/2}}{L} = |D_{ch}| \Delta \lambda_{1/2} = (8 \text{ ps nm}^{-1} \text{ km}^{-1})(2 \text{ nm}) = 16 \text{ ps km}^{-1}$$

Assuming a Gaussian light pulse shape, the RTZ bit rate $\times$ distance product ($BL$) is

$$BL = 0.59L/\Delta \tau_{1/2} = 0.59/(16 \text{ ps km}^{-1}) = 36.9 \text{ Gb s}^{-1} \text{ km}$$

The optical and electrical bandwidths for a 10 km fiber are

$$f_{op} = 0.75B = 0.75(36.9 \text{ Gb s}^{-1} \text{ km}) / (10 \text{ km}) = 2.8 \text{ GHz}$$

$$f_{el} = 0.70f_{op} = 1.9 \text{ GHz}$$