ECEN 5645
Introduction to Optoelectronics
Class Meeting 17

Graded Index Fiber Dispersion
Today’s Topics

• A multiplexed Fiber Link
• The Bitrate Hierarchy
  – Telecom
  – datacom
• Bitrates achievable to multimode fibers
  – Step Index
  – Graded Index
• Applications and examples
Assuming high enough SNR, rate is dispersion limited

Achievable rates are higher than single channel electrical rates – limited by TDM technology
Time Division Multiplexing (TDM)

- In TDM, bits from multiple streams are interleaved.
- N channel multiplex increases frequency N times if the input frame rates are to be same at receiver.
Telecom Transmission Rates

- Circuit Switching
- Digital Conversations
- TDM hierarchy of conversations
Circuit Switching

- The telephone network was originally analog and circuit switched, dialing set up the circuit and then 4 kHz of bandwidth was dedicated to the two way conversation.
Digital Conversations

\[ R = A \times S \times B \]

\[ R = 4\text{kHz} \times 2^{\text{samples/period}} \times 8\text{bits} \]

64kbps

- The bitrate \( R \) required for a single conversation is given by the analog bandwidth \( A \) times the sampling rate per period (SpP, Shannon requires 2 per period) times the number of bits per sample, \( \text{BpS} \), that determines the \# of levels \( = 2^{\text{BpS}} \)
Multiplexing Conversations to T1

- 24 conversations makes DS1 (T1)

\[
\left(8 \ \text{bits/channel} \times 24 \ \text{channels/frame} + 1 \ \text{framing bit/frame}\right) \times 8000 \ \text{frames/second} \\
= 1,544,000 \ \text{bits/second} \\
= 1.544 \ \text{Mbit/second}
\]
A problem with multiplexers is that they are inherently hot, like A2D’s as such devices throw away the energy that does not fit in the new slot – not yet a fundamental limitation but may be one.
## Multiplexing Conversations II

<table>
<thead>
<tr>
<th>Digital signal</th>
<th>Number of channels</th>
<th>Designation of carrier</th>
<th>Bandwidth in Megabits per second (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS0</td>
<td>1</td>
<td>None</td>
<td>0.064</td>
</tr>
<tr>
<td>DS1</td>
<td>24</td>
<td>T1</td>
<td>1.544</td>
</tr>
<tr>
<td>DS2</td>
<td>96</td>
<td>T2</td>
<td>6.312</td>
</tr>
<tr>
<td>DS3</td>
<td>672</td>
<td>T3</td>
<td>44.736</td>
</tr>
<tr>
<td>DS4</td>
<td>4032</td>
<td>T4</td>
<td>274.176</td>
</tr>
</tbody>
</table>

- Once one could switch digital streams, changing the rates was straightforward to a limit
The Synchronous Optical Network (SONET) Hierarchy

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>OC-1</td>
<td>51,84 Mbit/s</td>
<td>OC-24</td>
</tr>
<tr>
<td>OC-3</td>
<td>155,52 Mbit/s</td>
<td>OC-36</td>
</tr>
<tr>
<td>OC-9</td>
<td>466,56 Mbit/s</td>
<td>OC-48</td>
</tr>
<tr>
<td>OC-12</td>
<td>622,08 Mbit/s</td>
<td>OC-96</td>
</tr>
<tr>
<td>OC-18</td>
<td>933,12 Mbit/s</td>
<td>OC-192</td>
</tr>
</tbody>
</table>

Values of the SONET frame depending on the speed of the optical medium.

- SONET is an ITU standard – international
- OC-3, OC-12, OC-48 and OC-192 were achieved in rapid succession and are used in WDM/DWDM
Datacom Rates

- Are generally determined by the Ethernet card that is located in each computer that bridges the data link layer to the physical layer of open system interconnection model.
Ethernet Cards and Transmission Systems

- 10 Mbps – Aloha network
- 100 Mbps – FDDI (Fiber Distributed Data Interface) – used on CU campus circa 1995
- 1 Gbps – GbE (first VCSEL transmitters) – the first optical intra supercomputer links
- 10 Gbps – 10 GbE – present day supercomputer and data center short length VCSEL interconnects
- 25 Gbps – 100 GbE – 4 x 25 Gbps is likely to be the next generation but over longer length
Good time to insert a problem!

• Problem 2.17
Achievable Rates with MM Fiber

- RTZ OOK coding bit overlap
- Optical bandwidth
- Pulse shape and bitrate
- Maximum BL for a Gaussian like pulse shape
- Maximum BL for step index multimode fiber
- Maximum BL for parabolic index multimode fiber
NRZ and RTZ

Information:

| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

NRZ

RZ
A Gaussian output light pulse and some tolerable intersymbol interference between two consecutive output light pulses (y-axis in relative units). At time $t = \sigma$ from the pulse center, the relative magnitude is $e^{-1/2} = 0.607$ and full width root mean square (rms) spread is $\Delta \tau_{\text{rms}} = 2\sigma$. (The RTZ case)
An optical fiber link for transmitting analog signals and the effect of dispersion in the fiber on the bandwidth, $f_{op}$.
### Pulse Shape and Maximum Bit Rate

#### TABLE 2.4 Relationships between dispersion parameters, maximum bit rates, and bandwidths

<table>
<thead>
<tr>
<th>Dispersed pulse shape</th>
<th>Pulse shape and FWHM width, $\Delta \tau_{1/2}$</th>
<th>$\Delta \tau_{1/2}$ FWHM width</th>
<th>$B$ (RZ)</th>
<th>$f_{op}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian with rms deviation $\sigma$</td>
<td><img src="image1" alt="Gaussian pulse" /></td>
<td>$\Delta \tau_{1/2} = 2.353\sigma$</td>
<td>$0.25/\sigma$</td>
<td>$0.75B = 0.19/\sigma$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma = 0.425\Delta \tau_{1/2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular pulse with full-width $\Delta T$</td>
<td><img src="image2" alt="Triangular pulse" /></td>
<td>$\Delta \tau_{1/2} = 0.5\Delta T = (6^{1/2})\sigma$</td>
<td>$0.25/\sigma$</td>
<td>$0.99B = 0.247/\sigma$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma = \Delta \tau_{1/2}/2.45 = 0.408\Delta \tau_{1/2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular with full-width $\Delta T$</td>
<td><img src="image3" alt="Rectangular pulse" /></td>
<td>$\sigma = 0.289\Delta T = 0.289\Delta \tau_{1/2}$</td>
<td>$&lt;1/\Delta T$</td>
<td>$0.69B = 0.17/\sigma$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta \tau_{1/2} = \Delta T = (2)(3^{1/2})\sigma$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Note: RZ = Return-to-zero pulses.
Maximum Bitrate for Given Spreading

\[ B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta \tau_{1/2}} \]

\[ BL \approx \frac{0.5L}{\Delta \tau} \]

- Maximum bandwidth (bitrate) drops off linearly with the length
- Maximum bandwidth (bitrate) is inversely proportional to the spreading
Step Index Fiber Dispersion

\[
\frac{\Delta \tau}{L} \approx \frac{n_1 - n_2}{c} \approx \frac{NA^2}{2n_1 c} \approx \frac{n_1 \Delta}{c}
\]

\[
BL \approx \frac{n_1 c}{NA^2} \approx \frac{n_1 0.3 \text{MHz-Km}}{NA^2}
\]

- For a 0.2 NA glass fiber, the bandwidth is roughly 10 MHz-km
Parabolic Index Fiber

\[ \frac{\Delta^2}{L} \approx \frac{n_1 \Delta^2}{20 \sqrt{3} c} \approx \frac{\text{NA}^4}{80 n_1 \sqrt{3} c} \]

\[ \text{BL} \approx \frac{40 n_1 \sqrt{3} c}{\text{NA}^4} \approx \frac{20 \text{MHz-km}}{\text{NA}^4} \]

- The parabolic result is squared in delta with respect to the step index result.
- The result is too high, but not so very much too high – the BL product is presently approaches 10 GHz-km up from 100 MHz-km in 1990.
Table 2.5
Graded index multimode fibers

\[ d = \text{core diameter (\(\mu\text{m}\))}, \quad D = \text{cladding diameter (\(\mu\text{m}\))}. \] Typical properties at 850 nm. VCSEL is a vertical cavity surface emitting laser. \(\alpha\) is attenuation along the fiber. OM1, OM3 and OM4 are fiber standards for LAN data links (ethernet). \(\alpha\) are reported typical attenuation values. 10G and 40G networks represent data rates of 10 Gb s\(^{-1}\) and 40 Gb s\(^{-1}\) and correspond to 10 GbE (Gigabit Ethernet) and 40 GbE systems.

<table>
<thead>
<tr>
<th>MMF d/D</th>
<th>Compliance standard</th>
<th>Source</th>
<th>Typical (D_{ch}) (\text{ps nm}^{-1}\text{ km}^{-1})</th>
<th>Bandwidth (\text{MHz km})</th>
<th>NA</th>
<th>(\alpha) (\text{dB km}^{-1})</th>
<th>Reach in 10G and 40G networks</th>
</tr>
</thead>
<tbody>
<tr>
<td>50/125</td>
<td>OM4</td>
<td>VCSEL</td>
<td>−100</td>
<td>4700 (EMB)</td>
<td>0.200</td>
<td>&lt; 3</td>
<td>550 m (10G)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3500 (OFLBW)</td>
<td></td>
<td></td>
<td>150 m (40G)</td>
</tr>
<tr>
<td>50/125</td>
<td>OM3</td>
<td>VCSEL</td>
<td>−100</td>
<td>2000 (EMB)</td>
<td>0.200</td>
<td>&lt; 3</td>
<td>300 m (10G)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>500 (OFLBW)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62.5/125</td>
<td>OM1</td>
<td>LED</td>
<td>−117</td>
<td>200 (OFLBW)</td>
<td>0.275</td>
<td>&lt; 3</td>
<td>33 m (10G)</td>
</tr>
</tbody>
</table>
Good place for another problem

- Problem 2.18
Some Examples

- **Telecom**
  - Trunk lines
  - Long lines

- **Datacom**
  - Intra-rack (competing with copper)
  - Inter-rack
Example: Dispersion in a GRIN Fiber and Bit Rate

Graded index fiber. Diameter of 50 µm and a refractive index of \( n_1 = 1.4750, \Delta = 0.010 \).
The fiber is used in LANs at 850 nm with a vertical cavity surface emitting laser (VCSEL) that has very a narrow linewidth that is about 0.4 nm (FWHM). Assume that the chromatic dispersion at 850 nm is \(-100\) ps nm\(^{-1}\) km\(^{-1}\) as shown in Table 2.5. Assume the fiber has been optimized at 850 nm, and find the minimum rms dispersion. How many modes are there? What would be the upper limit on its bandwidth? What would be the bandwidth in practice?

Solution

Given \( \Delta \) and \( n_1 \), we can find \( n_2 \) from
\[
\Delta = 0.01 = (n_1 - n_2)/n_1 = (1.4750 - n_2)/1.4750.
\]
\[
\therefore n_2 = 1.4603.
\]
The \( V \)-number is then
\[
V = [(2\pi)(25 \, \mu\text{m})/(0.850 \, \mu\text{m})(1.4750^2-1.4603^2)^{1/2} = 38.39
\]
For the number of modes we can simply take \( \gamma = 2 \) and use
\[
M = (V^2/4) = (38.39^2/4) = 368 \text{ modes}
\]
The lowest intermodal dispersion for a profile optimized graded index fiber for a 1 km of fiber, \( L = 1 \text{ km} \), is
Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

\[
\frac{\sigma_{\text{intermode}}}{L} \approx \frac{n_1}{20\sqrt{3}c} \Delta^2 = \frac{1.4750}{20\sqrt{3} \left(3 \times 10^8\right)} (0.010)^2
\]

\[
= 14.20 \times 10^{-15} \text{ s m}^{-1} \text{ or } 14.20 \text{ ps km}^{-1}
\]

Assuming a triangular output light pulse and the relationship between \(\sigma\) and \(\Delta \tau_{1/2}\) given in Table 2.4, the intermodal spread \(\Delta \tau_{\text{intermode}}\) (FWHM) in the group delay over 1 km is

\[
\Delta \tau_{\text{intermode}} = (6^{1/2}) \sigma_{\text{intermode}} = (2.45)(14.20 \text{ ps}) = 34.8 \text{ ps}
\]

We also need the material dispersion at the operating wavelength over 1 km, which makes up the intramodal dispersion \(\Delta \tau_{\text{intramode}}\) (FWHM)

\[
\Delta \tau_{\text{intramode}} = L |D_{\text{ch}}| \Delta \lambda_{1/2} = (1 \text{ km})(-100 \text{ ps nm}^{-1} \text{ km}^{-1})(0.40 \text{ nm}) = 40.0 \text{ ps}
\]

\[
\Delta \tau^2 = \Delta \tau_{\text{intermode}}^2 + \Delta \tau_{\text{intramode}}^2 = (34.8)^2 + (40.0)^2 \quad \Rightarrow \quad \Delta \tau = 53.0 \text{ ps}
\]
Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

\[ B = \frac{0.25}{\sigma} = \frac{0.25}{0.408\Delta\tau} = \frac{0.61}{\Delta\tau} = \frac{0.61}{(53.0 \times 10^{-12}\text{ s})} = 11.5 \text{ Gb s}^{-1} \]

Optical bandwidth \( f_{\text{op}} = 0.99B = 11.4 \text{ GHz} \)

This is the upper limit since we assumed that the graded index fiber is perfectly optimized with \( \sigma_{\text{intermode}} \) being minimum. Small deviations around the optimum \( \gamma \) cause large increases in \( \sigma_{\text{intermode}} \), which would sharply reduce the bandwidth.
Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

If this were a multimode step-index fiber with the same $n_1$ and $n_2$, then the full dispersion (total spread) would roughly be

$$\Delta \tau \approx \frac{n_1 - n_2}{L} = \frac{n_1 \Delta}{c} = \frac{(1.475)(0.01)}{3 \times 10^8}$$

$$= 4.92 \times 10^{-11} \text{ s m}^{-1} \text{ or } 49.2 \text{ ns km}^{-1}$$

To calculate the $BL$ we use $\sigma_{\text{intermode}} \approx 0.29 \Delta \tau$

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} = \frac{0.25}{0.29(\Delta \tau / L)} = \frac{0.25}{(0.29)(49.2 \times 10^{-9} \text{ s km}^{-1})} = 17.5 \text{ Mb s}^{-1} \text{ km}$$

LANs now use graded index MMFs, and the step index MMFs are used mainly in low speed instrumentation.
Example: Dispersion in a graded-index fiber and bit rate

Consider a graded index fiber whose core has a diameter of 50 µm and a refractive index of $n_1 = 1.480$. The cladding has $n_2 = 1.460$. If this fiber is used at 1.30 µm with a laser diode that has very a narrow linewidth what will be the bit rate $\times$ distance product? Evaluate the $BL$ product if this were a multimode step index fiber.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.48 - 1.46)/1.48 = 0.0135$. Dispersion for 1 km of fiber is

$$\sigma_{\text{intermode}}/L = n_1 \Delta^2/[(20)(3^{1/2})c] = 2.6 \times 10^{-14} \text{ s m}^{-1} \text{ or } 0.026 \text{ ns km}^{-1}.$$ 

$$BL = 0.25/\sigma_{\text{intermode}} = 9.6 \text{ Gb s}^{-1} \text{ km}$$

We have ignored any material dispersion and, further, we assumed the index variation to perfectly follow the optimal profile which means that in practice $BL$ will be worse. (For example, a 15% variation in $\gamma$ from the optimal value can result in $\sigma_{\text{intermode}}$ and hence $BL$ that are more than 10 times worse.)

If this were a multimode step-index fiber with the same $n_1$ and $n_2$, then the full dispersion (total spread) would roughly be $6.67 \times 10^{-11}$ s m$^{-1}$ or 66.7 ns km$^{-1}$ and $BL = 12.9$ Mb s$^{-1}$ km

Note: Over long distances, the bit rate $\times$ distance product is not constant for multimode fibers and typically $B \propto L^{-\gamma}$ where $\gamma$ is an index between 0.5 and 1. The reason is that, due to various fiber imperfections, there is mode mixing which reduces the extent of spreading.