Double Exposure Non Destructive Testing

Consider an object, \( o(\vec{x}) \), that will undergo a slight deformation like a stressed part to a phase deformed state, \( o(\vec{z})e^{i\kappa \phi_d(\vec{z})} \). Record a double exposure.

\[
I_1(\vec{x}) = |r(\vec{x}) + o(\vec{x})|^2 = |r|^2 + |o|^2 + r^*o + o^*r
\]

\[
I_2(\vec{x}) = |r(\vec{x}) + o(\vec{x})|^2 = |r|^2 + |o|^2 + r^*oe^{i\kappa \phi_d(\vec{x})} + o^*e^{-i\kappa \phi_d(\vec{x})}r
\]

Total Exposure \( I = I_1 + I_2 \). Term of interest \( r^*o + r^*oe^{i\kappa \phi_d(\vec{x})} \)

Readout with reference beam \( r \) and observe in direction of object

\[
u = r(r^*o + r^*oe^{i\kappa \phi_d(\vec{x})}) = |r|^2 (o + oe^{i\kappa \phi_d(\vec{x})})
\]

\[
I = |\nu|^2 = |r|^4|o|^2(1 + e^{i\kappa \phi_d(\vec{x})})^2 = |r|^4|o|^2(1 + \cos k\phi_d) = |r|^4|o|^2 \cos^2 \frac{k\phi_d}{2}
\]

real Time NDT

First record a single exposure of the undeformed object

\[
I_1(\vec{x}) = |r(\vec{x}) + o(\vec{x})|^2 = |r|^2 + |o|^2 + r^*o + o^*r
\]

Deform object and readout with both \( r \) and \( o(\vec{x})e^{i\kappa \phi_d(\vec{x},t)} \) simultaneously (select terms of interest \( 1 + \eta r^*o \))

\[
(\eta + o(\vec{x})e^{i\kappa \phi_d(\vec{x},t)}) (1 + \eta r^*o) = \eta |r|^2 o(\vec{x}) + o(\vec{x})e^{i\kappa \phi_d(\vec{x},t)} + r^*o(\vec{x})e^{i\kappa \phi_d(\vec{x},t)}
\]

Intensity in image plane

\[
I(\vec{x}, t) = \eta^2 |r|^4|o|^2 + |o|^2 + \eta |r|^2|o|^2 \left| 1 + e^{i\kappa \phi_d(\vec{x},t)} \right|^2
\]

\[
= \eta^2 |r|^4|o|^2 + |o|^2 + \eta |r|^2|o|^2 \cos^2 \frac{k\phi_d(\vec{x},t)}{2}
\]

Holographic NDT of Beam deformation

Holographic NDT to find tire flaws

Figure 15.7 displays photographs of the holograms generated of a vibrating guitar surface. Time-average fringes characteristic of the vibrations are superimposed on the image.

Figure 15.8. Example of a hologram of a vibrating guitar surface. The holograms are characteristic of the first two vibrating modes of a guitar made by Giorgi, Hafn. The holograms were recorded by N. E. Meeks and K. A. Eronen at the Institute of Optical Research, Stockholm, Sweden.
Consider an object like a drumhead vibrating harmonically

\[ d(x, t) = A \cos \omega t \sin \frac{\pi x}{a} \]

A normally incident plane wave will experience a phase delay in reflection

\[ e^{-i\theta x}e^{-i\omega t} \Rightarrow e^{i\Delta \theta x}e^{i\omega t}e^{-i\Delta \omega t} \]

Interfering with a plane wave will produce a hologram

\[ I(x, t) = |\psi(x, t)|^2 = |s e^{i\Delta \theta x} e^{i\omega t} + r e^{-i\Delta \theta x} e^{-i\omega t}|^2 \]

Hologram time averages for many periods. Approximate as exactly integer (1) period.

\[ I_{avg}(x) = s^* r \int_0^T e^{i\Delta \theta x} e^{i\omega t} dt e^{-i\omega t} \]

Remember integral representation of Bessel function

\[ J_n(\alpha) = \frac{\alpha^n}{2^n} \int_0^{2\pi} e^{i\alpha \cos \theta} d\theta \]

\[ J_0(\alpha) = \frac{\alpha^n}{2^n} \int_0^{2\pi} e^{i\alpha \cos \theta} d\theta \]

A grating is imprinted with term of interest riding on carrier \( k_x \)

\[ I_{avg}(x) = s^* r \int_0^T e^{i\Delta \theta x} e^{i\omega t} dt e^{-i\omega t} \]

Amplitude DE \( \eta \propto s^* r \int_0^T e^{i\Delta \theta x} e^{i\omega t} dt e^{-i\omega t} \)

Intensity DE \( \eta^2 \propto |s^* r \int_0^T e^{i\Delta \theta x} e^{i\omega t} dt e^{-i\omega t}|^2 \)

Holographic Interferometry of a Guitar

2-color holography

2-color laser illumination

\[ \vec{E}(\vec{r}, t) = E_0 \hat{z}[e^{-i(\omega t + \vec{k}\cdot\vec{r})} e^{i\theta_0 \hat{z}} + e^{-i(\omega t - \vec{k}\cdot\vec{r})} e^{i\theta_0 \hat{z}}] \]

Each color will separately reflect off of object and propagate back to hologram to interfere with reference and expose hologram. Cross terms will wash out so we can look at each exposure separately as if we sequentially expose at two different wavelength. Object with reflectivity \( \sigma(x, y) \) and depth \( z(x, y) \) will reflect light towards hologram. Avoid complications with off-axis geometry by using on-axis illumination.
2-color Holographic Exposure

Reflected light incident on hologram after propagating back through distance $L$

$$\sigma(x, y, t) = R_{1|1} \{ a\sigma(x, y)e^{-i2\omega t}\}$$

interfere with reference $r(x, y, t) = r_0 e^{-i2\omega t + i\frac{2\pi}{\lambda}L}e^{-i\omega x}$ exposure at $\omega_1 = 2\pi (\nu_1 + \frac{\Delta \nu}{2})$

$$I_+(x, y) = \int_0^T \left| r_0 e^{-i2\omega t + i\frac{2\pi}{\lambda}L}e^{-i\omega x} \right|^2 dt$$

$$= T \left( |r_0|^2 + |a|^2 + r_0^* a e^{-i\omega x} R_{1|1} \{ \sigma(x, y)e^{i\frac{2\pi}{\lambda}L} + cc \} \right)$$

exposure at $\omega_2 = 2\pi (\nu_2 - \frac{\Delta \nu}{2})$

$$I_-(x, y) = \int_0^T \left| r_0 e^{-i2\omega t + i\frac{2\pi}{\lambda}L}e^{-i\omega x} \right|^2 dt$$

$$= T \left( |r_0|^2 + |a|^2 + r_0^* a e^{-i\omega x} R_{1|1} \{ \sigma(x, y)e^{i\frac{2\pi}{\lambda}L} + cc \} \right)$$

Same result for simultaneous exposure since cross terms will time average to zero

$$I(x, y) = \int_0^T \left( e^{-i\omega t + \omega_1} + e^{-i\omega t + \omega_2} \right) e^{i\omega x} + a\sigma(x, y) \left( e^{-i\omega t + \omega_1} + e^{-i\omega t + \omega_2} \right) \left( e^{-i\omega x} + e^{-i\omega x} \right) dt = I_+ + I_-$$

2 color holographic contouring example

![Image](image_url)

Figure 10-14  Photograph of a holographic image recorded using a two-frequency pulsed ruby laser. The 23-mm contour spacing is equal to the optical thickness of the resonant reflector in the ruby laser (10-43).

2 color holography

Developed hologram will add two exposures. Term of interest is indicated sideband

$$t(x, y) = \kappa T \left( a r_0 e^{-i\omega L} R_{1|1} \{ \sigma(x, y)e^{-i\omega t + \omega_1 + \omega_2} \} + a r_0^* e^{-i\omega L} R_{1|1} \{ \sigma(x, y)e^{-i\omega t + \omega_1 - \omega_2} \} \right)$$

$$= \kappa T a r_0 e^{-i\omega L} R_{1|1} \{ \sigma(x, y)e^{-i\omega t + \omega_1 + \omega_2} \}$$

Reilluminate hologram with same reference $r_1 e^{i\omega x}$. Diffraction field

$$E(x, y) = \kappa T r_1 r_0 a R_{1|1} \{ \sigma(x, y)e^{-i\omega t + \omega_1 + \omega_2} \}$$

Observe intensity when looking through hologram and focussing a distance $L$ behind hologram on object shows image of object reflectivity painted by depth contours.

$$I(x, y) \propto |\sigma(x, y)|^2 \cos \left( \frac{2\pi}{\lambda} \Delta \nu z \right)$$

Contour spacing for $\Delta \nu = 10$GHz $\frac{2\pi}{\lambda} \Delta \nu z = 1 \Rightarrow z = \frac{c}{\Delta \nu} = \frac{3 \times 10^8 \text{m/s}}{10 \times 10^12 \text{Hz}} = 1.5 \text{cm}$

Phase Conjugation

Time reversal: indistinguishable in CW case, can reverse pulses too (photon echoes)

Signal $E_s(\tau, t) = E_s(\tau) e^{-i\omega t} + cc$

Conjugate $E_c(\tau, t) = r E_s^*(\tau) e^{-i\omega t} + r^* E_s(\tau) e^{i\omega t}$

$$E_s = 6_0 A_0(\tau) e^{iK\tau}$$

$$E_c = 6_0^* A_0^*(\tau) e^{-iK\tau}$$

Example Application

Aberration Application

Dynamic Holography Interpretation of Phase Conjugation

Real-time Hologram

$$\Delta \epsilon(\tau) = \epsilon_0 \chi^{(3)} E_s E_s^* = \epsilon_0 \chi^{(3)} A_f A_c^* e^{iK\tau} e^{i\omega t} + cc$$

$$2 i k_0 \frac{\partial \Delta \epsilon}{\partial x} = \Delta \epsilon(\tau) E_0$$

$$E_c' = r A_c^*(\tau) e^{iK\tau} e^{-i\omega t - i\omega x}$$

$$L \lambda$$

$\chi^{(3)}$
Example of simulated volume hologram recording visualization using BEAMPROP

Phase conjugate recording

Geometry of simulated $k$-space and $\mathcal{K}$-space

recorded hologram in real and $\mathcal{K}$ space