Inclusion of Switching Loss in the Averaged Equivalent Circuit Model

The methods of Chapter 3 can be extended to include switching loss in the converter equivalent circuit model

- Include switching transitions in the converter waveforms
- Model effects of diode reverse recovery, etc.

To obtain tractable results, the waveforms during the switching transitions must usually be approximated

Things that can substantially change the results:

- Ringing caused by parasitic tank circuits
- Snubber circuits
- These are modeled in ECEN 5817, Resonant and Soft-Switching Phenomena in Power Electronics
Sketch the converter waveforms
- Including the switching transitions (idealizing assumptions are made to lead to tractable results)
- In particular, sketch inductor voltage, capacitor current, and input current waveforms

The usual steady-state relationships:
\[
\langle v_L \rangle = 0, \quad \langle i_C \rangle = 0, \quad \langle i_g \rangle = I_g
\]

Use the resulting equations to construct an equivalent circuit model, as usual
Buck Converter Example

- Ideal MOSFET, $p-n$ diode with reverse recovery
- Neglect semiconductor device capacitances, MOSFET switching times, etc.
- Neglect conduction losses
- Neglect ripple in inductor current and capacitor voltage
**Assumed waveforms**

Diode recovered charge $Q_r$, reverse recovery time $t_r$.

These waveforms assume that the diode voltage changes at the end of the reverse recovery transient:

- a “snappy” diode
- Voltage of soft-recovery diodes changes sooner
- Leads to a pessimistic estimate of induced switching loss
Inductor volt-second balance and capacitor charge balance

As usual: \( \langle v_L \rangle = 0 = DV_g - V \)

Also as usual: \( \langle i_C \rangle = 0 = I_L - V/R \)
Average input current

\[
\langle i_g \rangle = I_g = \frac{\text{area under curve}}{T_s} \\
= \frac{(DT_s I_L + t_r I_L + Q_r)}{T_s} \\
= DI_L + \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s}
\]
Construction of Equivalent Circuit Model

From inductor volt-second balance: $\langle v_L \rangle = 0 = DV_g - V$

From capacitor charge balance: $\langle i_C \rangle = 0 = I_L - V/R$
Input port of model

\[ \langle i_g \rangle = I_g = D I_L + t_r I_L / T_s + Q_r / T_s \]
Combine for complete model

The two independent current sources consume power

\[ V_g \left( \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s} \right) \]

equal to the switching loss induced by diode reverse recovery.
Solution of model

**Output:**

\[ V = D V_g \]

**Efficiency:**

\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \]

\[ P_{\text{out}} = V I_L \]
\[ P_{\text{in}} = V_g (D I_L + t_r I_L / T_s + Q_r / T_s) \]

Combine and simplify:

\[ \eta = 1 / [1 + f_s (t_r / D + Q_r R / D^2 V_g)] \]
Predicted Efficiency vs Duty Cycle

Switching frequency 100 kHz
Input voltage 24 V
Load resistance 15 Ω
Recovered charge 0.75 μCoul
Reverse recovery time 75 nsec

(no attempt is made here to model how the reverse recovery process varies with inductor current)

• Substantial degradation of efficiency
• Poor efficiency at low duty cycle
Boost Converter Example

Model same effects as in previous buck converter example:
- Ideal MOSFET, $p$–$n$ diode with reverse recovery
- Neglect semiconductor device capacitances, MOSFET switching times, etc.
- Neglect conduction losses
- Neglect ripple in inductor current and capacitor voltage
Boost converter

Transistor and diode waveforms have same shapes as in buck example, but depend on different quantities.
Inductor volt-second balance and average input current

As usual: \( \langle v_L \rangle = 0 = V_g - D'V \)

Also as usual: \( \langle i_g \rangle = I_L \)
Capacitor charge balance

\[ i_C = \langle i_d \rangle - V/R = 0 \]

\[ = - V/R + I_L(D'T_s - t_r)/T_s - Q_r/T_s \]

Collect terms: \( V/R = I_L(D'T_s - t_r)/T_s - Q_r/T_s \)
Construct model

The result is:

\[ I_g = I_L \]

\[ D' : 1 \]

\[ V (t_r I_L / T_s + Q_r / T_s) \]

The two independent current sources consume power equal to the switching loss induced by diode reverse recovery.
Predicted $V/V_g$ vs duty cycle

Switching frequency 100 kHz
Input voltage 24 V
Load resistance 60 Ω
Recovered charge 5 µCoul
Reverse recovery time 100 nsec
Inductor resistance $R_L = 0.3$ Ω
(inductor resistance also inserted into averaged model here)
Summary

The averaged modeling approach can be extended to include effects of switching loss. Transistor and diode waveforms are constructed, including the switching transitions. The effects of the switching transitions on the inductor, capacitor, and input current waveforms can then be determined. Inductor volt-second balance and capacitor charge balance are applied. Converter input current is averaged. Equivalent circuit corresponding to the averaged equations is constructed.
4.2.1. Power diodes

A power diode, under reverse-biased conditions:

![Diagram of a power diode with reverse bias. The diagram shows a depletion region and low doping concentration.]

- **Depletion region, reverse-biased**
- **Low doping concentration**
Forward-biased power diode

\[ \text{Forward-biased power diode} \]

\[ \text{conductivity modulation} \]

\[ \text{minority carrier injection} \]
Diode in OFF state:
reversed-biased, blocking voltage

- Diode is reverse-biased
- No stored minority charge: $q = 0$
- Depletion region blocks applied reverse voltage; charge is stored in capacitance of depletion region
Turn-on transient

The current $i(t)$ is determined by the converter circuit. This current supplies:

- charge to increase voltage across depletion region
- charge needed to support the on-state current
- charge to reduce on-resistance of $n^-$ region
Turn-off transient

Removal of stored minority charge $q$
Diode turn-off transient
continued

1. (1) Diode remains forward-biased.
2. (2) Remove stored charge in $n^-$ region
3. (3) $v(t)$
4. (4) Diode remains forward-biased.
   Remove stored charge in $n^-$ region
5. (5) Diode is reverse-biased.
   Charge depletion region capacitance.
6. (6) $i(t)$

$$\text{Area} \approx Q_r$$
The diode switching transients induce switching loss in the transistor

- Diode recovered stored charge $Q_r$ flows through transistor during transistor turn-on transition, inducing switching loss.
- $Q_r$ depends on diode on-state forward current, and on the rate-of-change of diode current during diode turn-off transition.
Types of power diodes

Standard recovery
Reverse recovery time not specified, intended for 50/60Hz

Fast recovery and ultra-fast recovery
Reverse recovery time and recovered charge specified
Intended for converter applications

Schottky diode
A majority carrier device
Essentially no recovered charge
Model with equilibrium $i$-$v$ characteristic, in parallel with depletion region capacitance
Restricted to low voltage (few devices can block 100V or more)
Paralleling diodes

Attempts to parallel diodes, and share the current so that $i_1 = i_2 = i/2$, generally don’t work.

*Reason*: thermal instability caused by temperature dependence of the diode equation.

Increased temperature leads to increased current, or reduced voltage.

One diode will hog the current.

To get the diodes to share the current, heroic measures are required:

- Select matched devices
- Package on common thermal substrate
- Build external circuitry that forces the currents to balance
Ringing induced by diode stored charge

- Diode is forward-biased while $i_L(t) > 0$
- Negative inductor current removes diode stored charge $Q_r$
- When diode becomes reverse-biased, negative inductor current flows through capacitor $C$.
- Ringing of $L$-$C$ network is damped by parasitic losses. Ringing energy is lost.

*see Section 4.3.3*
Energy associated with ringing

Recovered charge is

\[ Q_r = - \int_{t_2}^{t_3} i_L(t) \, dt \]

Energy stored in inductor during interval \( t_2 \leq t \leq t_3 \):

\[ W_L = \int_{t_2}^{t_3} v_L(t) \, i_L(t) \, dt \]

Applied inductor voltage during interval \( t_2 \leq t \leq t_3 \):

\[ v_L(t) = L \frac{di_L(t)}{dt} = -V_2 \]

Hence,

\[ W_L = \int_{t_2}^{t_3} L \frac{di_L(t)}{dt} \, i_L(t) \, dt = \int_{t_2}^{t_3} (-V_2) \, i_L(t) \, dt \]

\[ W_L = \frac{1}{2} L i_L^2(t_3) = V_2 Q_r \]