Introduction to Power Electronics
ECEN 4797/5797

Lecture 10
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$p$-n junction

Junction diode consisting of

- $p$-doped silicon
- $n$-doped silicon
- A $p$-$n$ junction where the $p$- and $n$-material meet

$p$ material contains mobile holes

$n$ material contains mobile electrons

$p$–n junction
Formation of depletion region
also called “space charge layer”

- At the junction, the concentrations of holes and electrons changes abruptly
- The holes and electrons diffuse in the direction of reducing concentration

These holes and electrons leave behind charged atoms—a “depletion region”
- An electric field forms in the vicinity of the junction
- This electric field constitutes an energy barrier that opposes diffusion
- The device comes to equilibrium when the voltage $v_o$ across the depletion region is enough to stop further diffusion of charges across the junction
The diode under reverse bias conditions

- Application of an external reverse voltage to the diode causes the depletion region to increase.
- The external voltage is blocked by the depletion region.
- Increasing the reverse voltage requires that charge is added to the depletion region.

"Junction capacitance": depletion region charge vs. voltage characteristic.
The diode under forward bias conditions

- When the diode voltage is positive, the depletion region voltage is not large enough to prevent diffusion of charge across the junction.
- Holes from the p-region diffuse across the junction, and become minority carriers in the n-region, whose energy state is high enough to enable them to conduct.
- Similarly, electrons from n-region diffuse across the junction and become minority carriers in the p-region.

![Diagram of diode under forward bias conditions]

Fundamentals of Power Electronics  4  Chapter 4: Switch realization
Minority-carrier stored charge in forward-biased diode

Under forward-biased conditions, a hole enters the p-material from the external circuit. It then either (a) diffuses across junction, then recombines with an electron in the n-region, or (b) recombines in the p-region with a minority-carrier electron.

The forward current of the diode consists entirely of recombination, either in the p- or n-region. The forward current continues as long as there is minority charge. To turn off the diode, the minority charge must be eliminated.
The diode equation:
\[ q(t) = Q_0 \left( e^{\lambda v(t)} - 1 \right) \]

Charge control equation:
\[ \frac{dq(t)}{dt} = i(t) - \frac{q(t)}{\tau_L} \]

with:
\[ \lambda = 1/(26 \text{ mV}) \text{ at } 300 \text{ K} \]
\[ \tau_L = \text{minority carrier lifetime} \]

(above equations don’t include current that charges depletion region capacitance)
(lumped-element charge control model with 1 lump)

In equilibrium: \( dq/dt = 0 \), and hence
\[ i(t) = \frac{q(t)}{\tau_L} = \frac{Q_0}{\tau_L} \left( e^{\lambda v(t)} - 1 \right) = I_0 \left( e^{\lambda v(t)} - 1 \right) \]

Charge-controlled behavior of the diode
Removal of stored charge during reverse recovery

Distribution of minority charge on one side of p-n junction during reverse recovery

Slope determines diffusion rate and hence current
Charge-control in the diode: Discussion

- The familiar $i$–$v$ curve of the diode is an equilibrium relationship that can be violated during transient conditions.
- During the turn-on and turn-off switching transients, the current deviates substantially from the equilibrium $i$–$v$ curve, because of change in the stored charge and change in the charge within the reverse-bias depletion region.
- The reverse-recovery time $t_r$ is the time required to remove the stored charge in the diode and enable it to block the full applied negative voltage. The area of the negative diode current during reverse recovery is the recovered charge $Q_r$. 
Inclusion of Switching Loss in the Averaged Equivalent Circuit Model

The methods of Chapter 3 can be extended to include switching loss in the converter equivalent circuit model

- Include switching transitions in the converter waveforms
- Model effects of diode reverse recovery, etc.

To obtain tractable results, the waveforms during the switching transitions must usually be approximated

Things that can substantially change the results:
- Ringing caused by parasitic tank circuits
- Snubber circuits
- These are modeled in ECEN 5817, Resonant and Soft-Switching Phenomena in Power Electronics
Sketch the converter waveforms

- Including the switching transitions (idealizing assumptions are made to lead to tractable results)
- In particular, sketch inductor voltage, capacitor current, and input current waveforms

The usual steady-state relationships:

\[ \langle v_L \rangle = 0, \langle i_C \rangle = 0, \langle i_g \rangle = I_g \]

Use the resulting equations to construct an equivalent circuit model, as usual
Buck Converter Example

- Ideal MOSFET, $p-n$ diode with reverse recovery
- Neglect semiconductor device capacitances, MOSFET switching times, etc.
- Neglect conduction losses
- Neglect ripple in inductor current and capacitor voltage
Assumed waveforms

Diode recovered charge $Q_r$, reverse recovery time $t_r$

These waveforms assume that the diode voltage changes at the end of the reverse recovery transient

- a “snappy” diode
- Voltage of soft-recovery diodes changes sooner
- Leads to a pessimistic estimate of induced switching loss
Inductor volt-second balance and capacitor charge balance

As usual: \( \langle v_L \rangle = 0 = DV_g - V \)

Also as usual: \( \langle i_C \rangle = 0 = I_L - V/R \)
Average input current

\[
\langle i_g \rangle = I_g = \frac{\text{area under curve}}{T_s} = \frac{DT_s I_L + t_r I_L + Q_r}{T_s} = DI_L + \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s}
\]
Construction of Equivalent Circuit Model

From inductor volt-second balance: \( \langle v_L \rangle = 0 = DV_g - V \)

From capacitor charge balance: \( \langle i_C \rangle = 0 = I_L - V/R \)
\[ \langle i_g \rangle = I_g = DI_L + t_r I_L / T_s + Q_r / T_s \]
Combine for complete model

The two independent current sources consume power

\[ V_g \left( \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s} \right) \]

equal to the switching loss induced by diode reverse recovery
Solution of model

**Output:**

\[ V = D V_g \]

**Efficiency:** \( \eta = \frac{P_{out}}{P_{in}} \)

\[ P_{out} = V I_L \]
\[ P_{in} = V_g (D I_L + t_r I_L/T_s + Q_r/T_s) \]

Combine and simplify:

\[ \eta = 1 / [1 + f_s (t_r/D + Q_r R/D^2 V_g)] \]
Predicted Efficiency vs Duty Cycle

Switching frequency 100 kHz
Input voltage 24 V
Load resistance 15 Ω
Recovered charge 0.75 µCoul
Reverse recovery time 75 nsec

(no attempt is made here to model how the reverse recovery process varies with inductor current)

• Substantial degradation of efficiency
• Poor efficiency at low duty cycle
Boost Converter Example

Model same effects as in previous buck converter example:
- Ideal MOSFET, $p-n$ diode with reverse recovery
- Neglect semiconductor device capacitances, MOSFET switching times, etc.
- Neglect conduction losses
- Neglect ripple in inductor current and capacitor voltage
Boost converter

Transistor and diode waveforms have same shapes as in buck example, but depend on different quantities.
Inductor volt-second balance and average input current

As usual: \( \langle v_L \rangle = 0 = V_g - D'V \)

Also as usual: \( \langle i_g \rangle = I_L \)
Capacitor charge balance

\[
\langle i_C \rangle = \langle i_d \rangle - V/R = 0
\]

\[
= -V/R + I_L(D'T_s - t_r)/T_s - Q_r/T_s
\]

Collect terms: \( V/R = I_L(D'T_s - t_r)/T_s - Q_r/T_s \)
Construct model

The result is:

$$I_g = I_L$$

$$D' : 1$$

The two independent current sources consume power

$$V \left( t_r I_L / T_s + Q_r / T_s \right)$$

equal to the switching loss induced by diode reverse recovery
Predicted $V/V_g$ vs duty cycle

Switching frequency 100 kHz
Input voltage 24 V
Load resistance 60 Ω
Recovered charge 5 μCoul
Reverse recovery time 100 nsec
Inductor resistance $R_L = 0.3 \, \Omega$
(inductor resistance also inserted into averaged model here)
Summary

The averaged modeling approach can be extended to include effects of switching loss.

Transistor and diode waveforms are constructed, including the switching transitions. The effects of the switching transitions on the inductor, capacitor, and input current waveforms can then be determined.

Inductor volt-second balance and capacitor charge balance are applied.

Converter input current is averaged.

Equivalent circuit corresponding to the averaged equations is constructed.
4.2.1. Power diodes

A power diode, under reverse-biased conditions:

![Diagram of a power diode with labels: low doping concentration and depletion region, reverse-biased]
Forward-biased power diode

\[ i \rightarrow \quad v \rightarrow \quad \text{conductivity modulation} \]

\[ \text{minority carrier injection} \]
Diode in OFF state:
reversed-biased, blocking voltage

- Diode is reverse-biased
- No stored minority charge: $q = 0$
- Depletion region blocks applied reverse voltage; charge is stored in capacitance of depletion region
Turn-on transient

The current $i(t)$ is determined by the converter circuit. This current supplies:

- charge to increase voltage across depletion region
- charge needed to support the on-state current
- charge to reduce on-resistance of $n^-$ region
Turn-off transient

Removal of stored minority charge $q$

Fundamentals of Power Electronics

Chapter 4: Switch realization
Diode turn-off transient
continued

(1) (2) (3) (4) (5) (6)

(4) Diode remains forward-biased.
Remove stored charge in $n^-$ region

(5) Diode is reverse-biased.
Charge depletion region capacitance.

Area $-Q_r$
The diode switching transients induce switching loss in the transistor

- Diode recovered stored charge $Q_r$ flows through transistor during transistor turn-on transition, inducing switching loss

- $Q_r$ depends on diode on-state forward current, and on the rate-of-change of diode current during diode turn-off transition
Types of power diodes

*Standard recovery*
Reverse recovery time not specified, intended for 50/60Hz

*Fast recovery and ultra-fast recovery*
Reverse recovery time and recovered charge specified
Intended for converter applications

*Schottky diode*
A majority carrier device
Essentially no recovered charge
Model with equilibrium $i-v$ characteristic, in parallel with depletion region capacitance
Restricted to low voltage (few devices can block 100V or more)
Paralleling diodes

Attempts to parallel diodes, and share the current so that $i_1 = i_2 = i/2$, generally don’t work.

*Reason:* thermal instability caused by temperature dependence of the diode equation.

Increased temperature leads to increased current, or reduced voltage.

One diode will hog the current.

To get the diodes to share the current, heroic measures are required:

- Select matched devices
- Package on common thermal substrate
- Build external circuitry that forces the currents to balance
Ringing induced by diode stored charge

see Section 4.3.3

- Diode is forward-biased while $i_L(t) > 0$
- Negative inductor current removes diode stored charge $Q_r$
- When diode becomes reverse-biased, negative inductor current flows through capacitor $C$.
- Ringing of $L$-$C$ network is damped by parasitic losses. Ringing energy is lost.
Energy associated with ringing

Recovered charge is

\[ Q_r = - \int_{t_2}^{t_3} i_L(t) \, dt \]

Energy stored in inductor during interval \( t_2 \leq t \leq t_3 \):

\[ W_L = \int_{t_2}^{t_3} v_L(t) \, i_L(t) \, dt \]

Applied inductor voltage during interval \( t_2 \leq t \leq t_3 \):

\[ v_L(t) = L \frac{di_L(t)}{dt} = -V_2 \]

Hence,

\[ W_L = \int_{t_2}^{t_3} L \frac{di_L(t)}{dt} \, i_L(t) \, dt = \int_{t_2}^{t_3} (-V_2) \, i_L(t) \, dt \]

\[ W_L = \frac{1}{2} L \, i_L^2(t_3) = V_2 \, Q_r \]