Introduction to Power Electronics
ECEN 4797/5797

Lecture 10
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4.2.1. Power diodes

A power diode, under reverse-biased conditions:

![Diagram of a power diode under reverse-biased conditions]

low doping concentration

depletion region, reverse-biased
Typical diode switching waveforms

\[ v(t) \]
\[ i(t) \]
\[ \frac{di}{dt} \]

(1) (2) (3) (4) (5) (6)
Forward-biased power diode

$i$

$v$

conductivity modulation

$p$  $n^-$  $n$

minority carrier injection
Charge-controlled behavior of the diode

The diode equation:

\[ q(t) = Q_0 \left( e^{\lambda v(t)} - 1 \right) \]

Charge control equation:

\[ \frac{dq(t)}{dt} = i(t) - \frac{q(t)}{\tau_L} \]

With:

\[ \lambda = 1/(26 \text{ mV}) \text{ at 300 K} \]

\[ \tau_L = \text{minority carrier lifetime} \]

(above equations don’t include current that charges depletion region capacitance)

In equilibrium: \( dq/dt = 0 \), and hence

\[ i(t) = \frac{q(t)}{\tau_L} = \frac{Q_0}{\tau_L} \left( e^{\lambda v(t)} - 1 \right) = I_0 \left( e^{\lambda v(t)} - 1 \right) \]
Charge-control in the diode: Discussion

- The familiar $i-v$ curve of the diode is an equilibrium relationship that can be violated during transient conditions.
- During the turn-on and turn-off switching transients, the current deviates substantially from the equilibrium $i-v$ curve, because of change in the stored charge and change in the charge within the reverse-bias depletion region.
- Under forward-biased conditions, the stored minority charge causes “conductivity modulation” of the resistance of the lightly-doped $n^-$ region, reducing the device on-resistance.
Removal of stored charge during reverse recovery

Distribution of minority charge on one side of p-n junction during reverse recovery

Slope determines diffusion rate and hence current
Diode in OFF state: reversed-biased, blocking voltage

- Diode is reverse-biased
- No stored minority charge: \( q = 0 \)
- Depletion region blocks applied reverse voltage; charge is stored in capacitance of depletion region
Turn-on transient

The current $i(t)$ is determined by the converter circuit. This current supplies:

- charge to increase voltage across depletion region
- charge needed to support the on-state current
- charge to reduce on-resistance of $n^-$ region
Turn-off transient

Removal of stored minority charge $q$

$i (< 0)$

$V$

$p$

$n^-$

$n$

Fundamentals of Power Electronics
Chapter 4: Switch realization
Diode turn-off transient
continued

(4) Diode remains forward-biased. Remove stored charge in $n^-$ region

(5) Diode is reverse-biased. Charge depletion region capacitance.
The diode switching transients induce switching loss in the transistor

- Diode recovered stored charge $Q_r$ flows through transistor during transistor turn-on transition, inducing switching loss

- $Q_r$ depends on diode on-state forward current, and on the rate-of-change of diode current during diode turn-off transition
Switching loss calculation

Energy lost in transistor:

\[ W_D = \int_{\text{switching transition}} v_A(t) i_A(t) \, dt \]

With abrupt-recovery diode:

\[ W_D \approx \int_{\text{switching transition}} V_g (i_L - i_B(t)) \, dt \]

\[ = V_g i_L t_r + V_g Q_r \]

- Often, this is the largest component of switching loss

Soft-recovery diode:

\((t_2 - t_1) >> (t_1 - t_0)\)

Abrupt-recovery diode:

\((t_2 - t_1) << (t_1 - t_0)\)
Inclusion of Switching Loss in the Averaged Equivalent Circuit Model

The methods of Chapter 3 can be extended to include switching loss in the converter equivalent circuit model

- Include switching transitions in the converter waveforms
- Model effects of diode reverse recovery, etc.

To obtain tractable results, the waveforms during the switching transitions must usually be approximated

Things that can substantially change the results:

- Ringing caused by parasitic tank circuits
- Snubber circuits
- These are modeled in ECEN 5817, Resonant and Soft-Switching Phenomena in Power Electronics
The Modeling Approach
Extension of Chapter 3 Methods

Sketch the converter waveforms
  – Including the switching transitions (idealizing assumptions are made to lead to tractable results)
  – In particular, sketch inductor voltage, capacitor current, and input current waveforms

The usual steady-state relationships:

\[ \langle v_L \rangle = 0, \langle i_C \rangle = 0, \langle i_g \rangle = I_g \]

Use the resulting equations to construct an equivalent circuit model, as usual
Buck Converter Example

- Ideal MOSFET, $p-n$ diode with reverse recovery
- Neglect semiconductor device capacitances, MOSFET switching times, etc.
- Neglect conduction losses
- Neglect ripple in inductor current and capacitor voltage
Assumed waveforms

Diode recovered charge $Q_r$, reverse recovery time $t_r$

These waveforms assume that the diode voltage changes at the end of the reverse recovery transient

- a “snappy” diode
- Voltage of soft-recovery diodes changes sooner
- Leads to a pessimistic estimate of induced switching loss
Inductor volt-second balance and capacitor charge balance

As usual: \( \langle v_L \rangle = 0 = DV_g - V \)

Also as usual: \( \langle i_C \rangle = 0 = I_L - V/R \)
Average input current

\[
\langle i_g \rangle = I_g = \frac{\text{(area under curve)}}{T_s}
\]

\[
= \frac{(DT_s I_L + t_r I_L + Q_r)}{T_s}
\]

\[
= DI_L + \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s}
\]
Construction of Equivalent Circuit Model

From inductor volt-second balance: \[ \langle v_L \rangle = 0 = D V_g - V \]
From capacitor charge balance: \[ \langle i_C \rangle = 0 = I_L - V/R \]
\[ \langle i_g \rangle = I_g = DI_L + \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s} \]
The two independent current sources consume power

\[ V_g \left( \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s} \right) \]

equal to the switching loss induced by diode reverse recovery
Solution of model

**Output:**

\[ V = D V_g \]

**Efficiency:**

\[ \eta = \frac{P_{out}}{P_{in}} \]

\[ P_{out} = V I_L \]

\[ P_{in} = V_g \left( D I_L + \frac{t_r I_L}{T_s} + \frac{Q_r}{T_s} \right) \]

Combine and simplify:

\[ \eta = \frac{1}{1 + f_s \left( \frac{t_r}{D} + \frac{Q_r R}{D^2 V_g} \right)} \]
Predicted Efficiency vs Duty Cycle

Switching frequency 100 kHz
Input voltage 24 V
Load resistance 15 Ω
Recovered charge 0.75 µCoul
Reverse recovery time 75 nsec

(no attempt is made here to model how the reverse recovery process varies with inductor current)

- Substantial degradation of efficiency
- Poor efficiency at low duty cycle
Boost Converter Example

Model same effects as in previous buck converter example:
- Ideal MOSFET, $p$–$n$ diode with reverse recovery
- Neglect semiconductor device capacitances, MOSFET switching times, etc.
- Neglect conduction losses
- Neglect ripple in inductor current and capacitor voltage
Boost converter

Transistor and diode waveforms have same shapes as in buck example, but depend on different quantities.
Inductor volt-second balance and average input current

As usual: $\langle v_L \rangle = 0 = V_g - D'V$

Also as usual: $\langle i_g \rangle = I_L$
Capacitor charge balance

\[ \langle i_C \rangle = \langle i_d \rangle - \frac{V}{R} = 0 \]
\[ = - \frac{V}{R} + \frac{I_L(D'T_s - t_r)}{T_s} - \frac{Q_r}{T_s} \]

Collect terms: \[ V/R = I_L(D'T_s - t_r)/T_s - Q_r/T_s \]
Construct model

The result is:

The two independent current sources consume power

\[ V \left( t_r I_L / T_s + Q_r / T_s \right) \]

equal to the switching loss induced by diode reverse recovery
Predicted $V/V_g$ vs duty cycle

Switching frequency 100 kHz
Input voltage 24 V
Load resistance 60 $\Omega$
Recovered charge 5 $\mu$Coul
Reverse recovery time 100 nsec
Inductor resistance $R_L = 0.3$ $\Omega$
(inductor resistance also inserted into averaged model here)
The averaged modeling approach can be extended to include effects of switching loss.

Transistor and diode waveforms are constructed, including the switching transitions. The effects of the switching transitions on the inductor, capacitor, and input current waveforms can then be determined.

Inductor volt-second balance and capacitor charge balance are applied.

Converter input current is averaged.

Equivalent circuit corresponding to the averaged equations is constructed.
Types of power diodes

**Standard recovery**
Reverse recovery time not specified, intended for 50/60Hz

**Fast recovery and ultra-fast recovery**
Reverse recovery time and recovered charge specified
Intended for converter applications

**Schottky diode**
A majority carrier device
Essentially no recovered charge
Model with equilibrium $i-v$ characteristic, in parallel with depletion region capacitance
Restricted to low voltage (few devices can block 100V or more)
Characteristics of several commercial power rectifier diodes

<table>
<thead>
<tr>
<th>Part number</th>
<th>Rated max voltage</th>
<th>Rated avg current</th>
<th>$V_F$ (typical)</th>
<th>$t_r$ (max)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fast recovery rectifiers</strong></td>
<td></td>
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<td></td>
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<tr>
<td>1N3913</td>
<td>400V</td>
<td>30A</td>
<td>1.1V</td>
<td>400ns</td>
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<td>SD453N25S20PC</td>
<td>2500V</td>
<td>400A</td>
<td>2.2V</td>
<td>2µs</td>
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<td><strong>Ultra-fast recovery rectifiers</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>MUR815</td>
<td>150V</td>
<td>8A</td>
<td>0.975V</td>
<td>35ns</td>
</tr>
<tr>
<td>MUR1560</td>
<td>600V</td>
<td>15A</td>
<td>1.2V</td>
<td>60ns</td>
</tr>
<tr>
<td>RHRU100120</td>
<td>1200V</td>
<td>100A</td>
<td>2.6V</td>
<td>60ns</td>
</tr>
<tr>
<td><strong>Schottky rectifiers</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MBR6030L</td>
<td>30V</td>
<td>60A</td>
<td>0.48V</td>
<td></td>
</tr>
<tr>
<td>444CNQ045</td>
<td>45V</td>
<td>440A</td>
<td>0.69V</td>
<td></td>
</tr>
<tr>
<td>30CPQ150</td>
<td>150V</td>
<td>30A</td>
<td>1.19V</td>
<td></td>
</tr>
</tbody>
</table>
Paralleling diodes

Attempts to parallel diodes, and share the current so that $i_1 = i_2 = i/2$, generally don’t work.

*Reason:* thermal instability caused by temperature dependence of the diode equation.

Increased temperature leads to increased current, or reduced voltage.

One diode will hog the current.

To get the diodes to share the current, heroic measures are required:

- Select matched devices
- Package on common thermal substrate
- Build external circuitry that forces the currents to balance
Ringing induced by diode stored charge

- Diode is forward-biased while $i_L(t) > 0$
- Negative inductor current removes diode stored charge $Q_r$
- When diode becomes reverse-biased, negative inductor current flows through capacitor $C$.
- Ringing of $L-C$ network is damped by parasitic losses. Ringing energy is lost.

see Section 4.3.3
Energy associated with ringing

Recovered charge is
\[ Q_r = - \int_{t_2}^{t_3} i_L(t) \, dt \]

Energy stored in inductor during interval \( t_2 \leq t \leq t_3 \):
\[ W_L = \int_{t_2}^{t_3} v_L(t) \, i_L(t) \, dt \]

Applied inductor voltage during interval \( t_2 \leq t \leq t_3 \):
\[ v_L(t) = L \frac{di_L(t)}{dt} = -V_2 \]

Hence,
\[ W_L = \int_{t_2}^{t_3} L \frac{di_L(t)}{dt} \, i_L(t) \, dt = \int_{t_2}^{t_3} (-V_2) \, i_L(t) \, dt \]

\[ W_L = \frac{1}{2} L \, i_L^2(t_3) = V_2 \, Q_r \]
4.2.2. The Power MOSFET

- Gate lengths approaching one micron
- Consists of many small enhancement-mode parallel-connected MOSFET cells, covering the surface of the silicon wafer
- Vertical current flow
- n-channel device is shown
MOSFET: Off state

- $p$-$n^-$ junction is reverse-biased
- off-state voltage appears across $n^-$ region
MOSFET: on state

- $p$-$n^-$ junction is slightly reverse-biased
- Positive gate voltage induces conducting channel
- Drain current flows through $n^-$ region and conducting channel
- On resistance = total resistances of $n^-$ region, conducting channel, source and drain contacts, etc.
MOSFET body diode

- $p$-$n^-$ junction forms an effective diode, in parallel with the channel
- negative drain-to-source voltage can forward-bias the body diode
- diode can conduct the full MOSFET rated current
- diode switching speed not optimized — body diode is slow, $Q_r$ is large
Typical MOSFET characteristics

- Off state: $V_{GS} < V_{th}$
- On state: $V_{GS} >> V_{th}$
- MOSFET can conduct peak currents well in excess of average current rating — characteristics are unchanged
- on-resistance has positive temperature coefficient, hence easy to parallel
A simple MOSFET equivalent circuit

- $C_{gs}$: large, essentially constant
- $C_{gd}$: small, highly nonlinear
- $C_{ds}$: intermediate in value, highly nonlinear
- Switching times determined by rate at which gate driver charges/discharges $C_{gs}$ and $C_{gd}$

\[
C_{ds}(v_{ds}) = \frac{C_0}{\sqrt{1 + \frac{v_{ds}}{V_0}}}
\]

\[
C_{ds}(v_{ds}) \approx C_0 \sqrt{\frac{V_0}{v_{ds}}} = \frac{C_0}{\sqrt{v_{ds}}}
\]
Switching loss caused by semiconductor output capacitances

*Buck converter example*

Energy lost during MOSFET turn-on transition (assuming linear capacitances):

\[ W_c = \frac{1}{2} (C_{ds} + C_j) V_g^2 \]
MOSFET nonlinear $C_{ds}$

Approximate dependence of incremental $C_{ds}$ on $v_{ds}$:

$$C_{ds}(v_{ds}) \approx C_0 \sqrt{\frac{V_0}{v_{ds}}} = \frac{C'_0}{\sqrt{v_{ds}}}$$

Energy stored in $C_{ds}$ at $v_{ds} = V_{DS}$:

$$W_{Cds} = \int v_{ds} i_C \, dt = \int_0^{V_{DS}} v_{ds} C_{ds}(v_{ds}) \, dv_{ds}$$

$$W_{Cds} = \int_0^{V_{DS}} C'_0(v_{ds}) \sqrt{v_{ds}} \, dv_{ds} = \frac{2}{3} C_{ds}(V_{DS}) \, V_{DS}^2$$

— same energy loss as linear capacitor having value $\frac{4}{3} C_{ds}(V_{DS})$
## Characteristics of several commercial power MOSFETs

<table>
<thead>
<tr>
<th>Part number</th>
<th>Rated max voltage</th>
<th>Rated avg current</th>
<th>$R_{on}$</th>
<th>$Q_g$ (typical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRFZ48</td>
<td>60V</td>
<td>50A</td>
<td>0.018Ω</td>
<td>110nC</td>
</tr>
<tr>
<td>IRF510</td>
<td>100V</td>
<td>5.6A</td>
<td>0.54Ω</td>
<td>8.3nC</td>
</tr>
<tr>
<td>IRF540</td>
<td>100V</td>
<td>28A</td>
<td>0.077Ω</td>
<td>72nC</td>
</tr>
<tr>
<td>APT10M25BNR</td>
<td>100V</td>
<td>75A</td>
<td>0.025Ω</td>
<td>171nC</td>
</tr>
<tr>
<td>IRF740</td>
<td>400V</td>
<td>10A</td>
<td>0.55Ω</td>
<td>63nC</td>
</tr>
<tr>
<td>MTM15N40E</td>
<td>400V</td>
<td>15A</td>
<td>0.3Ω</td>
<td>110nC</td>
</tr>
<tr>
<td>APT5025BN</td>
<td>500V</td>
<td>23A</td>
<td>0.25Ω</td>
<td>83nC</td>
</tr>
<tr>
<td>APT1001RBNR</td>
<td>1000V</td>
<td>11A</td>
<td>1.0Ω</td>
<td>150nC</td>
</tr>
</tbody>
</table>
MOSFET: conclusions

- A majority-carrier device: fast switching speed
- Typical switching frequencies: tens and hundreds of kHz
- On-resistance increases rapidly with rated blocking voltage
- Easy to drive
- The device of choice for blocking voltages less than 500V
- 1000V devices are available, but are useful only at low power levels (100W)
- Part number is selected on the basis of on-resistance rather than current rating
4.2.3. Bipolar Junction Transistor (BJT)

- Interdigitated base and emitter contacts
- Vertical current flow
- npn device is shown
- minority carrier device
- on-state: base-emitter and collector-base junctions are both forward-biased
- on-state: substantial minority charge in $p$ and $n^-$ regions, conductivity modulation
BJT switching times

\[ V_{CE}(t) = V_{CC} - i_C(t) \cdot R_L \]

\[ V_{BE}(t) = 0.7 \text{V} \]

\[ i_B(t) = I_{B1} \]

\[ i_C(t) = I_{Con} \]

\[ v_{BE}(t) = -V_{BE} \]

\[ v_{CE}(t) = V_{CE} \]

\[ v_{is}(t) = V_{is} \]

\[ v_{os}(t) = V_{os} \]

\[ t = 0 \]

\[ t = T \]
Ideal base current waveform

![Ideal base current waveform diagram](image)
Current crowding due to excessive $I_{B2}$

can lead to formation of hot spots and device failure
BJT characteristics

- Off state: $I_B = 0$
- On state: $I_B > I_C / \beta$
- Current gain $\beta$ decreases rapidly at high current. Device should not be operated at instantaneous currents exceeding the rated value
Breakdown voltages

$BV_{CBO}$: avalanche breakdown voltage of base-collector junction, with the emitter open-circuited

$BV_{CEO}$: collector-emitter breakdown voltage with zero base current

$BV_{sus}$: breakdown voltage observed with positive base current

In most applications, the off-state transistor voltage must not exceed $BV_{CEO}$. 

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Darlington-connected BJT

- Increased current gain, for high-voltage applications
- In a monolithic Darlington device, transistors $Q_1$ and $Q_2$ are integrated on the same silicon wafer
- Diode $D_1$ speeds up the turn-off process, by allowing the base driver to actively remove the stored charge of both $Q_1$ and $Q_2$ during the turn-off transition
Conclusions: BJT

- BJT has been replaced by MOSFET in low-voltage (<500V) applications
- BJT is being replaced by IGBT in applications at voltages above 500V
- A minority-carrier device: compared with MOSFET, the BJT exhibits slower switching, but lower on-resistance at high voltages
4.2.4. The Insulated Gate Bipolar Transistor (IGBT)

- A four-layer device
- Similar in construction to MOSFET, except extra $p$ region
- On-state: minority carriers are injected into $n^-$ region, leading to conductivity modulation
- Compared with MOSFET: slower switching times, lower on-resistance, useful at higher voltages (up to 1700V)
The IGBT

Symbol

Collector
Gate
Emitter

Equivalent circuit

Location of equivalent devices

Fundamentals of Power Electronics
Current tailing in IGBTs

\[ p_A(t) = v_A i_A \]

\[ p_{A(t)} = v_A i_A \]

\[ W_{off} \]

\[ i_A(t) \]

\[ v_A(t) \]

\[ i_L(t) \]

\[ v_B(t) \]

\[ V_g \]

\[ 0 \]

\[ t_0 \]

\[ t_1 \]

\[ t_2 \]

\[ t_3 \]
Switching loss due to current-tailing in IGBT

\[ P_{\text{sw}} = \frac{1}{T_s} \int_{t_0}^{t_1} p_A(t) \, dt = (W_{\text{on}} + W_{\text{off}}) \frac{f_s}{s} \]

Example: buck converter with IGBT

transistor turn-off transition
## Characteristics of several commercial devices

<table>
<thead>
<tr>
<th>Part number</th>
<th>Rated max voltage</th>
<th>Rated avg current</th>
<th>$V_F$ (typical)</th>
<th>$t_f$ (typical)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single-chip devices</strong></td>
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</tr>
<tr>
<td>HGTG32N60E2</td>
<td>600V</td>
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<td>2.4V</td>
<td>0.62µs</td>
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<td>1200V</td>
<td>30A</td>
<td>3.2A</td>
<td>0.58µs</td>
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<tr>
<td><strong>Multiple-chip power modules</strong></td>
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<td></td>
</tr>
<tr>
<td>CM400HA-12E</td>
<td>600V</td>
<td>400A</td>
<td>2.7V</td>
<td>0.3µs</td>
</tr>
<tr>
<td>CM300HA-24E</td>
<td>1200V</td>
<td>300A</td>
<td>2.7V</td>
<td>0.3µs</td>
</tr>
</tbody>
</table>
Conclusions: IGBT

- Becoming the device of choice in 500 to 1700V+ applications, at power levels of 1-1000kW
- Positive temperature coefficient at high current —easy to parallel and construct modules
- Forward voltage drop: diode in series with on-resistance. 2-4V typical
- Easy to drive —similar to MOSFET
- Slower than MOSFET, but faster than Darlington, GTO, SCR
- Typical switching frequencies: 3-30kHz
- IGBT technology is rapidly advancing:
  - 3300 V devices: HVIGBTs
  - 150 kHz switching frequencies in 600 V devices
4.2.5. Thyristors (SCR, GTO, MCT)

**The SCR**

**symbol**
- Anode (A)
- Gate (G)
- Cathode (K)

**equiv circuit**
- Anode
- Q₁
- Gate
- Q₂

**construction**
- K
- G
- K
- n
- p
- n
- Q₁
- Q₂
- A
The Silicon Controlled Rectifier (SCR)

- Positive feedback — a latching device
- A minority carrier device
- Double injection leads to very low on-resistance, hence low forward voltage drops attainable in very high voltage devices
- Simple construction, with large feature size
- Cannot be actively turned off
- A voltage-bidirectional two-quadrant switch
- 5000-6000V, 1000-2000A devices
Why the conventional SCR cannot be turned off via gate control

- Large feature size
- Negative gate current induces lateral voltage drop along gate-cathode junction
- Gate-cathode junction becomes reverse-biased only in vicinity of gate contact
The Gate Turn-Off Thyristor (GTO)

- An SCR fabricated using modern techniques — small feature size
- Gate and cathode contacts are highly interdigitated
- Negative gate current is able to completely reverse-bias the gate-cathode junction

Turn-off transition:
- Turn-off current gain: typically 2-5
- Maximum controllable on-state current: maximum anode current that can be turned off via gate control. GTO can conduct peak currents well in excess of average current rating, but cannot switch off
Summary: Thyristors

- The thyristor family: double injection yields lowest forward voltage drop in high voltage devices. More difficult to parallel than MOSFETs and IGBTs
- The SCR: highest voltage and current ratings, low cost, passive turn-off transition
- The GTO: intermediate ratings (less than SCR, somewhat more than IGBT). Slower than IGBT. Slower than MCT. Difficult to drive.
- The MCT: So far, ratings lower than IGBT. Slower than IGBT. Easy to drive. Second breakdown problems? Still an emerging device.
4.3. Switching loss

- Energy is lost during the semiconductor switching transitions, via several mechanisms:
  - Transistor switching times
  - Diode stored charge
  - Energy stored in device capacitances and parasitic inductances
- Semiconductor devices are *charge controlled*
- Time required to insert or remove the controlling charge determines switching times
4.3.1. Transistor switching with clamped inductive load

**Buck converter example**

\[ v_B(t) = v_A(t) - V_g \]
\[ i_A(t) + i_B(t) = i_L \]

**transistor turn-off transition**

\[ W_{off} = \frac{1}{2} V_g i_L (t_2 - t_0) \]
Switching loss induced by transistor turn-off transition

Energy lost during transistor turn-off transition:

\[ W_{\text{off}} = \frac{1}{2} V_s i_L (t_2 - t_0) \]

Similar result during transistor turn-on transition.
Average power loss:

\[ P_{\text{sw}} = \frac{1}{T_s} \int_{\text{switching transitions}} p_A(t) \, dt = (W_{\text{on}} + W_{\text{off}}) f_s \]
Switching loss due to current-tailing in IGBT

Example: buck converter with IGBT

transistor turn-off transition

\[ P_{sw} = \frac{1}{T_s} \int_{t_0}^{t_1} p_A(t) \, dt = (W_{on} + W_{off}) \, f_s \]
4.3.2. Diode recovered charge

- Diode recovered stored charge $Q_r$ flows through transistor during transistor turn-on transition, inducing switching loss.

- $Q_r$ depends on diode on-state forward current, and on the rate-of-change of diode current during diode turn-off transition.
Switching loss calculation

Energy lost in transistor:

\[ W_D = \int_{\text{switching transition}} v_A(t) \, i_A(t) \, dt \]

With abrupt-recovery diode:

\[ W_D \approx \int_{\text{switching transition}} V_g \, (i_L - i_B(t)) \, dt \]

\[ = V_g \, i_L \, t_r + V_g \, Q_r \]

- Often, this is the largest component of switching loss

Soft-recovery diode:

\( (t_2 - t_1) \gg (t_1 - t_0) \)

Abrupt-recovery diode:

\( (t_2 - t_1) \ll (t_1 - t_0) \)
4.3.3. Device capacitances, and leakage, package, and stray inductances

- Capacitances that appear effectively in parallel with switch elements are shorted when the switch turns on. Their stored energy is lost during the switch turn-on transition.

- Inductances that appear effectively in series with switch elements are open-circuited when the switch turns off. Their stored energy is lost during the switch turn-off transition.

Total energy stored in linear capacitive and inductive elements:

\[ W_C = \sum_{\text{capacitive elements}} \frac{1}{2} C_i V_i^2 \quad W_L = \sum_{\text{inductive elements}} \frac{1}{2} L_j I_j^2 \]
Example: semiconductor output capacitances

Buck converter example

Energy lost during MOSFET turn-on transition (assuming linear capacitances):

\[ W_C = \frac{1}{2} (C_{ds} + C_j) V_g^2 \]
MOSFET nonlinear $C_{ds}$

Approximate dependence of incremental $C_{ds}$ on $v_{ds}$:

$$C_{ds}(v_{ds}) \approx C_0 \sqrt{\frac{V_0}{v_{ds}}} = \frac{C'_0}{\sqrt{v_{ds}}}$$

Energy stored in $C_{ds}$ at $v_{ds} = V_{DS}$:

$$W_{Cds} = \int v_{ds} \, i_C \, dt = \int_0^{V_{DS}} v_{ds} \, C_{ds}(v_{ds}) \, dv_{ds}$$

$$W_{Cds} = \int_0^{V_{DS}} C'_0(v_{ds}) \sqrt{v_{ds}} \, dv_{ds} = \frac{2}{3} C_{ds}(V_{DS}) \, V_{DS}^2$$

— same energy loss as linear capacitor having value $\frac{4}{3} C_{ds}(V_{DS})$
Some other sources of this type of switching loss

Schottky diode
- Essentially no stored charge
- Significant reverse-biased junction capacitance

Transformer leakage inductance
- Effective inductances in series with windings
- A significant loss when windings are not tightly coupled

Interconnection and package inductances
- Diodes
- Transistors
- A significant loss in high current applications
Ringing induced by diode stored charge

- Diode is forward-biased while $i_L(t) > 0$
- Negative inductor current removes diode stored charge $Q_r$
- When diode becomes reverse-biased, negative inductor current flows through capacitor $C$.
- Ringing of $L-C$ network is damped by parasitic losses. Ringing energy is lost.
Energy associated with ringing

Recovered charge is

\[ Q_r = - \int_{t_2}^{t_3} i_L(t) \, dt \]

Energy stored in inductor during interval \( t_2 \leq t \leq t_3 \):

\[ W_L = \int_{t_2}^{t_3} v_L(t) \, i_L(t) \, dt \]

Applied inductor voltage during interval \( t_2 \leq t \leq t_3 \):

\[ v_L(t) = L \frac{d i_L(t)}{dt} = -V_2 \]

Hence,

\[ W_L = \int_{t_2}^{t_3} L \frac{d i_L(t)}{dt} i_L(t) \, dt = \int_{t_2}^{t_3} (-V_2) i_L(t) \, dt \]

\[ W_L = \frac{1}{2} L i_L^2(t_3) = V_2 Q_r \]
4.3.4. Efficiency vs. switching frequency

Add up all of the energies lost during the switching transitions of one switching period:

\[ W_{tot} = W_{on} + W_{off} + W_D + W_C + W_L + ... \]

Average switching power loss is

\[ P_{sw} = W_{tot} f_{sw} \]

Total converter loss can be expressed as

\[ P_{loss} = P_{cond} + P_{fixed} + W_{tot} f_{sw} \]

where \[ P_{fixed} = \text{fixed losses (independent of load and } f_{sw}) \]
\[ P_{cond} = \text{conduction losses} \]
Efficiency vs. switching frequency

$$P_{\text{loss}} = P_{\text{cond}} + P_{\text{fixed}} + W_{\text{tot}} f_{\text{sw}}$$

Switching losses are equal to the other converter losses at the critical frequency

$$f_{\text{crit}} = \frac{P_{\text{cond}} + P_{\text{fixed}}}{W_{\text{tot}}}$$

This can be taken as a rough upper limit on the switching frequency of a practical converter. For $f_{\text{sw}} > f_{\text{crit}}$, the efficiency decreases rapidly with frequency.
Summary of chapter 4

1. How an SPST ideal switch can be realized using semiconductor devices depends on the polarity of the voltage which the devices must block in the off-state, and on the polarity of the current which the devices must conduct in the on-state.

2. Single-quadrant SPST switches can be realized using a single transistor or a single diode, depending on the relative polarities of the off-state voltage and on-state current.

3. Two-quadrant SPST switches can be realized using a transistor and diode, connected in series (bidirectional-voltage) or in anti-parallel (bidirectional-current). Several four-quadrant schemes are also listed here.

4. A “synchronous rectifier” is a MOSFET connected to conduct reverse current, with gate drive control as necessary. This device can be used where a diode would otherwise be required. If a MOSFET with sufficiently low $R_{on}$ is used, reduced conduction loss is obtained.
Summary of chapter 4

5. Majority carrier devices, including the MOSFET and Schottky diode, exhibit very fast switching times, controlled essentially by the charging of the device capacitances. However, the forward voltage drops of these devices increases quickly with increasing breakdown voltage.

6. Minority carrier devices, including the BJT, IGBT, and thyristor family, can exhibit high breakdown voltages with relatively low forward voltage drop. However, the switching times of these devices are longer, and are controlled by the times needed to insert or remove stored minority charge.

7. Energy is lost during switching transitions, due to a variety of mechanisms. The resulting average power loss, or switching loss, is equal to this energy loss multiplied by the switching frequency. Switching loss imposes an upper limit on the switching frequencies of practical converters.
8. The diode and inductor present a “clamped inductive load” to the transistor. When a transistor drives such a load, it experiences high instantaneous power loss during the switching transitions. An example where this leads to significant switching loss is the IGBT and the “current tail” observed during its turn-off transition.

9. Other significant sources of switching loss include diode stored charge and energy stored in certain parasitic capacitances and inductances. Parasitic ringing also indicates the presence of switching loss.