Introduction to Power Electronics
ECEN 4797/5797

Lecture 13
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4.2.3. Bipolar Junction Transistor (BJT)

- Interdigitated base and emitter contacts
- Vertical current flow
- npn device is shown
- minority carrier device
- on-state: base-emitter and collector-base junctions are both forward-biased
- on-state: substantial minority charge in $p$ and $n^-$ regions, conductivity modulation
BJT switching times
Ideal base current waveform
Current crowding due to excessive $I_{B2}$

can lead to formation of hot spots and device failure.
BJT characteristics

- Off state: $I_B = 0$
- On state: $I_B > I_C / \beta$
- Current gain $\beta$ decreases rapidly at high current. Device should not be operated at instantaneous currents exceeding the rated value.
Darlington-connected BJT

- Increased current gain, for high-voltage applications
- In a monolithic Darlington device, transistors $Q_1$ and $Q_2$ are integrated on the same silicon wafer
- Diode $D_1$ speeds up the turn-off process, by allowing the base driver to actively remove the stored charge of both $Q_1$ and $Q_2$ during the turn-off transition
Conclusions: BJT

- BJT has been replaced by MOSFET in low-voltage (<500V) applications
- BJT is being replaced by IGBT in applications at voltages above 500V
- A minority-carrier device: compared with MOSFET, the BJT exhibits slower switching, but lower on-resistance at high voltages
4.2.4. The Insulated Gate Bipolar Transistor (IGBT)

- A four-layer device
- Similar in construction to MOSFET, except extra $p$ region
- On-state: minority carriers are injected into $n^-$ region, leading to conductivity modulation
- Compared with MOSFET: slower switching times, lower on-resistance, useful at higher voltages (up to 1700V)
The IGBT

Symbol

Location of equivalent devices

Equivalent circuit
Current tailing in IGBTs

The figure shows the waveforms of an IGBT and a diode. The IGBT waveform includes the voltage $v_A(t)$ and the current $i_A(t)$. The diode waveform includes the voltage $v_B(t)$ and the current $i_B(t)$. The figure also illustrates the current tail, which is the area $W_{off}$.

Mathematically, the current tailing in IGBTs can be described by the equation:

$$\int_{t_0}^{t_3} i_A(t) \, dt = \int_{t_0}^{t_3} v_A(t) \, dt$$

Where $i_A(t)$ is the instantaneous current, $v_A(t)$ is the voltage across the IGBT, and the integration is performed over the time interval from $t_0$ to $t_3$. The area $W_{off}$ represents the energy dissipation during the switching process.
Switching loss due to current-tailing in IGBT

**Example: buck converter with IGBT**

Transistor turn-off transition

\[
P_{sw} = \frac{1}{T_s} \int_{\text{switching transitions}} p_A(t) \, dt = (W_{on} + W_{off}) f_s
\]
## Characteristics of several commercial devices

<table>
<thead>
<tr>
<th>Part number</th>
<th>Rated max voltage</th>
<th>Rated avg current</th>
<th>$V_F$ (typical)</th>
<th>$t_f$ (typical)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single-chip devices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HGTG32N60E2</td>
<td>600V</td>
<td>32A</td>
<td>2.4V</td>
<td>0.62µs</td>
</tr>
<tr>
<td>HGTG30N120D2</td>
<td>1200V</td>
<td>30A</td>
<td>3.2A</td>
<td>0.58µs</td>
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<tr>
<td><strong>Multiple-chip power modules</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM400HA-12E</td>
<td>600V</td>
<td>400A</td>
<td>2.7V</td>
<td>0.3µs</td>
</tr>
<tr>
<td>CM300HA-24E</td>
<td>1200V</td>
<td>300A</td>
<td>2.7V</td>
<td>0.3µs</td>
</tr>
</tbody>
</table>
Conclusions: IGBT

- Becoming the device of choice in 500 to 1700V+ applications, at power levels of 1-1000kW
- Positive temperature coefficient at high current —easy to parallel and construct modules
- Forward voltage drop: diode in series with on-resistance. 2-4V typical
- Easy to drive —similar to MOSFET
- Slower than MOSFET, but faster than Darlington, GTO, SCR
- Typical switching frequencies: 3-30kHz
- IGBT technology is rapidly advancing:
  - 3300 V devices: HVIGBTs
  - 150 kHz switching frequencies in 600 V devices
An IGBT and a silicon diode operate in a buck converter, with the IGBT waveforms illustrated in Fig. 4.57. The converter operates with input voltage $V_g = 400$ V, output voltage $V = 200$ V, and load current $I = 10$ A.

(a) Estimate the total energy lost during the switching transitions.

(b) The forward voltage drop of the IGBT is 2.5 V, and the diode has a forward voltage drop 1.5 V. All other sources of conduction loss and fixed loss can be neglected. Estimate the semiconductor conduction loss.

(c) Sketch the converter efficiency over the range of switching frequencies $1$ kHz $\leq f_s \leq 100$ kHz, and label numerical values.

Fig. 4.57 IGBT voltage and current waveforms, Problem 4.7.

Two MOSFETs are employed as current-bidirectional two-quadrant switches in a bidirectional battery charger/discharger based on the dc-dc buck converter. This converter interfaces a 16 V battery to a 28 V main power bus. The maximum battery current is 40 A. The MOSFETs have on-resistances of 35 mΩ.
Chapter 5. The Discontinuous Conduction Mode

5.1. Origin of the discontinuous conduction mode, and mode boundary

5.2. Analysis of the conversion ratio $M(D, K)$

5.3. Boost converter example

5.4. Summary of results and key points
Introduction to
Discontinuous Conduction Mode (DCM)

- Occurs because switching ripple in inductor current or capacitor voltage causes polarity of applied switch current or voltage to reverse, such that the current- or voltage-unidirectional assumptions made in realizing the switch are violated.
- Commonly occurs in dc-dc converters and rectifiers, having single-quadrant switches. May also occur in converters having two-quadrant switches.
- Typical example: dc-dc converter operating at light load (small load current). Sometimes, dc-dc converters and rectifiers are purposely designed to operate in DCM at all loads.
- Properties of converters change radically when DCM is entered:
  - $M$ becomes load-dependent
  - Output impedance is increased
  - Dynamics are altered
  - Control of output voltage may be lost when load is removed
5.1. Origin of the discontinuous conduction mode, and mode boundary

Buck converter example, with single-quadrant switches

Minimum diode current is \((I - \Delta i_L)\)

Dc component \(I = V/R\)

Current ripple is

\[
\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}
\]

Note that \(I\) depends on load, but \(\Delta i_L\) does not.
Reduction of load current

Increase $R$, until $I = \Delta i_L$

Minimum diode current is $(I - \Delta i_L)$

Dc component $I = V/R$

Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} \cdot DT_s = \frac{V_g \cdot DD'T_s}{2L}$$

Note that $I$ depends on load, but $\Delta i_L$ does not.
Further reduce load current

Increase $R$ some more, such that $I < \Delta i_L$

Minimum diode current is $(I - \Delta i_L)$

Dc component $I = V/R$

Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

Note that $I$ depends on load, but $\Delta i_L$ does not.
The load current continues to be positive and non-zero.
Mode boundary

\[ I > \Delta i_L \quad \text{for CCM} \]
\[ I < \Delta i_L \quad \text{for DCM} \]

Insert buck converter expressions for \( I \) and \( \Delta i_L \):

\[ \frac{DV_g}{R} < \frac{DD'T_sV_g}{2L} \]

Simplify:

\[ \frac{2L}{RT_s} < D' \]

This expression is of the form

\[ K < K_{\text{crit}}(D) \quad \text{for DCM} \]

where 
\[ K = \frac{2L}{RT_s} \quad \text{and} \quad K_{\text{crit}}(D) = D' \]
$K$ and $K_{\text{crit}}$ vs. $D$

for $K < 1$:

- $K < K_{\text{crit}}$: DCM
- $K = \frac{2L}{RT_s}$

for $K > 1$:

- $K > K_{\text{crit}}$: CCM
- $K = \frac{2L}{RT_s}$
Critical load resistance $R_{\text{crit}}$

Solve $K_{\text{crit}}$ equation for load resistance $R$:

\[
R < R_{\text{crit}}(D) \quad \text{for CCM}
\]
\[
R > R_{\text{crit}}(D) \quad \text{for DCM}
\]

where \[ R_{\text{crit}}(D) = \frac{2L}{D'T_s} \]
Summary: mode boundary

\[ K > K_{\text{crit}}(D) \quad \text{or} \quad R < R_{\text{crit}}(D) \quad \text{for CCM} \]

\[ K < K_{\text{crit}}(D) \quad \text{or} \quad R > R_{\text{crit}}(D) \quad \text{for DCM} \]

Table 5.1. CCM-DCM mode boundaries for the buck, boost, and buck-boost converters

<table>
<thead>
<tr>
<th>Converter</th>
<th>( K_{\text{crit}}(D) )</th>
<th>( \max_{0 \leq D \leq 1}(K_{\text{crit}}) )</th>
<th>( R_{\text{crit}}(D) )</th>
<th>( \min_{0 \leq D \leq 1}(R_{\text{crit}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>((1 - D))</td>
<td>1</td>
<td>(\frac{2L}{(1 - D)T_s})</td>
<td>(\frac{2L}{T_s})</td>
</tr>
<tr>
<td>Boost</td>
<td>(D(1 - D)^2)</td>
<td>(\frac{4}{27})</td>
<td>(\frac{2L}{D(1 - D)^2T_s})</td>
<td>(\frac{27L}{2T_s})</td>
</tr>
<tr>
<td>Buck-boost</td>
<td>((1 - D)^2)</td>
<td>1</td>
<td>(\frac{2L}{(1 - D)^2T_s})</td>
<td>(\frac{2L}{T_s})</td>
</tr>
</tbody>
</table>
5.2. Analysis of the conversion ratio $M(D,K)$

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance

$$\langle v_L \rangle = \frac{1}{T_s} \int_{0}^{T_s} v_L(t) \, dt = 0$$

Capacitor charge balance

$$\langle i_C \rangle = \frac{1}{T_s} \int_{0}^{T_s} i_C(t) \, dt = 0$$

Small ripple approximation sometimes applies:

$$v(t) \approx V \quad \text{because} \quad \Delta v << V$$

$$i(t) \approx I \quad \text{is a poor approximation when} \quad \Delta i > I$$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.
Example: Analysis of DCM buck converter $M(D,K)$

Subinterval 1

Subinterval 2

Subinterval 3
Subinterval 1

\[ v_L(t) = V_g - v(t) \]
\[ i_C(t) = i_L(t) - v(t) / R \]

Small ripple approximation for \( v(t) \) (but not for \( i(t) \)):

\[ v_L(t) \approx V_g - V \]
\[ i_C(t) \approx i_L(t) - V / R \]
Subinterval 2

\[ v_L(t) = -v(t) \]
\[ i_C(t) = i_L(t) - v(t) / R \]

Small ripple approximation for \( v(t) \) but not for \( i(t) \):

\[ v_L(t) \approx -V \]
\[ i_C(t) \approx i_L(t) - V / R \]
Subinterval 3

\[ v_L = 0, \quad i_L = 0 \]
\[ i_C(t) = i_L(t) - \frac{v(t)}{R} \]

Small ripple approximation:

\[ v_L(t) = 0 \]
\[ i_C(t) = -\frac{V}{R} \]
Inductor volt-second balance

Volt-second balance:

\[
\langle v_L(t) \rangle = D_1(V_g - V) + D_2(-V) + D_3(0) = 0
\]

Solve for \( V \):

\[
V = V_g \frac{D_1}{D_1 + D_2}
\]

note that \( D_2 \) is unknown
Capacitor charge balance

node equation:

\[ i_L(t) = i_C(t) + \frac{V}{R} \]

capacitor charge balance:

\[ \langle i_C \rangle = 0 \]

hence

\[ \langle i_L \rangle = \frac{V}{R} \]

must compute dc component of inductor current and equate to load current (for this buck converter example)
Inductor current waveform

peak current:

$$i_L(D_1T_s) = i_{pk} = \frac{V_g - V}{L} D_1 T_s$$

average current:

$$\langle i_L \rangle = \frac{1}{T_s} \int_0^{T_s} i_L(t) \, dt$$

triangle area formula:

$$\int_0^{T_s} i_L(t) \, dt = \frac{1}{2} i_{pk} (D_1 + D_2) T_s$$

$$\langle i_L \rangle = (V_g - V) \frac{D_1 T_s}{2L} (D_1 + D_2)$$

$$V = \frac{D_1 T_s}{2L} (D_1 + D_2) (V_g - V)$$
Solution for $V$

Two equations and two unknowns ($V$ and $D_2$):

$$V = V_g \frac{D_1}{D_1 + D_2}$$  
(from inductor volt-second balance)

$$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2) (V_g - V)$$  
(from capacitor charge balance)

Eliminate $D_2$, solve for $V$:

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = \frac{2L}{RT_s}$

valid for $K < K_{crit}$
Buck converter $M(D,K)$

$$M = \begin{cases} 
D & \text{for } K > K_{\text{crit}} \\
\frac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{\text{crit}}
\end{cases}$$

- $K = 0.01$
- $K = 0.1$
- $K = 0.5$
- $K \geq 1$
5.3. Boost converter example

Mode boundary:

\[ I > \Delta i_L \quad \text{for CCM} \]
\[ I < \Delta i_L \quad \text{for DCM} \]

Previous CCM soln:

\[ I = \frac{V_g}{D^2 R} \]
\[ \Delta i_L = \frac{V_g}{2L} DT_s \]
Mode boundary

\[
\frac{V_g}{D^2R} > \frac{DT_s V_g}{2L} \quad \text{for CCM}
\]

\[
\frac{2L}{RT_s} > DD^2 \quad \text{for CCM}
\]

\[
K > K_{crit}(D) \quad \text{for CCM}
\]

\[
K < K_{crit}(D) \quad \text{for DCM}
\]

where \( K = \frac{2L}{RT_s} \) and \( K_{crit}(D) = DD^2 \)

\[K_{crit}\left(\frac{1}{3}\right) = \frac{4}{27}\]
Mode boundary

\[ K > K_{\text{crit}}(D) \quad \text{for CCM} \]
\[ K < K_{\text{crit}}(D) \quad \text{for DCM} \]

where \( K = \frac{2L}{RT_s} \) and \( K_{\text{crit}}(D) = DD'^2 \)
Conversion ratio: DCM boost

Fundamentals of Power Electronics
Subinterval 1

\[ v_L(t) = V_g \]
\[ i_C(t) = -v(t) / R \]

Small ripple approximation for \( v(t) \) (but not for \( i(t) \)):

\[ v_L(t) \approx V_g \]
\[ i_C(t) \approx -V / R \]

\[ 0 < t < D_1 T_s \]
Subinterval 2

\[ v_L(t) = V_g - v(t) \]
\[ i_C(t) = i(t) - \frac{v(t)}{R} \]

Small ripple approximation for \( v(t) \) but not for \( i(t) \):

\[ v_L(t) \approx V_g - V \]
\[ i_C(t) \approx i(t) - \frac{V}{R} \]

\[ D_1 T_s < t < (D_1 + D_2) T_s \]
Subinterval 3

\[ v_L = 0, \quad i = 0 \]
\[ i_c(t) = -\frac{v(t)}{R} \]

Small ripple approximation:

\[ v_L(t) = 0 \]
\[ i_c(t) = -\frac{V}{R} \]

\[ (D_1 + D_2)T_s < t < T_s \]
Inductor volt-second balance

Volt-second balance:

\[ D_1 V_g + D_2(V_g - V) + D_3(0) = 0 \]

Solve for \( V \):

\[ V = \frac{D_1 + D_2}{D_2} V_g \]

note that \( D_2 \) is unknown
Capacitor charge balance

node equation:

\[ i_D(t) = i_C(t) + v(t) / R \]

capacitor charge balance:

\[ \langle i_C \rangle = 0 \]

hence

\[ \langle i_D \rangle = V / R \]

must compute dc component of diode current and equate to load current (for this boost converter example)
Inductor and diode current waveforms

peak current:

\[ i_{pk} = \frac{V_g}{L} D_1 T_s \]

average diode current:

\[ \langle i_D \rangle = \frac{1}{T_s} \int_0^{T_s} i_D(t) \, dt \]

triangle area formula:

\[ \int_0^{T_s} i_D(t) \, dt = \frac{1}{2} i_{pk} D_2 T_s \]
Equate diode current to load current

average diode current:

\[
\langle i_D \rangle = \frac{1}{T_s} \left( \frac{1}{2} i_{pk} D_2 T_s \right) = \frac{V_g D_1 D_2 T_s}{2L}
\]

equate to dc load current:

\[
\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}
\]
Solution for $V$

Two equations and two unknowns ($V$ and $D_2$):

$$V = \frac{D_1 + D_2}{D_2} V_g$$  \hspace{1cm} \text{(from inductor volt-second balance)}

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}$$  \hspace{1cm} \text{(from capacitor charge balance)}

Eliminate $D_2$, solve for $V$. From volt-sec balance eqn:

$$D_2 = D_1 \frac{V_g}{V - V_g}$$

Substitute into charge balance eqn, rearrange terms:

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$
Solution for $V$

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

Note that one root leads to positive $V$, while other leads to negative $V$. Select positive root:

$$\frac{V}{V_g} = M(D_1,K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where

$$K = \frac{2L}{RT_s}$$

valid for

$$K < K_{crit}(D)$$

Transistor duty cycle $D = \text{interval 1 duty cycle } D_1$
Boost converter characteristics

Approximate $M$ in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

for $K > K_{crit}$

$$M = \begin{cases} 
\frac{1}{1 - D} & \text{for } K > K_{crit} \\
\frac{1 + \sqrt{1 + 4D^2 / K}}{2} & \text{for } K < K_{crit}
\end{cases}$$
Summary of DCM characteristics

<table>
<thead>
<tr>
<th>Converter</th>
<th>$K_{crit}(D)$</th>
<th>DCM $M(D,K)$</th>
<th>DCM $D_2(D,K)$</th>
<th>CCM $M(D)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>$(1 - D)$</td>
<td>$\frac{2}{1 + \sqrt{1 + 4K/D^2}}$</td>
<td>$\frac{K}{D} M(D,K)$</td>
<td>$D$</td>
</tr>
<tr>
<td>Boost</td>
<td>$D(1 - D)^2$</td>
<td>$\frac{1 + \sqrt{1 + 4D^2/K}}{2}$</td>
<td>$\frac{K}{D} M(D,K)$</td>
<td>$\frac{1}{1 - D}$</td>
</tr>
<tr>
<td>Buck-boost</td>
<td>$(1 - D)^2$</td>
<td>$-\frac{D}{\sqrt{K}}$</td>
<td>$\sqrt{K}$</td>
<td>$-\frac{D}{1 - D}$</td>
</tr>
</tbody>
</table>

with $K = 2L / RT_s$. DCM occurs for $K < K_{crit}$. 

Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters
Summary of DCM characteristics

- DCM buck and boost characteristics are asymptotic to $M = 1$ and to the DCM buck-boost characteristic.

- DCM buck-boost characteristic is linear.

- CCM and DCM characteristics intersect at mode boundary. Actual $M$ follows characteristic having larger magnitude.

- DCM boost characteristic is nearly linear.
Summary of key points

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.

2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.

3. The dc conversion ratio $M$ of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.
Summary of key points

4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.

5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.