8.1.7. The low-\(Q\) approximation

Given a second-order denominator polynomial, of the form

\[
G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

When the roots are real, i.e., when \(Q < 0.5\), then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

\[
G(s) = \left(\frac{1}{1 + \frac{s}{\omega_1}}\right) \left(\frac{1}{1 + \frac{s}{\omega_2}}\right)
\]

This is a particularly desirable approach when \(Q \ll 0.5\), i.e., when the corner frequencies \(\omega_1\) and \(\omega_2\) are well separated.
An example

A problem with this procedure is the complexity of the quadratic formula used to find the corner frequencies.

R-L-C network example:

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + \frac{sL}{R} + s^2LC} \]

Use quadratic formula to factor denominator. Corner frequencies are:

\[ \omega_1, \omega_2 = \frac{L/R \pm \sqrt{(L/R)^2 - 4LC}}{2LC} \]
Factoring the denominator

\[ \omega_1, \omega_2 = \frac{L/R \pm \sqrt{(L/R)^2 - 4LC}}{2LC} \]

This complicated expression yields little insight into how the corner frequencies \( \omega_1 \) and \( \omega_2 \) depend on \( R, L, \) and \( C \).

When the corner frequencies are well separated in value, it can be shown that they are given by the much simpler (approximate) expressions

\[ \omega_1 \approx \frac{R}{L}, \quad \omega_2 \approx \frac{1}{RC} \]

\( \omega_1 \) is then independent of \( C \), and \( \omega_2 \) is independent of \( L \).

These simpler expressions can be derived via the Low-\( Q \) Approximation.
Derivation of the Low-\(Q\) Approximation

Given

\[
G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

Use quadratic formula to express corner frequencies \(\omega_1\) and \(\omega_2\) in terms of \(Q\) and \(\omega_0\) as:

\[
\omega_1 = \omega_0 \frac{1 - \sqrt{1 - 4Q^2}}{2Q} \quad \omega_2 = \omega_0 \frac{1 + \sqrt{1 - 4Q^2}}{2Q}
\]
Corner frequency $\omega_2$

\[ \omega_2 = \frac{\omega_0}{Q} \left( 1 + \sqrt{1 - 4Q^2} \right) \]

can be written in the form

\[ \omega_2 = \frac{\omega_0}{Q} F(Q) \]

where

\[ F(Q) = \frac{1}{2} \left( 1 + \sqrt{1 - 4Q^2} \right) \]

For small $Q$, $F(Q)$ tends to 1. We then obtain

\[ \omega_2 \approx \frac{\omega_0}{Q} \quad \text{for} \quad Q \ll \frac{1}{2} \]

For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.
Corner frequency $\omega_1$

$$\omega_1 = \frac{\omega_0}{Q} \frac{1 - \sqrt{1 - 4Q^2}}{2}$$

can be written in the form

$$\omega_1 = \frac{Q \omega_0}{F(Q)}$$

where

$$F(Q) = \frac{1}{2} \left( 1 + \sqrt{1 - 4Q^2} \right)$$

For small $Q$, $F(Q)$ tends to 1. We then obtain

$$\omega_1 \approx Q \omega_0 \quad \text{for} \quad Q \ll \frac{1}{2}$$

For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.
The Low-\(Q\) Approximation

\[ f_1 = \frac{Qf_0}{F(Q)} \]

\[ f_2 = \frac{f_0F(Q)}{Q} \approx \frac{f_0}{Q} \]

\( \| G \|_{\text{dB}} \)

0 dB

\(-20\text{dB/decade}\)

\(-40\text{dB/decade}\)
R-L-C Example

For the previous example:

\[
G(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{1 + s \frac{R}{L} + s^2 LC}
\]

\[
f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \sqrt{LC}}
\]

\[
Q = R \sqrt{\frac{C}{L}}
\]

Use of the Low-\(Q\) Approximation leads to

\[
\omega_1 \approx Q \omega_0 = R \sqrt{\frac{C}{L}} \frac{1}{\sqrt{LC}} = \frac{R}{L}
\]

\[
\omega_2 \approx \frac{\omega_0}{Q} = \frac{1}{R \sqrt{\frac{C}{L}}} \frac{1}{\sqrt{LC}} = \frac{1}{RC}
\]
8.1.8. Approximate Roots of an Arbitrary-Degree Polynomial

Generalize the low-Q approximation to obtain approximate factorization of the \( n^{th}\)-order polynomial

\[
P(s) = 1 + a_1 s + a_2 s^2 + \cdots + a_n s^n
\]

It is desired to factor this polynomial in the form

\[
P(s) = \left(1 + \tau_1 s\right) \left(1 + \tau_2 s\right) \cdots \left(1 + \tau_n s\right)
\]

When the roots are real and well separated in value, then approximate analytical expressions for the time constants \( \tau_1, \tau_2, \ldots, \tau_n \) can be found, that typically are simple functions of the circuit element values.

**Objective:** find a general method for deriving such expressions. Include the case of complex root pairs.
Derivation of method

Multiply out factored form of polynomial, then equate to original form (equate like powers of $s$):

- $a_1 = \tau_1 + \tau_2 + \cdots + \tau_n$
- $a_2 = \tau_1(\tau_2 + \cdots + \tau_n) + \tau_2(\tau_3 + \cdots + \tau_n) + \cdots$
- $a_3 = \tau_1\tau_2(\tau_3 + \cdots + \tau_n) + \tau_2\tau_3(\tau_4 + \cdots + \tau_n) + \cdots$
- $\vdots$
- $a_n = \tau_1\tau_2\tau_3\cdots\tau_n$

- Exact system of equations relating roots to original coefficients
- Exact general solution is hopeless
- Under what conditions can solution for time constants be easily approximated?
Approximation of time constants
when roots are real and well separated

\[
\begin{align*}
    a_1 &= \tau_1 + \tau_2 + \cdots + \tau_n \\
    a_2 &= \tau_1(\tau_2 + \cdots + \tau_n) + \tau_2(\tau_3 + \cdots + \tau_n) + \cdots \\
    a_3 &= \tau_1\tau_2(\tau_3 + \cdots + \tau_n) + \tau_2\tau_3(\tau_4 + \cdots + \tau_n) + \cdots \\
    &\vdots \\
    a_n &= \tau_1\tau_2\cdots\tau_n
\end{align*}
\]

Suppose that roots are real and well-separated, and are arranged in decreasing order of magnitude:

\[
\left|\tau_1\right| >> \left|\tau_2\right| >> \cdots >> \left|\tau_n\right|
\]

Then the first term of each equation is dominant

⇒ Neglect second and following terms in each equation above
Approximation of time constants when roots are real and well separated

System of equations:
(only first term in each equation is included)

\[ a_1 \approx \tau_1 \]
\[ a_2 \approx \tau_1 \tau_2 \]
\[ a_3 \approx \tau_1 \tau_2 \tau_3 \]
\[ \vdots \]
\[ a_n = \tau_1 \tau_2 \tau_3 \cdots \tau_n \]

Solve for the time constants:

\[ \tau_1 \approx a_1 \]
\[ \tau_2 \approx \frac{a_2}{a_1} \]
\[ \tau_3 \approx \frac{a_3}{a_2} \]
\[ \vdots \]
\[ \tau_n \approx \frac{a_n}{a_{n-1}} \]
Result
when roots are real and well separated

If the following inequalities are satisfied

\[ |a_1| >> \frac{a_2}{a_1} >> \frac{a_3}{a_2} >> \ldots >> \frac{a_n}{a_{n-1}} \]

Then the polynomial \( P(s) \) has the following approximate factorization

\[ P(s) = \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s\right) \left(1 + \frac{a_3}{a_2} s\right) \ldots \left(1 + \frac{a_n}{a_{n-1}} s\right) \]

• If the \( a_n \) coefficients are simple analytical functions of the element values \( L, C, \) etc., then the roots are similar simple analytical functions of \( L, C, \) etc.
• Numerical values are used to justify the approximation, but analytical expressions for the roots are obtained
When two roots are not well separated
then leave their terms in quadratic form

Suppose inequality \( k \) is not satisfied:

\[
|a_1| >> \left| \frac{a_2}{a_1} \right| >> \ldots >> \left| \frac{a_k}{a_{k-1}} \right| >> \left| \frac{a_{k+1}}{a_k} \right| >> \ldots >> \left| \frac{a_n}{a_{n-1}} \right|
\]

(not satisfied)

Then leave the terms corresponding to roots \( k \) and \((k + 1)\) in quadratic form, as follows:

\[
P(s) = \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s\right) \ldots \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2\right) \ldots \left(1 + \frac{a_n}{a_{n-1}} s\right)
\]

This approximation is accurate provided

\[
|a_1| >> \left| \frac{a_2}{a_1} \right| >> \ldots >> \left| \frac{a_k}{a_{k-1}} \right| >> \left| \frac{a_{k-2}a_{k+1}}{a_k^2a_{k-1}^2} \right| >> \left| \frac{a_{k+2}}{a_{k+1}} \right| >> \ldots >> \left| \frac{a_n}{a_{n-1}} \right|
\]
When the first inequality is violated
A special case for quadratic roots

When inequality 1 is not satisfied:

$$\begin{align*}
|a_1| &> |\frac{a_2}{a_1}| >> |\frac{a_3}{a_2}| >> \cdots >> |\frac{a_n}{a_{n-1}}| \\
\uparrow \\
\text{not satisfied}
\end{align*}$$

Then leave the first two roots in quadratic form, as follows:

$$P(s) \approx \left(1 + a_1 s + a_2 s^2\right) \left(1 + \frac{a_3}{a_2} s\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

This approximation is justified provided

$$\begin{align*}
|\frac{a_2^2}{a_3}| >> |a_1| >> |\frac{a_3}{a_2}| >> |\frac{a_4}{a_3}| >> \cdots >> |\frac{a_n}{a_{n-1}}|
\end{align*}$$
Other cases

- When several isolated inequalities are violated
  - Leave the corresponding roots in quadratic form
  - See next two slides

- When several adjacent inequalities are violated
  - Then the corresponding roots are close in value
  - Must use cubic or higher-order roots
Leaving adjacent roots in quadratic form

In the case when inequality \( k \) is not satisfied:

\[
\begin{vmatrix}
\frac{a_1}{a_1} & \frac{a_2}{a_1} & \ldots & \frac{a_k}{a_{k-1}} & \frac{a_{k+1}}{a_k} & \ldots & \frac{a_n}{a_{n-1}}
\end{vmatrix}
\]

Then leave the corresponding roots in quadratic form:

\[
P(s) \approx \left(1 + \frac{a_2}{a_1} s\right) \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_k} s^2\right) \ldots \left(1 + \frac{a_n}{a_{n-1}} s\right)
\]

This approximation is accurate provided that

\[
\begin{vmatrix}
\frac{a_1}{a_1} & \frac{a_2}{a_1} & \ldots & \frac{a_k}{a_{k-1}} & \frac{a_{k+1}}{a_k} & \ldots & \frac{a_n}{a_{n-1}}
\end{vmatrix}
\]

(derivation is similar to the case of well-separated roots)
When the first inequality is not satisfied

The formulas of the previous slide require a special form for the case when the first inequality is not satisfied:

\[
\begin{align*}
|a_1| & > \left| \frac{a_2}{a_1} \right| \Rightarrow \left| \frac{a_3}{a_2} \right| \Rightarrow \cdots \Rightarrow \left| \frac{a_n}{a_{n-1}} \right|
\end{align*}
\]

We should then use the following form:

\[
P(s) = \left(1 + a_1 s + a_2 s^2\right) \left(1 + \frac{a_3}{a_2} s\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)
\]

The conditions for validity of this approximation are:

\[
\begin{align*}
\left| \frac{a_2}{a_3} \right| & > \left| a_1 \right| \Rightarrow \left| \frac{a_3}{a_2} \right| \Rightarrow \left| \frac{a_4}{a_3} \right| \Rightarrow \cdots \Rightarrow \left| \frac{a_n}{a_{n-1}} \right|
\end{align*}
\]
Example
Damped input EMI filter

\[ G(s) = \frac{i_g(s)}{i_c(s)} = \frac{1 + s \frac{L_1 + L_2}{R}}{1 + s \frac{L_1 + L_2}{R} + s^2 L_1 C + s^3 \frac{L_1 L_2 C}{R}} \]
Example
Approximate factorization of a third-order denominator

The filter transfer function from the previous slide is

\[
G(s) = \frac{i_g(s)}{i_c(s)} = \frac{1 + s \frac{L_1 + L_2}{R}}{1 + s \frac{L_1 + L_2}{R} + s^2 L_1 C + s^3 \frac{L_1 L_2 C}{R}}
\]

—contains a third-order denominator, with the following coefficients:

\[
a_1 = \frac{L_1 + L_2}{R}
\]
\[
a_2 = L_1 C
\]
\[
a_3 = \frac{L_1 L_2 C}{R}
\]
Real roots case

Factorization as three real roots:

\[
\left(1 + s \frac{L_1 + L_2}{R}\right)\left(1 + sRC \frac{L_1}{L_1 + L_2}\right)\left(1 + s \frac{L_2}{R}\right)
\]

This approximate analytical factorization is justified provided

\[
\frac{L_1 + L_2}{R} \gg RC \frac{L_1}{L_1 + L_2} \gg \frac{L_2}{R}
\]

Note that these inequalities cannot be satisfied unless \(L_1 \gg L_2\). The above inequalities can then be further simplified to

\[
\frac{L_1}{R} \gg RC \gg \frac{L_2}{R}
\]

And the factored polynomial reduces to

\[
\left(1 + s \frac{L_1}{R}\right)\left(1 + sRC\right)\left(1 + s \frac{L_2}{R}\right)
\]

*Illustrates in a simple way how the roots depend on the element values*
When the second inequality is violated

\[
\frac{L_1 + L_2}{R} \gg \frac{RC}{L_1 + L_2} \quad \text{not satisfied}
\]

Then leave the second and third roots in quadratic form:

\[
P(s) = \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s + \frac{a_3}{a_1} s^2\right)
\]

which is

\[
\left(1 + s \frac{L_1 + L_2}{R}\right) \left(1 + sRC \frac{L_1}{L_1 + L_2} + s^2 \frac{L_1}{L_1 + L_2} C\right)
\]
Validity of the approximation

This is valid provided

\[ \frac{L_1 + L_2}{R} \gg RC \quad \frac{L_1}{L_1 + L_2} \gg \frac{L_1 \parallel L_2}{L_1 + L_2} RC \quad \text{(use } a_0 = 1) \]

These inequalities are equivalent to

\[ L_1 \gg L_2, \quad \text{and} \quad \frac{L_1}{R} \gg RC \]

It is no longer required that \( RC \gg L_2/R \)

The polynomial can therefore be written in the simplified form

\[ \left( 1 + s \frac{L_1}{R} \right) \left( 1 + sRC + s^2L_2C \right) \]
When the first inequality is violated

\[
\frac{L_1 + L_2}{R} \gg RC \frac{L_1}{L_1 + L_2} \gg \frac{L_2}{R}
\]

Then leave the first and second roots in quadratic form:

\[
P(s) = \left(1 + a_1 s + a_2 s^2\right) \left(1 + \frac{a_3}{a_2} s\right)
\]

which is

\[
\left(1 + s \frac{L_1 + L_2}{R} + s^2 L_1 C\right) \left(1 + s \frac{L_2}{R}\right)
\]
Validity of the approximation

This is valid provided

\[ \frac{L_1 RC}{L_2} \gg \frac{L_1 + L_2}{R} \gg \frac{L_2}{R} \]

These inequalities are equivalent to

\[ L_1 \gg L_2, \quad \text{and} \quad RC \gg \frac{L_2}{R} \]

It is no longer required that \( L_1/R \gg RC \)

The polynomial can therefore be written in the simplified form

\[ \left( 1 + s \frac{L_1}{R} + s^2 L_1 C \right) \left( 1 + s \frac{L_2}{R} \right) \]
8.2. Analysis of converter transfer functions

8.2.1. Example: transfer functions of the buck-boost converter
8.2.2. Transfer functions of some basic CCM converters
8.2.3. Physical origins of the right half-plane zero in converters
8.2.1. Example: transfer functions of the buck-boost converter

Small-signal ac model of the buck-boost converter, derived in Chapter 7:
Definition of transfer functions

The converter contains two inputs, \( \hat{d}(s) \) and \( \hat{v}_g(s) \) and one output, \( \hat{v}(s) \)

Hence, the ac output voltage variations can be expressed as the superposition of terms arising from the two inputs:

\[
\hat{v}(s) = G_{vd}(s) \hat{d}(s) + G_{vg}(s) \hat{v}_g(s)
\]

The control-to-output and line-to-output transfer functions can be defined as

\[
G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g(s) = 0} \quad \text{and} \quad G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\hat{d}(s) = 0}
\]
Derivation of line-to-output transfer function $G_{vg}(s)$

Set $\hat{d}$ sources to zero:

![Diagram showing derivation process]

Push elements through transformers to output side:

$$\hat{v}_g(s) \left(-\frac{D}{D'}\right)$$
Derivation of transfer functions

Use voltage divider formula to solve for transfer function:

\[ G_{v_g}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} \bigg|_{d(s) = 0} = -\frac{D}{D^2} \frac{R\parallel \frac{1}{sC}}{\frac{sL}{D^2} + \left(R\parallel \frac{1}{sC}\right)} \]

Expand parallel combination and express as a rational fraction:

\[ G_{v_g}(s) = -\frac{D}{D^2} \frac{\frac{R}{1 + sRC}}{\frac{sL}{D^2} + \frac{R}{1 + sRC}} \]

\[ = -\left(\frac{D}{D^2}\right) \frac{R}{R + \frac{sL}{D^2} + \frac{s^2RLC}{D^2}} \]

We aren’t done yet! Need to write in normalized form, where the coefficient of \( s^0 \) is 1, and then identify salient features.
Derivation of transfer functions

Divide numerator and denominator by $R$. Result: the line-to-output transfer function is

$$G_{vg}(s) = \frac{\dot{v}(s)}{\ddot{v}(s)} \bigg|_{\dot{a}(t)=0} = \left( -\frac{D}{D'} \right) \frac{1}{1 + s \frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}}$$

which is of the following standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + s \frac{s}{Q \omega_0} + \left( \frac{s}{\omega_0} \right)^2}$$
Salient features of the line-to-output transfer function

Equate standard form to derived transfer function, to determine expressions for the salient features:

\[ G_{g0} = -\frac{D}{D'} \]

\[ \frac{1}{\omega_0^2} = \frac{LC}{D'^2} \quad \omega_0 = \frac{D'}{\sqrt{LC}} \]

\[ \frac{1}{Q\omega_0} = \frac{L}{D'^2R} \quad Q = D'R \sqrt{\frac{C}{L}} \]
Derivation of control-to-output transfer function $G_{vd}(s)$

In small-signal model, set $\hat{v}_g$ source to zero:

$$\begin{align*}
\left( V_g - V \right) \hat{d}(s) + v(s) - L \hat{d}(s)
\end{align*}$$

Push all elements to output side of transformer:

$$\begin{align*}
\frac{V_g - V}{D} \hat{d}(s) + \frac{L}{D^2} \hat{d}(s) + C \hat{v}(s) + R \hat{v}(s)
\end{align*}$$

There are two $\hat{d}$ sources. One way to solve the model is to use superposition, expressing the output $\hat{v}$ as a sum of terms arising from the two sources.
Superposition

With the voltage source only:

\[ v(s) = \left( -\frac{V_s - V}{D} \right) \frac{R \parallel \frac{1}{sC}}{\frac{L}{D^2} + \frac{R \parallel \frac{1}{sC}}{sL}} \]

With the current source alone:

\[ i(s) = I \left( \frac{sL}{D^2} \parallel \frac{R}{sC} \right) \]

Total:

\[ G_{vd}(s) = \left( -\frac{V_s - V}{D} \right) \frac{R \parallel \frac{1}{sC}}{\frac{L}{D^2} + \frac{sL}{D^2} \parallel \frac{R}{sC}} + I \left( \frac{R}{sC} \right) \]
Control-to-output transfer function

Express in normalized form:

\[
G_{vd}(s) = \frac{\bar{v}(s)}{\bar{d}(s)} \bigg|_{\bar{r}_g(s) = 0} = \left( -\frac{V_g - V}{D^2} \right) \frac{\left( 1 - s \frac{LI}{V_g - V} \right)}{\left( 1 + s \frac{L}{D^2 R} + s^2 \frac{LC}{D^2} \right)}
\]

This is of the following standard form:

\[
G_{vd}(s) = G_{d0} \frac{\left( 1 - \frac{s}{\omega_z} \right)}{\left( 1 + \frac{s}{Q\omega_0} + \left( \frac{s}{\omega_0} \right)^2 \right)}
\]
Salient features of control-to-output transfer function

\[ G_{d0} = - \frac{V_g - V}{D'} = - \frac{V_g}{D'^2} = \frac{V}{DD'} \]

\[ \omega_c = \frac{V_g - V}{LT} = \frac{D' R}{D' L} \quad \text{(RHP)} \]

\[ \omega_0 = \frac{D'}{\sqrt{LC}} \]

\[ Q = D' R \sqrt{\frac{C}{L}} \]

— Simplified using the dc relations: \[ V = - \frac{D}{D'} V_g \]
\[ I = - \frac{V}{D' R} \]
Plug in numerical values

Suppose we are given the following numerical values:

\[ D = 0.6 \]
\[ R = 10\Omega \]
\[ V_g = 30V \]
\[ L = 160\mu H \]
\[ C = 160\mu F \]

Then the salient features have the following numerical values:

\[ |G_{g0}| = \frac{D}{D} = 1.5 \Rightarrow 3.5 \text{ dB} \]
\[ |G_{d0}| = \frac{|V|}{DD'} = 187.5 \text{ V} \Rightarrow 45.5 \text{ dBV} \]
\[ f_0 = \frac{\omega_0}{2\pi} = \frac{D'}{2\pi\sqrt{LC}} = 400 \text{ Hz} \]
\[ Q = D'R\sqrt{\frac{C}{L}} = 4 \Rightarrow 12 \text{ dB} \]
\[ f_z = \frac{\omega_z}{2\pi} = \frac{D'^2R}{2\pi DL} = 2.65 \text{ kHz} \]
Bode plot: control-to-output transfer function

\[ G_{vd} = 187 \text{ V} \implies 45.5 \text{ dBV} \]
\[ Q = 4 \implies 12 \text{ dB} \]
\[ f_0 = 400 \text{ Hz} \]
\[ 10^{\frac{1}{2}Q} f_0 = 533 \text{ Hz} \]
\[ f_z = 2.6 \text{ kHz} \text{ RHP} \]
\[ 10 f_z = 26 \text{ kHz} \]
\[ f_z/10 = 260 \text{ Hz} \]
\[ 10^{\frac{1}{2}Q} f_0 = 300 \text{ Hz} \]
\[ f_0/20 = 20 \text{ Hz} \]

Fundamentals of Power Electronics
Bode plot: line-to-output transfer function

\[ |G_{vg}| \]

\[ \angle G_{vg} \]

- \( G_{g0} = 1.5 \Rightarrow 3.5 \text{ dB} \)
- \( Q = 4 \Rightarrow 12 \text{ dB} \)
- \( f_0 = 400 \text{ Hz} \)
- \( f_0 \text{ Hz} \)
- \( 10^{-1/2Q} f_0 = 300 \text{ Hz} \)
- \( 10^{1/2Q} f_0 = 533 \text{ Hz} \)
- \( 40 \text{ dB/decade} \)
- \( 0^\circ \)
- \( -180^\circ \)
- \( 10 \text{ Hz} \)
- \( 1 \text{ kHz} \)
- \( 10 \text{ kHz} \)
- \( 100 \text{ kHz} \)

Fundamentals of Power Electronics
8.2.2. Transfer functions of some basic CCM converters

<table>
<thead>
<tr>
<th>Converter</th>
<th>$G_{g0}$</th>
<th>$G_{d0}$</th>
<th>$\omega_0$</th>
<th>$Q$</th>
<th>$\omega_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>buck</td>
<td>$D$</td>
<td>$\frac{V}{D}$</td>
<td>$\frac{1}{\sqrt{L C}}$</td>
<td>$R \sqrt{\frac{C}{L}}$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>boost</td>
<td>$\frac{1}{D}$</td>
<td>$\frac{V}{D}$</td>
<td>$\frac{D'}{\sqrt{L C}}$</td>
<td>$D'R \sqrt{\frac{C}{L}}$</td>
<td>$\frac{D'^2 R}{L}$</td>
</tr>
<tr>
<td>buck-boost</td>
<td>$-\frac{D'}{D'^2}$</td>
<td>$\frac{V}{D'D'^2}$</td>
<td>$\frac{D'}{\sqrt{L C}}$</td>
<td>$D'R \sqrt{\frac{C}{L}}$</td>
<td>$\frac{D'^2 R}{DL}$</td>
</tr>
</tbody>
</table>

where the transfer functions are written in the standard forms

$$G_{vd}(s) = G_{g0} \frac{\left(1 - \frac{s}{\omega_0}\right)}{1 + \frac{s}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
8.2.3. Physical origins of the right half-plane zero

\[ G(s) = \left( 1 - \frac{s}{\omega_0} \right) \]

- phase reversal at high frequency
- transient response: output initially tends in wrong direction
Two converters whose CCM control-to-output transfer functions exhibit RHP zeroes

\[ \langle i_o \rangle_{T_s} = d' \langle i_i \rangle_{T_s} \]

**Boost**

**Buck-boost**
Waveforms, step increase in duty cycle

\[ \langle i_D \rangle_{T_s} = d' \langle i_L \rangle_{T_s} \]

- Increasing \( d(t) \) causes the average diode current to initially decrease.
- As inductor current increases to its new equilibrium value, average diode current eventually increases.