Example 1: \[ G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} \]

with \( G_0 = 40 \Rightarrow 32 \text{ dB} \), \( f_1 = \frac{\omega_1}{2\pi} = 100 \text{ Hz} \), \( f_2 = \frac{\omega_2}{2\pi} = 2 \text{ kHz} \)
Example 2

Determine the transfer function $A(s)$ corresponding to the following asymptotes:

$\parallel A \parallel$

$\parallel A_0 \parallel_{\text{dB}}$

$+20 \, \text{dB/dec}$

$\angle A$

$0^\circ$

$\frac{f_1}{10}$

$-45^\circ/\text{dec}$

$10f_1$

$\frac{f_2}{10}$

$-90^\circ$

$10f_2$

$0^\circ$
Example 2, continued

One solution:

\[
A(s) = A_0 \left( \frac{1 + \frac{s}{\omega_1}}{1 + \frac{s}{\omega_2}} \right)
\]

Analytical expressions for asymptotes:

For \( f < f_1 \)

\[
A_0 \left( \frac{1 + \frac{s}{\omega_1}}{1 + \frac{s}{\omega_2}} \right) \bigg|_{s = j\omega_0} = A_0 \frac{1}{1} = A_0
\]

For \( f_1 < f < f_2 \)

\[
A_0 \left( \frac{\frac{\omega}{\omega_1} + \frac{s}{\omega_1}}{1 + \frac{s}{\omega_2}} \right) \bigg|_{s = j\omega_0} = A_0 \frac{\frac{\omega}{\omega_1}}{1} = A_0 \frac{\omega_0}{\omega_1} = A_0 \frac{f}{f_i}
\]
Example 2, continued

For \( f > f_2 \)

\[
A_0 \begin{pmatrix}
\frac{s}{\omega_1} \\
\frac{s}{\omega_2}
\end{pmatrix}
\bigg|_{s = jo} = A_0 \begin{pmatrix}
\frac{\omega_2}{\omega_1} \\
\frac{\omega_1}{\omega_2}
\end{pmatrix}
\bigg|_{s = jo} = A_0 \frac{\omega_2}{\omega_1} = A_0 \frac{f_2}{f_1}
\]

So the high-frequency asymptote is

\[
A_\infty = A_0 \frac{f_2}{f_1}
\]

Another way to express \( A(s) \): use inverted poles and zeroes, and express \( A(s) \) directly in terms of \( A_\infty \)

\[
A(s) = A_\infty \frac{1 + \frac{\omega_1}{s}}{1 + \frac{\omega_2}{s}}
\]
8.1.6 Quadratic pole response: resonance

Example

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s \frac{L}{R} + s^2 L C} \]

Second-order denominator, of the form

\[ G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \]

with \( a_1 = \frac{L}{R} \) and \( a_2 = LC \)

How should we construct the Bode diagram?
Approach 1: factor denominator

\[ G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \]

We might factor the denominator using the quadratic formula, then construct Bode diagram as the combination of two real poles:

\[ G(s) = \frac{1}{\left(1 - \frac{s}{s_1}\right)\left(1 - \frac{s}{s_2}\right)} \]

with

\[ s_1 = -\frac{a_1}{2a_2} \left[1 - \sqrt{1 - \frac{4a_2}{a_1^2}}\right] \]

\[ s_2 = -\frac{a_1}{2a_2} \left[1 + \sqrt{1 - \frac{4a_2}{a_1^2}}\right] \]

- If \( 4a_2 \leq a_1^2 \), then the roots \( s_1 \) and \( s_2 \) are real. We can construct Bode diagram as the combination of two real poles.
- If \( 4a_2 > a_1^2 \), then the roots are complex. In Section 8.1.1, the assumption was made that \( \omega_0 \) is real; hence, the results of that section cannot be applied and we need to do some additional work.
$G(j\omega)$ and $\| G(j\omega) \|$ 

Let $s = j\omega$:

$$G(j\omega) = \frac{1}{1 + j \frac{\omega}{\omega_0}} = \frac{1 - j \frac{\omega}{\omega_0}}{1 + (\frac{\omega}{\omega_0})^2}$$

Magnitude is

$$\| G(j\omega) \| = \sqrt{[\text{Re} (G(j\omega))]^2 + [\text{Im} (G(j\omega))]^2}$$

$$= \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_0})^2}}$$

Magnitude in dB:

$$\| G(j\omega) \|_{\text{db}} = -20 \log_{10} \left( \sqrt{1 + (\frac{\omega}{\omega_0})^2} \right) \text{ dB}$$
Approach 2: Define a standard normalized form for the quadratic case

\[ G(s) = \frac{1}{1 + 2\zeta \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

- When the coefficients of \(s\) are real and positive, then the parameters \(\zeta, \omega_0,\) and \(Q\) are also real and positive.
- The parameters \(\zeta, \omega_0,\) and \(Q\) are found by equating the coefficients of \(s\).
- The parameter \(\omega_0\) is the angular corner frequency, and we can define \(f_0 = \omega_0/2\pi\).
- The parameter \(\zeta\) is called the damping factor. \(\zeta\) controls the shape of the exact curve in the vicinity of \(f = f_0\). The roots are complex when \(\zeta < 1\).
- The alternative form, the parameter \(Q\) is called the quality factor. \(Q\) also controls the shape of the exact curve in the vicinity of \(f = f_0\). The roots are complex when \(Q > 0.5\).
The $Q$-factor

In a second-order system, $\zeta$ and $Q$ are related according to

$$Q = \frac{1}{2\zeta}$$

$Q$ is a measure of the dissipation in the system. A more general definition of $Q$, for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}$$

For a second-order passive system, the two equations above are equivalent. We will see that $Q$ has a simple interpretation in the Bode diagrams of second-order transfer functions.
Analytical expressions for $f_0$ and $Q$

Two-pole low-pass filter example: we found that

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s \frac{L}{R} + s^2 L C}$$

Equate coefficients of like powers of $s$ with the standard form

$$G(s) = \frac{1}{1 + \frac{s}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Result:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \sqrt{LC}}$$

$$Q = R \sqrt{\frac{C}{L}}$$
Magnitude asymptotes, quadratic form

In the form

\[ G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

let \( s = j\omega \) and find magnitude: \[ |G(j\omega)| = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}} \]

Asymptotes are
\[ \|G\| \to 1 \quad \text{for} \quad \omega << \omega_0 \]
\[ \|G\| \to \left(\frac{f}{f_0}\right)^{-2} \quad \text{for} \quad \omega >> \omega_0 \]
Deviation of exact curve from magnitude asymptotes

\[ \| G(j\omega) \| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}} \]

At \( \omega = \omega_0 \), the exact magnitude is

\[ \| G(j\omega_0) \| = Q \quad \text{or, in dB:} \quad \| G(j\omega_0) \|_{dB} = |Q|_{dB} \]

The exact curve has magnitude \( Q \) at \( f = f_0 \). The deviation of the exact curve from the asymptotes is \( |Q|_{dB} \).
Phase asymptotes

\[ \angle G(j\omega) = -\tan^{-1}\left[ \frac{\frac{1}{Q} \left( \frac{\omega}{\omega_0} \right)}{1 - \left( \frac{\omega}{\omega_0} \right)^2} \right] \]

- Low frequency asymptote of 0°
- High frequency asymptote of -180°
- Change from 0° to -180° becomes more sharp as \( Q \) is increased

Increasing \( Q \)

--

Fundamentals of Power Electronics

Chapter 8: Converter Transfer Functions
Mid-frequency phase asymptote

Match slope at \( f = f_0 \): or Choose same approximation as in real pole case:

\[
\begin{align*}
\angle G &= 0^\circ \\
\angle G &= -90^\circ \\
\angle G &= -180^\circ
\end{align*}
\]

\[
\begin{align*}
f_a &= \left(e^{\pi/2}\right)^{-1/2Q} f_0 \\
f_b &= \left(e^{\pi/2}\right)^{1/2Q} f_0
\end{align*}
\]

\[
\begin{align*}
f_a &= 10^{-1/2Q} f_0 \\
f_b &= 10^{1/2Q} f_0
\end{align*}
\]
Two-pole response: exact curves
8.1.7. The low-\(Q\) approximation

Given a second-order denominator polynomial, of the form

\[
G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

When the roots are real, i.e., when \(Q < 0.5\), then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

\[
G(s) = \frac{1}{\left(1 + \frac{s}{\omega_1}\right) \left(1 + \frac{s}{\omega_2}\right)}
\]

This is a particularly desirable approach when \(Q \ll 0.5\), i.e., when the corner frequencies \(\omega_1\) and \(\omega_2\) are well separated.
An example

A problem with this procedure is the complexity of the quadratic formula used to find the corner frequencies.

R-L-C network example:

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2LC} \]

Use quadratic formula to factor denominator. Corner frequencies are:

\[ \omega_1, \omega_2 = \frac{L / R \pm \sqrt{(L / R)^2 - 4LC}}{2LC} \]
Factoring the denominator

\[ \omega_1, \omega_2 = \frac{L / R \pm \sqrt{(L / R)^2 - 4LC}}{2LC} \]

This complicated expression yields little insight into how the corner frequencies \( \omega_1 \) and \( \omega_2 \) depend on \( R, L, \) and \( C \).

When the corner frequencies are well separated in value, it can be shown that they are given by the much simpler (approximate) expressions

\[ \omega_1 \approx \frac{R}{L}, \quad \omega_2 \approx \frac{1}{RC} \]

\( \omega_1 \) is then independent of \( C \), and \( \omega_2 \) is independent of \( L \).

These simpler expressions can be derived via the Low-\( Q \) Approximation.
Derivation of the Low-\(Q\) Approximation

Given

\[ G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

Use quadratic formula to express corner frequencies \(\omega_1\) and \(\omega_2\) in terms of \(Q\) and \(\omega_0\) as:

\[
\omega_1 = \frac{\omega_0}{Q} \frac{1 - \sqrt{1 - 4Q^2}}{2} \quad \quad \quad \quad \quad \omega_2 = \frac{\omega_0}{Q} \frac{1 + \sqrt{1 - 4Q^2}}{2}
\]
Corner frequency $\omega_2$

\[
\omega_2 = \frac{\omega_0}{Q} \left( 1 + \frac{\sqrt{1 - 4Q^2}}{2} \right)
\]

can be written in the form

\[
\omega_2 = \frac{\omega_0}{Q} F(Q)
\]

where

\[
F(Q) = \frac{1}{2} \left( 1 + \sqrt{1 - 4Q^2} \right)
\]

For small $Q$, $F(Q)$ tends to 1. We then obtain

\[
\omega_2 \approx \frac{\omega_0}{Q} \quad \text{for} \quad Q \ll \frac{1}{2}
\]

For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.
Corner frequency $\omega_1$

$$\omega_1 = \frac{\omega_0}{Q} \frac{1 - \sqrt{1 - 4Q^2}}{2}$$

can be written in the form

$$\omega_1 = \frac{Q \omega_0}{F(Q)}$$

where

$$F(Q) = \frac{1}{2} \left( 1 + \sqrt{1 - 4Q^2} \right)$$

For small $Q$, $F(Q)$ tends to 1. We then obtain

$$\omega_1 \approx Q \omega_0 \quad \text{for} \quad Q \ll \frac{1}{2}$$

For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.
The Low-Q Approximation

\[ f_1 = \frac{Qf_0}{F(Q)} \]

\[ f_2 = \frac{f_0F(Q)}{Q} \]

\[ \| G \|_{\text{dB}} \]

0dB

-20dB/decade

-40dB/decade
For the previous example:

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2LC} \]

\[ f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \]

\[ Q = R\sqrt{\frac{C}{L}} \]

Use of the Low-\(Q\) Approximation leads to

\[ \omega_1 \approx Q \omega_0 = R \sqrt{\frac{C}{L}} \frac{1}{\sqrt{LC}} = \frac{R}{L} \]

\[ \omega_2 \approx \frac{\omega_0}{Q} = \frac{1}{\sqrt{LC}} \frac{1}{R \sqrt{\frac{C}{L}}} = \frac{1}{RC} \]
8.1.8. Approximate Roots of an Arbitrary-Degree Polynomial

Generalize the low-$Q$ approximation to obtain approximate factorization of the $n^{th}$-order polynomial

$$P(s) = 1 + a_1 s + a_2 s^2 + \cdots + a_n s^n$$

It is desired to factor this polynomial in the form

$$P(s) = \left( 1 + \tau_1 s \right) \left( 1 + \tau_2 s \right) \cdots \left( 1 + \tau_n s \right)$$

When the roots are real and well separated in value, then approximate analytical expressions for the time constants $\tau_1, \tau_2, \ldots \tau_n$ can be found, that typically are simple functions of the circuit element values.

**Objective:** find a general method for deriving such expressions. Include the case of complex root pairs.
Derivation of method

Multiply out factored form of polynomial, then equate to original form (equate like powers of $s$):

- $a_1 = \tau_1 + \tau_2 + \cdots + \tau_n$
- $a_2 = \tau_1(\tau_2 + \cdots + \tau_n) + \tau_2(\tau_3 + \cdots + \tau_n) + \cdots$
- $a_3 = \tau_1\tau_2(\tau_3 + \cdots + \tau_n) + \tau_2\tau_3(\tau_4 + \cdots + \tau_n) + \cdots$
- $\vdots$
- $a_n = \tau_1\tau_2\cdots\tau_n$

- Exact system of equations relating roots to original coefficients
- Exact general solution is hopeless
- Under what conditions can solution for time constants be easily approximated?
Approximation of time constants
when roots are real and well separated

\[
a_1 = \tau_1 + \tau_2 + \cdots + \tau_n
\]

**System of equations:**
\[
a_2 = \tau_1(\tau_2 + \cdots + \tau_n) + \tau_2(\tau_3 + \cdots + \tau_n) + \cdots
\]
\[
a_3 = \tau_1\tau_2(\tau_3 + \cdots + \tau_n) + \tau_2\tau_3(\tau_4 + \cdots + \tau_n) + \cdots
\]
\[\vdots\]
\[
a_n = \tau_1\tau_2\cdots\tau_n
\]

Suppose that roots are real and well-separated, and are arranged in decreasing order of magnitude:

\[
|\tau_1| >> |\tau_2| >> \cdots >> |\tau_n|
\]

Then the first term of each equation is dominant

\[\Rightarrow\textrm{ Neglect second and following terms in each equation above}\]
Approximation of time constants when roots are real and well separated

**System of equations:**

(only first term in each equation is included)

\[ a_1 \approx \tau_1 \]
\[ a_2 \approx \tau_1 \tau_2 \]
\[ a_3 \approx \tau_1 \tau_2 \tau_3 \]
\[ \vdots \]
\[ a_n = \tau_1 \tau_2 \tau_3 \cdots \tau_n \]

**Solve for the time constants:**

\[ \tau_1 \approx a_1 \]
\[ \tau_2 \approx \frac{a_2}{a_1} \]
\[ \tau_3 \approx \frac{a_3}{a_2} \]
\[ \vdots \]
\[ \tau_n \approx \frac{a_n}{a_{n-1}} \]
Result
when roots are real and well separated

If the following inequalities are satisfied

\[ |a_1| >> \frac{a_2}{a_1} >> \frac{a_3}{a_2} >> \ldots >> \frac{a_n}{a_{n-1}} \]

Then the polynomial \( P(s) \) has the following approximate factorization

\[ P(s) = \left( 1 + a_1 s \right) \left( 1 + \frac{a_2}{a_1} s \right) \left( 1 + \frac{a_3}{a_2} s \right) \ldots \left( 1 + \frac{a_n}{a_{n-1}} s \right) \]

- If the \( a_n \) coefficients are simple analytical functions of the element values \( L, C, \) etc., then the roots are similar simple analytical functions of \( L, C, \) etc.
- Numerical values are used to justify the approximation, but analytical expressions for the roots are obtained
When two roots are not well separated
then leave their terms in quadratic form

Suppose inequality \( k \) is not satisfied:

\[
|a_1| >> \left| \frac{a_2}{a_1} \right| >> \cdots >> \left| \frac{a_k}{a_{k-1}} \right| \gg \left| \frac{a_{k+1}}{a_k} \right| >> \cdots >> \left| \frac{a_n}{a_{n-1}} \right|
\]

Then leave the terms corresponding to roots \( k \) and \((k + 1)\) in quadratic form, as follows:

\[
P(s) = \left(1 + a_1 s \right) \left(1 + \frac{a_2}{a_1} s \right) \cdots \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2 \right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s \right)
\]

This approximation is accurate provided

\[
|a_1| >> \left| \frac{a_2}{a_1} \right| >> \cdots >> \left| \frac{a_k}{a_{k-1}} \right| >> \left| \frac{a_{k-2} a_{k+1}}{a_k a_{k+1}} \right| >> \left| \frac{a_{k+2}}{a_{k+1}} \right| >> \cdots >> \left| \frac{a_n}{a_{n-1}} \right|
\]
When the first inequality is violated
A special case for quadratic roots

When inequality 1 is not satisfied:

\[
\begin{align*}
\left| a_1 \right| & \not\geq \left| \frac{a_2}{a_1} \right| \gg \left| \frac{a_3}{a_2} \right| \gg \ldots \gg \left| \frac{a_n}{a_{n-1}} \right| \\
\uparrow \\
\text{not satisfied}
\end{align*}
\]

Then leave the first two roots in quadratic form, as follows:

\[
P(s) \approx \left(1 + a_1 s + a_2 s^2\right) \left(1 + \frac{a_3}{a_2} s\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)
\]

This approximation is justified provided

\[
\left| \frac{a_2}{a_3} \right| \gg \left| a_1 \right| \gg \left| \frac{a_3}{a_2} \right| \gg \left| \frac{a_4}{a_3} \right| \gg \ldots \gg \left| \frac{a_n}{a_{n-1}} \right|
\]
Other cases

- When several isolated inequalities are violated
  - Leave the corresponding roots in quadratic form
  - See next two slides

- When several adjacent inequalities are violated
  - Then the corresponding roots are close in value
  - Must use cubic or higher-order roots
Leaving adjacent roots in quadratic form

In the case when inequality $k$ is not satisfied:

$$\left| \frac{a_1}{a_1} \right| > \left| \frac{a_2}{a_1} \right| > \ldots > \left| \frac{a_k}{a_{k-1}} \right| > \left| \frac{a_{k+1}}{a_k} \right| > \ldots > \left| \frac{a_n}{a_{n-1}} \right|$$

Then leave the corresponding roots in quadratic form:

$$P(s) \approx \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s\right) \ldots \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_k} s^2\right) \ldots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

This approximation is accurate provided that

$$\left| \frac{a_1}{a_1} \right| > \left| \frac{a_2}{a_1} \right| > \ldots > \left| \frac{a_k}{a_{k-1}} \right| > \left| \frac{a_{k+1}}{a_k} \right| > \ldots > \left| \frac{a_n}{a_{n-1}} \right|$$

(derivation is similar to the case of well-separated roots)
When the first inequality is not satisfied

The formulas of the previous slide require a special form for the case when the first inequality is not satisfied:

\[
\left| a_1 \right| \gg \left| \frac{a_2}{a_1} \right| \gg \left| \frac{a_3}{a_2} \right| \gg \ldots \gg \left| \frac{a_n}{a_{n-1}} \right|
\]

We should then use the following form:

\[
P(s) = \left(1 + a_1 s + a_3 s^2\right)\left(1 + \frac{a_3}{a_2} s \right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s \right)
\]

The conditions for validity of this approximation are:

\[
\left| \frac{a_2}{a_3} \right| \gg \left| a_1 \right| \gg \left| \frac{a_3}{a_2} \right| \gg \left| \frac{a_4}{a_3} \right| \gg \ldots \gg \left| \frac{a_n}{a_{n-1}} \right|
\]
Example
Damped input EMI filter

\[
G(s) = \frac{i_g(s)}{i_c(s)} = \frac{1 + s \frac{L_1 + L_2}{R}}{1 + s \frac{L_1 + L_2}{R} + s^2 L_1 C + s^3 \frac{L_1 L_2 C}{R}}
\]
Example
Approximate factorization of a third-order denominator

The filter transfer function from the previous slide is

\[ G(s) = \frac{i_g(s)}{i_c(s)} = \frac{1 + s \frac{L_1 + L_2}{R}}{1 + s \frac{L_1 + L_2}{R} + s^2 L_1 C + s^3 \frac{L_1 L_2 C}{R}} \]

—contains a third-order denominator, with the following coefficients:

\[ a_1 = \frac{L_1 + L_2}{R} \]
\[ a_2 = L_1 C \]
\[ a_3 = \frac{L_1 L_2 C}{R} \]
Real roots case

Factorization as three real roots:

\[
\left(1 + s \frac{L_1 + L_2}{R}\right) \left(1 + sRC \frac{L_1}{L_1 + L_2}\right) \left(1 + s \frac{L_2}{R}\right)
\]

This approximate analytical factorization is justified provided

\[
\frac{L_1 + L_2}{R} \gg RC \frac{L_1}{L_1 + L_2} \gg \frac{L_2}{R}
\]

Note that these inequalities cannot be satisfied unless \(L_1 \gg L_2\). The above inequalities can then be further simplified to

\[
\frac{L_1}{R} \gg RC \gg \frac{L_2}{R}
\]

And the factored polynomial reduces to

\[
\left(1 + s \frac{L_1}{R}\right) \left(1 + sRC\right) \left(1 + s \frac{L_2}{R}\right)
\]

• Illustrates in a simple way how the roots depend on the element values
When the second inequality is violated

\[
\frac{L_1 + L_2}{R} \gg RC \quad \frac{L_1}{L_1 + L_2} \quad \star \quad \frac{L_2}{R}
\]

not satisfied

Then leave the second and third roots in quadratic form:

\[
P(s) = \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s + \frac{a_3}{a_1} s^2\right)
\]

which is

\[
\left(1 + s \frac{L_1 + L_2}{R}\right) \left(1 + sRC \frac{L_1}{L_1 + L_2} + s^2 L_1\|L_2 C\right)
\]
Validity of the approximation

This is valid provided

\[
\frac{L_1 + L_2}{R} \gg RC \quad \frac{L_1}{L_1 + L_2} \gg \frac{L_1 || L_2}{L_1 + L_2} \quad RC \quad \text{(use } a_0 = 1)\]

These inequalities are equivalent to

\[
L_1 \gg L_2, \quad \text{and} \quad \frac{L_1}{R} \gg RC
\]

It is no longer required that \(RC \gg L_2/R\)

The polynomial can therefore be written in the simplified form

\[
\left(1 + s \frac{L_1}{R}\right) \left(1 + sRC + s^2 L_2 C\right)
\]
When the first inequality is violated

\[ \frac{L_1 + L_2}{R} \gg RC \frac{L_1}{L_1 + L_2} \gg \frac{L_2}{R} \]

Then leave the first and second roots in quadratic form:

\[ P(s) = \left(1 + a_1 s + a_2 s^2\right) \left(1 + \frac{a_3}{a_2} s\right) \]

which is

\[ \left(1 + s \frac{L_1 + L_2}{R} + s^2 L_1 C\right) \left(1 + s \frac{L_2}{R}\right) \]
Validity of the approximation

This is valid provided

\[
\frac{L_1RC}{L_2} \gg \frac{L_1 + L_2}{R} \gg \frac{L_2}{R}
\]

These inequalities are equivalent to

\[
L_1 \gg L_2, \quad \text{and} \quad RC \gg \frac{L_2}{R}
\]

It is no longer required that \( L_1/R \gg RC \)

The polynomial can therefore be written in the simplified form

\[
\left( 1 + s \frac{L_1}{R} + s^2L_1C \right) \left( 1 + s \frac{L_2}{R} \right)
\]