Introduction to Power Electronics
ECEN 4797/5797

Lecture 27
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Impedance graph paper
Voltage divider transfer function
Division of asymptotes

Two ways to construct the transfer function $H(s)$:

\[
\frac{\hat{v}_2(s)}{\hat{v}_1(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{Z_2}{Z_{in}}
\]

\[
\frac{\hat{v}_2(s)}{\hat{v}_1(s)} = \frac{Z_2Z_1}{Z_1 + \frac{1}{Z_1}} = \frac{Z_{out}}{Z_1}
\]
Voltage divider transfer function

Division of asymptotes

\[ + \quad \frac{L}{C} \quad + \quad v_2(s) \quad - \quad H(s) \quad \frac{Z_{\text{out}}}{Z_1 Z_2} \quad \{ \quad \{ \quad Z_{\text{in}} v_1(s) \quad \frac{L}{C} v_2(s) \quad \frac{Z_{\text{out}}}{Z_1 Z_2} \quad \} \quad \} \quad \]

Two ways to construct the transfer function \( H(s) \):
Construction of transfer function

\[
\frac{\hat{v}_2(s)}{\hat{v}_1(s)} = \frac{Z_2 Z_1}{Z_1 + Z_2} \frac{1}{Z_{\text{out}}} = \frac{Z_{\text{out}}}{Z_1}
\]

\[
\omega L = 1
\]

\[
|H| = \left| \frac{Z_{\text{out}}}{Z_1} \right|
\]

\[
Q = R / R_0
\]

\[
\| Z_{\text{out}} \| = \| Z_1 \| = \omega L
\]

\[
\frac{1}{\omega C} = \frac{1}{\omega^2 LC}
\]
Fundamentals of Power Electronics

Chapter 8: Converter Transfer Functions

Transfer functions predicted by canonical model

Diagram of circuit components with labels and equations.
Output impedance $Z_{out}$: set sources to zero

\[ Z_{out} = Z_1 \parallel Z_2 \]
Graphical construction of output impedance

\[ \frac{1}{\omega C} \]

\[ R \]

\[ \| Z_I \| = \omega L_e \]

\[ Q = \frac{R}{R_0} \]

\[ f_0 \]

\[ \| Z_{out} \| \]
Graphical construction of filter effective transfer function

\[ Q = \frac{R}{R_0} \]

\[ \omega L_e \frac{\omega L_e}{\omega} = 1 \]

\[ \left( \frac{1}{\omega C} \right) \left( \frac{\omega L_e}{\omega} \right) = \frac{1}{\omega^2 L_e C} \]

\[ \| H_e \| = \frac{Z_{out}}{Z_1} \]
Boost and buck-boost converters: $L_e = L / D' \, ^2$
8.4. Measurement of ac transfer functions and impedances

**Network Analyzer**

<table>
<thead>
<tr>
<th>Injection source</th>
<th>Measured inputs</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_z ) magnitude</td>
<td>( v_x ) input</td>
<td>( \frac{v_z}{v_x} ) 17.3 dB</td>
</tr>
<tr>
<td>( v_z ) frequency</td>
<td>( v_y ) input</td>
<td>( -134.7^\circ )</td>
</tr>
<tr>
<td>( v_z ) output</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data bus to computer
Swept sinusoidal measurements

- Injection source produces sinusoid \( \hat{v}_z \) of controllable amplitude and frequency

- Signal inputs \( \hat{v}_x \) and \( \hat{v}_y \) perform function of narrowband tracking voltmeter:
  Component of input at injection source frequency is measured
  Narrowband function is essential: switching harmonics and other noise components are removed

- Network analyzer measures
  \[
  \left| \frac{\hat{v}_y}{\hat{v}_x} \right| \quad \text{and} \quad \angle \frac{\hat{v}_y}{\hat{v}_x}
  \]
Measurement of an ac transfer function

- Potentiometer establishes correct quiescent operating point
- Injection sinusoid coupled to device input via dc blocking capacitor
- Actual device input and output voltages are measured as \( \hat{v}_x \) and \( \hat{v}_y \)
- Dynamics of blocking capacitor are irrelevant

\[
\frac{\hat{v}_y(s)}{\hat{v}_x(s)} = G(s)
\]
Measurement of an output impedance

\[ Z(s) = \frac{\dot{v}(s)}{\dot{i}(s)} \]

\[ Z_{out}(s) = \frac{\dot{v}_y(s)}{\dot{i}_{out}(s)} \text{ with } \text{ac input} = 0 \]
Measurement of output impedance

- Treat output impedance as transfer function from output current to output voltage:

\[
Z(s) = \frac{\hat{v}(s)}{i(s)} \quad Z_{\text{out}}(s) = \frac{\hat{v}_{\text{out}}(s)}{i_{\text{out}}(s)}
\]

- Potentiometer at device input port establishes correct quiescent operating point

- Current probe produces voltage proportional to current; this voltage is connected to network analyzer channel \( \hat{v} \)

- Network analyzer result must be multiplied by appropriate factor, to account for scale factors of current and voltage probes
Measurement of small impedances

Grounding problems cause measurement to fail:

Injection current can return to analyzer via two paths. Injection current which returns via voltage probe ground induces voltage drop in voltage probe, corrupting the measurement. Network analyzer measures

$$Z + (1 - k) Z_{\text{probe}} = Z + Z_{\text{probe}} \parallel Z_{\text{rc}}$$

For an accurate measurement, require

$$|Z| >> \left|\frac{Z_{\text{probe}} \parallel Z_{\text{rc}}}{Z_{\text{probe}} \parallel Z_{\text{rc}}}\right|$$
Improved measurement: add isolation transformer

Injection current must now return entirely through transformer. No additional voltage is induced in voltage probe ground connection.
Chapter 9. Controller Design

9.1. Introduction

9.2. Effect of negative feedback on the network transfer functions
   9.2.1. Feedback reduces the transfer function from disturbances to the output
   9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$ and the closed-loop transfer functions
Controller design

9.4. Stability
   9.4.1. The phase margin test
   9.4.2. The relation between phase margin and closed-loop damping factor
   9.4.3. Transient response vs. damping factor

9.5. Regulator design
   9.5.1. Lead (PD) compensator
   9.5.2. Lag (PI) compensator
   9.5.3. Combined (PID) compensator
   9.5.4. Design example
9.1. Introduction

Output voltage of a switching converter depends on duty cycle $d$, input voltage $v_g$, and load current $i_{load}$. 

\[ v(t) = f(v_g, i_{load}, d) \]
The dc regulator application

Objective: maintain constant output voltage $v(t) = V$, in spite of disturbances in $v_g(t)$ and $i_{load}(t)$.

Typical variation in $v_g(t)$: 100Hz or 120Hz ripple, produced by rectifier circuit.

Load current variations: a significant step-change in load current, such as from 50% to 100% of rated value, may be applied.

A typical output voltage regulation specification: $5V \pm 0.1V$.

Circuit elements are constructed to some specified tolerance. In high volume manufacturing of converters, all output voltages must meet specifications.
The dc regulator application

So we cannot expect to set the duty cycle to a single value, and obtain a given constant output voltage under all conditions.

Negative feedback: build a circuit that automatically adjusts the duty cycle as necessary, to obtain the specified output voltage with high accuracy, regardless of disturbances or component tolerances.
Negative feedback: a switching regulator system

Diagram of a switching regulator system with the following components:

- **Power input**: Voltage source $v_g$
- **Switching converter**: Transistor gate driver, Pulse-width modulator $G_c(s)$, Compensator $H(s)$, Sensor gain $H(v)$
- **Load**: Current source $i_{load}$
- **Error signal**: $v_e$
- **Reference input**: Voltage source $v_{ref}$

The diagram illustrates the flow of signals and components within the switching regulator system, highlighting the negative feedback mechanism.
Negative feedback

\[ v(t) = f(v_g, i_{load}, d) \]

Switching converter

\[ v(t) \]

Error signal

Reference input

Compensator

Pulse-width modulator

Sensor gain

Control input

Disturbances
9.2. Effect of negative feedback on the network transfer functions

Small signal model: open-loop converter

Output voltage can be expressed as

\[ v(s) = G_{vd}(s) \hat{d}(s) + G_{vg}(s) \hat{v}_g(s) - Z_{out}(s) \hat{i}_{load}(s) \]

where

\[ G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g = 0, \hat{i}_{load} = 0} \]

\[ G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\hat{d} = 0, \hat{i}_{load} = 0} \]

\[ Z_{out}(s) = -\left. \frac{\hat{v}(s)}{\hat{i}_{load}(s)} \right|_{\hat{d} = 0, \hat{v}_g = 0} \]
Voltage regulator system small-signal model

- **Use small-signal converter model**
- **Perturb and linearize remainder of feedback loop:**

\[ v_{\text{ref}}(t) = V_{\text{ref}} + v_{\text{ref}}(t) \]

\[ v_e(t) = V_e + v_e(t) \]

*etc.*
Regulator system small-signal block diagram
Solution of block diagram

Manipulate block diagram to solve for $\tilde{v}(s)$. Result is

$$\tilde{v} = \tilde{v}_{\text{ref}} \frac{G_c G_{vd} / V_M}{1 + HG_c G_{vd} / V_M} + \tilde{v}_{g} \frac{G_{vg}}{1 + HG_c G_{vd} / V_M} - \tilde{i}_{\text{load}} \frac{Z_{out}}{1 + HG_c G_{vd} / V_M}$$

which is of the form

$$\tilde{v} = \tilde{v}_{\text{ref}} \frac{1}{H} \frac{T}{1+T} + \tilde{v}_{g} \frac{G_{vg}}{1+T} - \tilde{i}_{\text{load}} \frac{Z_{out}}{1+T}$$

with $T(s) = H(s) G_c(s) G_{vd}(s) / V_M$ = "loop gain"

Loop gain $T(s)$ = products of the gains around the negative feedback loop.
9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

\[ G_{vg}(s) = \frac{\ddot{v}(s)}{\ddot{v}_g(s)} \bigg|_{\dot{i}_{load} = 0, i_{load} = 0} \]

With addition of negative feedback, the line-to-output transfer function becomes:

\[ \frac{\ddot{v}(s)}{\ddot{v}_g(s)} \bigg|_{\ddot{v}_{ref} = 0, \dot{i}_{load} = 0} = \frac{G_{vg}(s)}{1 + T(s)} \]

Feedback reduces the line-to-output transfer function by a factor of \( \frac{1}{1 + T(s)} \).

If \( T(s) \) is large in magnitude, then the line-to-output transfer function becomes small.
Closed-loop output impedance

Original (open-loop) output impedance:

\[
Z_{\text{out}}(s) = -\frac{\hat{v}(s)}{\hat{i}_{\text{load}}(s)} \bigg|_{\hat{v}_{\text{g}} = 0, \hat{v}_{\text{ref}} = 0}
\]

With addition of negative feedback, the output impedance becomes:

\[
\frac{\hat{v}(s)}{-\hat{i}_{\text{load}}(s)} \bigg|_{\hat{v}_{\text{ref}} = 0} = \frac{Z_{\text{out}}(s)}{1 + T(s)}
\]

Feedback reduces the output impedance by a factor of

\[
\frac{1}{1 + T(s)}
\]

If \( T(s) \) is large in magnitude, then the output impedance is greatly reduced in magnitude.
9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop.

Closed-loop transfer function from \( v_{\text{ref}} \) to \( v(s) \) is:

\[
\left. \frac{v(s)}{v_{\text{ref}}(s)} \right|_{i_{\text{load}} = 0} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}
\]

If the loop gain is large in magnitude, i.e., \( \|T\| >> 1 \), then \((1+T) \approx T \) and \( T/(1+T) \approx T/T = 1 \). The transfer function then becomes

\[
\frac{v(s)}{v_{\text{ref}}(s)} \approx \frac{1}{H(s)}
\]

which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

\[
\frac{V}{V_{\text{ref}}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} \approx \frac{1}{H(0)}
\]
9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$

Example

$$T(s) = T_0 \frac{1 + \frac{s}{\omega_c}}{1 + \frac{s}{Q \omega_p^1} + \left(\frac{s}{\omega_p^1}\right)^2 \left(1 + \frac{s}{\omega_p^2}\right)}$$

At the crossover frequency $f_c$, $\| T \| = 1$
Approximating $1/(1+T)$ and $T/(1+T)$

$$\frac{T}{1 + T} \approx \begin{cases} 1 & \text{for } \| T \| >> 1 \\ T & \text{for } \| T \| << 1 \end{cases}$$

$$\frac{1}{1 + T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$
Example: construction of \( T/(1+T) \)

\[
\frac{T}{1+T} = \begin{cases} 
1 & \text{for } ||T|| >> 1 \\
\frac{1}{T} & \text{for } ||T|| << 1 
\end{cases}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
\text{Frequency} & 0 \text{ dB} & -20 \text{ dB} & -40 \text{ dB} & -60 \text{ dB} & -80 \text{ dB} \\
\hline
1 \text{ Hz} & f_p & (||T||) & f_c & f_z & f_{p2} \\
10 \text{ Hz} & & & & & \\
100 \text{ Hz} & & & & & \\
1 \text{ kHz} & & & & & \\
10 \text{ kHz} & & & & & \\
100 \text{ kHz} & & & & & \\
\hline
\end{array}
\]

Crossover frequency

- 20 dB/decade
- 40 dB/decade
Example: analytical expressions for approximate reference to output transfer function

At frequencies sufficiently less that the crossover frequency, the loop gain $T(s)$ has large magnitude. The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{\text{ref}}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{1}{H(s)}$$

This is the desired behavior: the output follows the reference according to the ideal gain $1/H(s)$. The feedback loop works well at frequencies where the loop gain $T(s)$ has large magnitude.

At frequencies above the crossover frequency, $\|T\| < 1$. The quantity $\frac{T}{1+T}$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{\text{ref}}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s)G_{\text{vd}}(s)}{V_M}$$

This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\|T\| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.
Same example: construction of \( \frac{1}{1+T} \)

\[
\frac{1}{1+T(s)} = \begin{cases} 
\frac{1}{T(s)} & \text{for } \| T \| \gg 1 \\
1 & \text{for } \| T \| \ll 1
\end{cases}
\]

\( f_p \) dB

\( f_c \) dB

\( f_z \) dB

Crossover frequency

\( Q_{\text{dB}} \)

\( |T_0|_{\text{dB}} \)

\( f_{\text{p1}} \)

\( f_{\text{p2}} \)

\( f_{\text{c}} \)

\( f_{\text{z}} \)

\(-40 \text{ dB/decade}\)

\(+20 \text{ dB/decade}\)

\(-20 \text{ dB/decade}\)

\(+40 \text{ dB/decade}\)

\(-80 \text{ dB}\)

\(-60 \text{ dB}\)

\(-40 \text{ dB}\)

\(-20 \text{ dB}\)

\(0 \text{ dB}\)

\(20 \text{ dB}\)

\(40 \text{ dB}\)

\(60 \text{ dB}\)

\(80 \text{ dB}\)

1 Hz 10 Hz 100 Hz 1 kHz 10 kHz 100 kHz

\( f \)
Interpretation: how the loop rejects disturbances

Below the crossover frequency: $f < f_c$ and $\|T\| > 1$
Then $1/(1+T) \approx 1/T$, and disturbances are reduced in magnitude by $1/\|T\|$.

Above the crossover frequency: $f > f_c$ and $\|T\| < 1$
Then $1/(1+T) \approx 1$, and the feedback loop has essentially no effect on disturbances.

$$\frac{1}{1+T(s)} = \begin{cases} \frac{1}{T(s)} & \text{for } \|T\| >> 1 \\ 1 & \text{for } \|T\| << 1 \end{cases}$$
Terminology: open-loop vs. closed-loop

Original transfer functions, before introduction of feedback (“open-loop transfer functions”):

\[ G_{vd}(s) \quad G_{vg}(s) \quad Z_{out}(s) \]

Upon introduction of feedback, these transfer functions become (“closed-loop transfer functions”):

\[ \frac{1}{H(s)} \quad \frac{T(s)}{1 + T(s)} \quad \frac{G_{vg}(s)}{1 + T(s)} \quad \frac{Z_{out}(s)}{1 + T(s)} \]

The loop gain:

\[ T(s) \]