9.5.4. Design example

\[ v(t) - v_g(t) = 28 \text{ V} \]

\[ v_{ref} = 5 \text{ V} \]

\[ f_s = 100 \text{ kHz} \]

\[ V_M = 4 \text{ V} \]

\[ R = 3 \Omega \]

\[ L = 50 \mu\text{H} \]

\[ C = 500 \mu\text{F} \]

\[ i_{load} \]

\[ H(s) \]

\[ v_e \]

\[ G_c(s) \]

\[ H_v \]

\[ v_c \]

\[ \delta \]

\[ \text{Transistor gate driver} \]

\[ \text{Pulse-width modulator} \]

\[ \text{Compensator} \]
Quiescent operating point

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage</td>
<td>$V_g = 28\text{V}$</td>
</tr>
<tr>
<td>Output</td>
<td>$V = 15\text{V}, I_{load} = 5\text{A}, R = 3\text{Ω}$</td>
</tr>
<tr>
<td>Quiescent duty cycle</td>
<td>$D = 15/28 = 0.536$</td>
</tr>
<tr>
<td>Reference voltage</td>
<td>$V_{\text{ref}} = 5\text{V}$</td>
</tr>
<tr>
<td>Quiescent value of control voltage</td>
<td>$V_c = DV_M = 2.14\text{V}$</td>
</tr>
<tr>
<td>Gain $H(s)$</td>
<td>$H = V_{\text{ref}}/V = 5/15 = 1/3$</td>
</tr>
</tbody>
</table>
Small-signal model

\[ \hat{V}_s(s) - \frac{V}{D} \dot{d} - \frac{1}{D} \dot{d} + L \dot{v} = 0 \]

\[ V_D = 2 \]

\[ V_R = \frac{d}{d} \]

Error signal

\[ \hat{v}_{ref} (= 0) \]

Compensator

\[ G_c(s) \]

\[ \frac{1}{V_M} \]

\[ V_M = 4 \ V \]

\[ H(s) \hat{v}(s) \]

\[ H = \frac{1}{3} \]

\[ T(s) \]

Fundamentals of Power Electronics

Chapter 9: Controller design
Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s \frac{L}{R} + s^2 LC}$$

standard form:

$$G_{vd}(s) = G_{d0} \frac{1}{1 + s \frac{1}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28V$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \sqrt{LC}} = 1kHz$$

$$Q_0 = R \sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5dB$$

$G_{d0} = 28V \Rightarrow 29dBV$

$Q_0 = 9.5 \Rightarrow 19.5dB$

$10^{-1/2} f_0 = 900Hz$

$10^{1/2} f_0 = 1.1kHz$
Open-loop line-to-output transfer function and output impedance

\[ G_{vg}(s) = D \frac{1}{1 + s \frac{L}{R} + s^2 LC} \]

—same poles as control-to-output transfer function
standard form:

\[ G_{vg}(s) = G_{g0} \frac{1}{1 + s \frac{Q_0}{\omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]

Output impedance:

\[ Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s \frac{L}{R} + s^2 LC} \]
System block diagram

\[
T(s) = G_c(s) \left( \frac{1}{V_M} \right) G_vd(s) H(s)
\]

\[
T(s) = \frac{G_c(s) H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left( \frac{s}{\omega_0} \right)^2}
\]

\[
\hat{v}_\text{ref} (=0) \rightarrow \hat{v}_e(s) \rightarrow G_c(s) \rightarrow \frac{1}{V_M} \rightarrow V_M = 4 \text{ V} \rightarrow \hat{d}(s) \rightarrow G_vd(s) \rightarrow Z_{\text{out}}(s) \rightarrow \text{Converter power stage} \rightarrow \hat{v}(s)
\]

Load current variation

\[
\hat{v}_g(s) \rightarrow G_vg(s) \rightarrow \text{Converter power stage} \rightarrow \hat{v}(s)
\]

Load current variation

\[
\hat{v}_g(s) \rightarrow G_vg(s) \rightarrow \text{Converter power stage} \rightarrow \hat{v}(s)
\]

Ac line variation

\[
\hat{v}_g(s) \rightarrow G_vg(s) \rightarrow \text{Converter power stage} \rightarrow \hat{v}(s)
\]
Uncompensated loop gain (with $G_c = 1$)

With $G_c = 1$, the loop gain is

$$T_u(s) = \frac{T_{u0}}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H V}{D V_M} = 2.33 \Rightarrow 7.4 \text{ dB}$$

$f_c = 1.8 \text{ kHz}$, $\phi_m = 5^\circ$
Lead compensator design

- Obtain a crossover frequency of 5 kHz, with phase margin of 52°
- $T_u$ has phase of approximately $-180°$ at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of $+52°$ at 5 kHz
- $T_u$ has magnitude of $-20.6$ dB at 5 kHz
- Lead compensator gain should have magnitude of $+20.6$ dB at 5 kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz}) \sqrt{\frac{1 - \sin (52°)}{1 + \sin (52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz}) \sqrt{\frac{1 + \sin (52°)}{1 - \sin (52°)}} = 14.5\text{kHz}$$

- Compensator dc gain should be

$$G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$$
$Q$ vs. $\varphi_m$

![Graph showing $Q$ vs. $\varphi_m$](image)

- $Q = 1 \Rightarrow 0$ dB
- $Q = 0.5 \Rightarrow -6$ dB
- $Q = 0.1 \Rightarrow -10$ dB
- $Q = 0.01 \Rightarrow -15$ dB
- $Q = 0.001 \Rightarrow -20$ dB

$\varphi_m = 52^\circ$

$\varphi_m = 76^\circ$
9.5.1. Lead (PD) compensator

\[ G_c(s) = G_{c0} \left( \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right) \]

Improves phase margin

\[ \| G_c \| \quad G_{c0} \sqrt{\frac{f_p}{f_z}} \quad f_p \]

\[ f_z = \sqrt{f_z f_p} \]

\[ \angle G_c \quad 0^\circ \quad f_z/10 \quad 10f_z \]

\[ +45^\circ/\text{decade} \quad f_p/10 \quad -45^\circ/\text{decade} \]
Lead compensator: maximum phase lead

\[ f_{\text{ymax}} = \sqrt{f_p f_z} \]

\[ \angle G_c(f_{\text{ymax}}) = \tan^{-1}\left( \frac{\sqrt{f_p} - \sqrt{f_z}}{2} \right) \]

\[ \frac{f_p}{f_c} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)} \]
Lead compensator design

To optimally obtain a compensator phase lead of $\theta$ at frequency $f_c$, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin (\theta)}{1 + \sin (\theta)}}$$

$$f_p = f_c \sqrt{\frac{1 + \sin (\theta)}{1 - \sin (\theta)}}$$

If it is desired that the magnitude of the compensator gain at $f_c$ be unity, then $G_{c0}$ should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$
Lead compensator Bode plot

\[ G_c = G_{c_0} \frac{f_p}{f_z} \sqrt{f_z} \]

- \[ f_z = \sqrt{f_z f_p} \]
- \[ G_{c_0} \]

Frequency range: 1 Hz to 100 kHz

- 0 dB
- -20 dB
- -40 dB

Phase angles:
- 0°
- 90°
- -90°
- -180°
Loop gain, with lead compensator

\[ T(s) = T_{w0} G_{c0} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \left( 1 + \frac{s}{Q_0 \omega_0} + \left( \frac{s}{\omega_0} \right)^2 \right) \]

\[ \| T \| \text{ at } f_0 = 1 \text{ kHz, } Q_0 = 9.5 \text{ dB} \]

\[ \angle T \text{ at } f_0 = 1 \text{ kHz, } Q_0 = 9.5 \text{ dB} \]
\[ \frac{1}{1 + T} \], with lead compensator

- need more low-frequency loop gain
- hence, add inverted zero (PID controller)
Improved compensator (PID)

\[ G_c(s) = G_{cm} \frac{1 + \frac{s}{\omega_c}}{1 + \frac{\omega_p}{s}} \]

- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose \( f_L \) to be \( f_p/10 \), so that phase margin is unchanged
$T(s)$ and $1/(1+T(s))$, with PID compensator
Open-loop line-to-output transfer function and output impedance

\[
G_{vg}(s) = D \frac{1}{1 + s \frac{L}{R} + s^2 LC}
\]

—same poles as control-to-output transfer function

standard form:

\[
G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

Output impedance:

\[
Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s \frac{L}{R} + s^2 LC}
\]
Line-to-output transfer function

\[
\frac{\hat{v}}{\hat{v}_k} = G_{vg}(0) = D
\]

Open-loop || \( G_{vg} \) ||

\[
\frac{D}{T_{sd} G_{cm}}
\]

Closed-loop || \( G_{vg} \) ||

\[
\frac{G_{vg}}{1 + T}
\]

- 40 dB/decade
- 20 dB/decade

f_0, f_L, f_z, f_c

-40 dB/decade
9.6. Measurement of loop gains

Objective: experimentally determine loop gain $T(s)$, by making measurements at point $A$

Correct result is

$$T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s)$$
Conventional approach: break loop, measure $T(s)$ as conventional transfer function

\[
T_m(s) = \frac{\hat{v}_y(s)}{\hat{v}_x(s)} \bigg|_{v_{ref} = 0, \hat{v}_y = 0}
\]

\[
T_m(s) = G_1(s) G_2(s) H(s)
\]
Measured vs. actual loop gain

Actual loop gain:

\[ T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s) \]

Measured loop gain:

\[ T_m(s) = G_1(s) G_2(s) H(s) \]

Express \( T_m \) as function of \( T \):

\[ T_m(s) = T(s) \left( 1 + \frac{Z_1(s)}{Z_2(s)} \right) \]

\[ T_m(s) \approx T(s) \quad \text{provided that} \quad \|Z_2\| >> \|Z_1\| \]

Fundamentals of Power Electronics 62 Chapter 9: Controller design
Discussion

• Breaking the loop disrupts the loading of block 2 on block 1.
  A suitable injection point must be found, where loading is not significant.

• Breaking the loop disrupts the dc biasing and quiescent operating point.
  A potentiometer must be used, to correctly bias the input to block 2.
  In the common case where the dc loop gain is large, it is very difficult to correctly set the dc bias.

• It would be desirable to avoid breaking the loop, such that the biasing circuits of the system itself set the quiescent operating point.
9.6.1. Voltage injection

- Ac injection source $v_z$ is connected between blocks 1 and 2
- Dc bias is determined by biasing circuits of the system itself
- Injection source does modify loading of block 2 on block 1
Voltage injection: measured transfer function $T_v(s)$

Network analyzer measures

$$T_v(s) = \frac{\hat{v}_y(s)}{\hat{v}_x(s)} \bigg|_{\hat{v}_{\text{ref}} = 0, \hat{v}_y = 0}$$

Solve block diagram:

$$\hat{v}_x(s) = -H(s) G_2(s) \hat{v}_y(s)$$

$$\hat{v}_y(s) = G_1(s) \hat{v}_x(s) - \hat{i}(s) Z_1(s)$$

Hence

$$\hat{v}_x(s) = -\hat{v}_x(s) G_2(s) H(s) G_1(s) - \hat{i}(s) Z_1(s)$$

with

$$\hat{i}(s) = \frac{\hat{v}_x(s)}{Z_2(s)}$$

Substitute:

$$\hat{v}_y(s) = \hat{v}_x(s) \left( G_1(s) G_2(s) H(s) + \frac{Z_1(s)}{Z_2(s)} \right)$$

which leads to the measured gain

$$T_v(s) = G_1(s) G_2(s) H(s) + \frac{Z_1(s)}{Z_2(s)}$$
Comparison of $T_v(s)$ with $T(s)$

Actual loop gain is

$$T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s)$$

Gain measured via voltage injection:

$$T_v(s) = G_1(s) \frac{Z_1(s)}{Z_2(s)}$$

Express $T_v(s)$ in terms of $T(s)$:

$$T_v(s) = T(s) \left( 1 + \frac{Z_1(s)}{Z_2(s)} \right) + \frac{Z_1(s)}{Z_2(s)}$$

Condition for accurate measurement:

$$T_v(s) = T(s) \quad \text{provided} \quad (i) \quad \left\| Z_1(s) \right\| < < \left\| Z_2(s) \right\|, \quad \text{and} \quad (ii) \quad \left\| T(s) \right\| >> \left\| \frac{Z_1(s)}{Z_2(s)} \right\|$$
Example: voltage injection

\[ Z_1(s) = 50\Omega \]
\[ Z_2(s) = 500\Omega \]
\[ \frac{Z_1(s)}{Z_2(s)} = 0.1 \Rightarrow -20\text{dB} \]
\[ \left( 1 + \frac{Z_1(s)}{Z_2(s)} \right) = 1.1 \Rightarrow 0.83\text{dB} \]

suppose actual \( T(s) = \left( \frac{10^4}{1 + \frac{s}{2\pi 10\text{Hz}}} \right) \left( \frac{1 + \frac{s}{2\pi 100\text{kHz}}}{1 + \frac{s}{2\pi 10\text{Hz}}} \right) \)
Example: measured $T_v(s)$ and actual $T(s)$

$$T_v(s) = T(s) \left( 1 + \frac{Z_1(s)}{Z_2(s)} \right) + \frac{Z_1(s)}{Z_2(s)}$$
9.6.2. Current injection

\[ T_i(s) = \left. \frac{i_z(s)}{i_x(s)} \right|_{v_{ref} = 0, v_g = 0} \]

\[ h(s) = 0 \]

[Diagram of current injection with blocks and equations]
Current injection

It can be shown that

\[ T_i(s) = T(s) \left( 1 + \frac{Z_2(s)}{Z_1(s)} \right) + \frac{Z_2(s)}{Z_1(s)} \]

Conditions for obtaining accurate measurement:

1. \[ |Z_2(s)| \ll |Z_1(s)| \], and
2. \[ |T(s)| \gg \left| \frac{Z_2(s)}{Z_1(s)} \right| \]

Injection source impedance \( Z_s \) is irrelevant. We could inject using a Thevenin-equivalent voltage source:
9.6.3. Measurement of unstable systems

- Injection source impedance $Z_s$ does not affect measurement.
- Increasing $Z_s$ reduces loop gain of circuit, tending to stabilize system.
- Original (unstable) loop gain is measured (not including $Z_s$), while circuit operates stably.

\[ H(s) \]

$T_v(s)$

$G_1(s)\hat{v}_c(s) + \hat{v}_c(s)$

$Z_1(s)$

$R_{ext}$

$L_{ext}$

$Z_2(s)$

$G_2(s)\hat{v}_c(s) = \hat{v}(s)$