9.5.4. Design example

![Diagram of a power electronics system with a transistor gate driver, pulse-width modulator, compensator, and sensor gain.]
Quiescent operating point

Input voltage

\[ V_g = 28 \text{V} \]

Output

\[ V = 15 \text{V}, \; I_{\text{load}} = 5 \text{A}, \; R = 3 \Omega \]

Quiescent duty cycle

\[ D = \frac{15}{28} = 0.536 \]

Reference voltage

\[ V_{\text{ref}} = 5 \text{V} \]

Quiescent value of control voltage

\[ V_c = DV_M = 2.14 \text{V} \]

Gain \( H(s) \)

\[ H = \frac{V_{\text{ref}}}{V} = \frac{5}{15} = \frac{1}{3} \]
Small-signal model

\[ V_D^2 \frac{d^2}{ds^2} V_R \]

\[ V_M = 4 \text{ V} \]

\[ H(s) \]

\[ H = \frac{1}{s} \]
Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s \frac{L}{R} + s^2 L C}$$

standard form:

$$G_{vd}(s) = G_{d0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R \sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$

$G_{d0} = 28\text{V}$

$Q_0 = 9.5 \Rightarrow 19.5\text{dB}$

$f_0 = 1.1\text{kHz}$

$10^{-1/2} f_0 = 900\text{Hz}$

$10^{1/2} f_0 = 1.1\text{kHz}$
Open-loop line-to-output transfer function and output impedance

\[ G_{\text{vg}}(s) = D \frac{1}{1 + s \frac{L}{R} + s^2 LC} \]

—same poles as control-to-output transfer function
standard form:

\[ G_{\text{vg}}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]

Output impedance:

\[ Z_{\text{out}}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s \frac{L}{R} + s^2 LC} \]
System block diagram

\[ T(s) = G_c(s) \left( \frac{1}{V_M} \right) G_{vd}(s) H(s) \]

\[ T(s) = \frac{G_c(s) H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]

\[ \hat{v}_{ref} (=0) \]

\[ \hat{v}_c(s) \]

\[ V_M = 4 \text{ V} \]

\[ \frac{1}{V_M} \]

\[ \hat{d}(s) \]

\[ G_c(s) \]

\[ G_{vd}(s) \]

\[ H(s) = \frac{1}{3} \]

\[ \hat{v}(s) \]

\[ \hat{i}_{load}(s) \]

\[ \hat{v}_g(s) \]

\[ Z_{out}(s) \]

\[ \text{ac line variation} \]

\[ \text{Converter power stage} \]

\[ \text{Load current variation} \]

\[ \text{Duty cycle variation} \]

\[ \text{Load current variation} \]

\[ \text{Converter power stage} \]
Uncompensated loop gain (with $G_c = 1$)

With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{HV}{DV_M} = 2.33 \Rightarrow 7.4\, \text{dB}$$

$f_c = 1.8\, \text{kHz}$, $\varphi_m = 5^\circ$
Lead compensator design

- Obtain a crossover frequency of 5 kHz, with phase margin of 52°
- $T_u$ has phase of approximately –180° at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of +52° at 5 kHz
- $T_u$ has magnitude of –20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of +20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be
  \[ f_z = (5 \text{kHz}) \sqrt{\frac{1 - \sin(52^\circ)}{1 + \sin(52^\circ)}} = 1.7\text{kHz} \]
  \[ f_p = (5 \text{kHz}) \sqrt{\frac{1 + \sin(52^\circ)}{1 - \sin(52^\circ)}} = 14.5\text{kHz} \]
- Compensator dc gain should be
  \[ G_{c0} = \left( \frac{f_z}{f_0} \right)^2 \frac{1}{T_u0} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB} \]
$Q$ vs. $\varphi_m$

![Graph showing the relationship between $Q$ and $\varphi_m$. The graph indicates $Q = 1 \Rightarrow 0$ dB at $\varphi_m = 52^\circ$. $Q = 0.5 \Rightarrow -6$ dB at $\varphi_m = 76^\circ$.](image)
9.5.1. Lead (PD) compensator

\[ G_c(s) = G_{c0} \left( \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right) \]

Improves phase margin

\[ f_{z} = \frac{f_p}{\sqrt{f_z f_{q_{max}}}} \]

\[ 0^\circ, \frac{f_z}{10}, f_{p}/10, 10f_z \]

\[ +45^\circ/\text{decade}, -45^\circ/\text{decade} \]
Lead compensator: maximum phase lead

\[ f_{q_{\text{max}}} = \sqrt{f_p f_z} \]

\[ \angle G_c(f_{q_{\text{max}}}) = \tan^{-1}\left(\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}\right) \]

\[ \frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)} \]
Lead compensator design

To optimally obtain a compensator phase lead of $\theta$ at frequency $f_c$, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

$$f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at $f_c$ be unity, then $G_{c0}$ should be chosen as

$$G_{c0} = \frac{f_z}{f_p}$$
Lead compensator Bode plot

\[ f_c = f_z \]

\[ G_c = G_{c0} \sqrt{\frac{f_p}{f_z}} \]

\[ f_z = \sqrt{f_p f_z} \]

\[ f_z / 10 \]

\[ 10 f_z \]

\[ 0^\circ \]

\[ -90^\circ \]

\[ -180^\circ \]

\[ 1 \text{ Hz} \quad 10 \text{ Hz} \quad 100 \text{ Hz} \quad 1 \text{ kHz} \quad 10 \text{ kHz} \quad 100 \text{ kHz} \]
Loop gain, with lead compensator

\[ T(s) = T_u G_c \frac{1 + \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_p}\right) \left(1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)} \]

\[ \| T \| \quad \| T \| \quad T_0 = 8.6 \Rightarrow 18.7 \text{ dB} \]
\[ Q_0 = 9.5 \Rightarrow 19.5 \text{ dB} \]

\[ f_0 = 1 \text{ kHz} \quad f_c = 1.7 \text{ kHz} \quad f_p = 5 \text{ kHz} \quad f = 14 \text{ kHz} \]

\[ f = 900 \text{ Hz} \quad f = 17 \text{ kHz} \]

\[ m = 52 \]

\[ f = 1.1 \text{ kHz} \quad f = 1.4 \text{ kHz} \quad f = 170 \text{ Hz} \]

\[ f = 5 \text{ kHz} \quad f = 1.7 \text{ kHz} \quad f = 10 \text{ kHz} \]
$1/(1+T)$, with lead compensator

- need more low-frequency loop gain
- hence, add inverted zero (PID controller)
Improved compensator (PID)

\[ G_c(s) = G_{cm} \left( \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right) \left( 1 + \frac{s}{t_z} \right) \]

- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose \( f_L \) to be \( f_p/10 \), so that phase margin is unchanged
$T(s)$ and $1/(1+T(s))$, with PID compensator
Open-loop line-to-output transfer function and output impedance

\[ G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2LC} \]

—same poles as control-to-output transfer function standard form:

\[ G_{vg}(s) = G_{g0} \frac{1}{1 + s\frac{s}{\Omega_0^2\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

Output impedance:

\[ Z_{out}(s) = R \parallel sC \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2LC} \]
Fundamentals of Power Electronics Chapter 9: Controller design

Line-to-output transfer function

\[
\frac{\hat{y}}{\hat{v}_k} = G_{vg}(0) = D
\]

Open-loop \( G_{vg} \)

Closed-loop \( \frac{G_{vg}}{1 + T} \)

-40 dB/decade

-40 dB/decade

f

1 Hz 10 Hz 100 Hz 1 kHz 10 kHz 100 kHz
9.6. Measurement of loop gains

Objective: experimentally determine loop gain $T(s)$, by making measurements at point $A$

Correct result is

$$T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s)$$
Fundamentals of Power Electronics

Chapter 9: Controller design

Conventional approach: break loop, measure $T(s)$ as conventional transfer function

$$T_m(s) = \frac{\hat{v}_y(s)}{\hat{v}_y(s)} \bigg|_{v_{\text{ref}} = 0, \hat{v}_g = 0}$$

$$T_m(s) = G_1(s) G_2(s) H(s)$$
Measured vs. actual loop gain

Actual loop gain:
\[ T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s) \]

Measured loop gain:
\[ T_m(s) = G_1(s) G_2(s) H(s) \]

Express \( T_m \) as function of \( T \):
\[ T_m(s) = T(s) \left( 1 + \frac{Z_1(s)}{Z_2(s)} \right) \]

\[ T_m(s) \approx T(s) \quad \text{provided that} \quad \left| \frac{Z_2}{Z_1} \right| \gg 1 \]
Discussion

• Breaking the loop disrupts the loading of block 2 on block 1.
  A suitable injection point must be found, where loading is not significant.

• Breaking the loop disrupts the dc biasing and quiescent operating point.
  A potentiometer must be used, to correctly bias the input to block 2.
  In the common case where the dc loop gain is large, it is very difficult to correctly set the dc bias.

• It would be desirable to avoid breaking the loop, such that the biasing circuits of the system itself set the quiescent operating point.
9.6.1. Voltage injection

- Ac injection source $v_z$ is connected between blocks 1 and 2
- Dc bias is determined by biasing circuits of the system itself
- Injection source does modify loading of block 2 on block 1
Voltage injection: measured transfer function $T_v(s)$

Network analyzer measures

$$T_v(s) = \frac{\hat{v}_y(s)}{\hat{v}_x(s)} \bigg|_{v_{\text{ref}} = 0, i_y = 0}$$

Solve block diagram:

$$\hat{v}_x(s) = -H(s) G_2(s) \hat{v}_y(s)$$

$$-\hat{v}_x(s) = G_1(s) \hat{v}_x(s) - i(s) Z_1(s)$$

Hence

$$-\hat{v}_x(s) = -\hat{v}_x(s) G_2(s) H(s) G_1(s) - i(s) Z_1(s)$$

with

$$i(s) = \frac{\hat{v}_x(s)}{Z_2(s)}$$

Substitute:

$$\hat{v}_y(s) = \hat{v}_y(s) \left( G_1(s) G_2(s) H(s) + \frac{Z_1(s)}{Z_2(s)} \right)$$

which leads to the measured gain

$$T_v(s) = G_1(s) G_2(s) H(s) + \frac{Z_1(s)}{Z_2(s)}$$
Comparison of $T_v(s)$ with $T(s)$

Actual loop gain is

$$T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s)$$

Gain measured via voltage injection:

$$T_v(s) = G_1(s) G_2(s) H(s) + \frac{Z_1(s)}{Z_2(s)}$$

Express $T_v(s)$ in terms of $T(s)$:

$$T_v(s) = T(s) \left( 1 + \frac{Z_2(s)}{Z_2(s)} \right) + \frac{Z_1(s)}{Z_2(s)}$$

Condition for accurate measurement:

$$T_v(s) = T(s) \quad \text{provided} \quad (i) \quad \left| Z_1(s) \right| \ll \left| Z_2(s) \right|, \quad \text{and} \quad (ii) \quad \left| T(s) \right| \gg \left| \frac{Z_1(s)}{Z_2(s)} \right|$$
Example: voltage injection

\[ Z_1(s) = 50\Omega \]
\[ Z_2(s) = 500\Omega \]
\[ \frac{Z_1(s)}{Z_2(s)} = 0.1 \Rightarrow -20\text{dB} \]
\[ \left(1 + \frac{Z_1(s)}{Z_2(s)}\right) = 1.1 \Rightarrow 0.83\text{dB} \]

suppose actual \( T(s) = \frac{10^4}{\left(1 + \frac{s}{2\pi \text{10Hz}}\right)\left(1 + \frac{s}{2\pi \text{100kHz}}\right)} \)
Example: measured $T_v(s)$ and actual $T(s)$

\[
T_v(s) = T(s) \left( 1 + \frac{Z_1(s)}{Z_2(s)} \right) + \frac{Z_1(s)}{Z_2(s)}
\]
9.6.2. Current injection

\[ T_i(s) = \frac{\dot{i}_y(s)}{\dot{i}_z(s)} \bigg|_{v_{ref} = 0, v_e = 0} \]

Block 1

\[ G_1(s) \hat{v}_e(s) + Z_1(s) \hat{i}_y(s) = \hat{i}_z(s) \]

Block 2

\[ G_2(s) \hat{i}_x(s) = v(s) \]

\[ H(s) \]

\[ T_i(s) \]
Current injection

It can be shown that

$$T_i(s) = T(s) \left( 1 + \frac{Z_2(s)}{Z_1(s)} \right) + \frac{Z_2(s)}{Z_1(s)}$$

Conditions for obtaining accurate measurement:

(i) \[ \| Z_2(s) \| \ll \| Z_1(s) \|, \text{ and} \]

(ii) \[ |T(s)| \gg \left\| \frac{Z_2(s)}{Z_1(s)} \right\| \]

Injection source impedance \( Z_s \) is irrelevant. We could inject using a Thevenin-equivalent voltage source:
9.6.3. Measurement of unstable systems

- Injection source impedance $Z_s$ does not affect measurement.
- Increasing $Z_s$ reduces loop gain of circuit, tending to stabilize system.
- Original (unstable) loop gain is measured (not including $Z_s$), while circuit operates stably.

\[ H(s) + \frac{Z(s)G(s)}{Z(s) + L(s)} = T(s) \]