Introduction to Power Electronics
ECEN 4797/5797

Lecture 33
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Improved compensator (PID)

\[ G_c(s) = G_{cm} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_L}} \left( 1 + \frac{s}{\omega_p} \right) \]

- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose \( f_L \) to be \( f_p/10 \), so that phase margin is unchanged
Part III. Magnetics

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Chapter 13 Basic Magnetics Theory

13.5 Several Types of Magnetic Devices, Their $B-H$ Loops, and Core vs. Copper Loss

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13.5.3 Transformer
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13.6 Summary of Key Points
13.1 Review of Basic Magnetics

13.1.1 Basic relationships

Faraday’s law

\[ \nu(t) \leftrightarrow B(t), \Phi(t) \]

Terminal characteristics

Ampere’s law

\[ i(t) \leftrightarrow H(t), \mathcal{F}(t) \]

Core characteristics
Basic quantities

**Magnetic quantities**

- Length $\ell$
- Magnetic field $H$
- MMF $\mathcal{F} = H\ell$
- Total flux $\Phi$
- Flux density $B$

**Electrical quantities**

- Length $\ell$
- Electric field $E$
- Voltage $V = E\ell$
- Total current $I$
- Current density $J$

Equations:

- Voltage: $V = E\ell$
- Current: $I = J\ell$
- Flux density: $B = \frac{\Phi}{A_c}$
- MMF: $\mathcal{F} = H\ell$
Magnetic field $H$ and magnetomotive force $\mathcal{F}$

Magnetomotive force (MMF) $\mathcal{F}$ between points $x_1$ and $x_2$ is related to the magnetic field $H$ according to

$$\mathcal{F} = \int_{x_1}^{x_2} H \cdot dl$$

**Example: uniform magnetic field of magnitude $H$**

**Analogous to electric field of strength $E$, which induces voltage (EMF) $V$:**

$$V = El$$
Flux density $B$ and total flux $\Phi$

The total magnetic flux $\Phi$ passing through a surface of area $A_c$ is related to the flux density $B$ according to

$$\Phi = \int_{\text{surface } S} B \cdot dA$$

**Example: uniform flux density of magnitude $B$**

**Analogous to electrical conductor current density of magnitude $J$, which leads to total conductor current $I$:**
Faraday’s law

Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to

$$v(t) = \frac{d\Phi(t)}{dt}$$

For uniform flux distribution, $\Phi(t) = B(t)A_c$ and hence

$$v(t) = A_c \frac{dB(t)}{dt}$$
Lenz’s law

The voltage $v(t)$ induced by the changing flux $\Phi(t)$ is of the polarity that tends to drive a current through the loop to counteract the flux change.

**Example: a shorted loop of wire**

- Changing flux $\Phi(t)$ induces a voltage $v(t)$ around the loop
- This voltage, divided by the impedance of the loop conductor, leads to current $i(t)$
- This current induces a flux $\Phi'(t)$, which tends to oppose changes in $\Phi(t)$
Ampere’s law

The net MMF around a closed path is equal to the total current passing through the interior of the path:

\[ \oint_{\text{closed path}} H \cdot dl = \text{total current passing through interior of path} \]

Example: magnetic core. Wire carrying current \( i(t) \) passes through core window.

- Illustrated path follows magnetic flux lines around interior of core
- For uniform magnetic field strength \( H(t) \), the integral (MMF) is \( H(t)l_m \). So

\[ \mathcal{F}(t) = H(t)l_m = i(t) \]
Ampere’s law: discussion

- Relates magnetic field strength $H(t)$ to winding current $i(t)$
- We can view winding currents as sources of MMF
- Previous example: total MMF around core, $\Phi(t) = H(t)l_m$, is equal to the winding current MMF $i(t)$
- The total MMF around a closed loop, accounting for winding current MMF’s, is zero
Core material characteristics:
the relation between $B$ and $H$

$B = \mu_0 H$

$\mu_0 = \text{permeability of free space}$
$= 4\pi \cdot 10^{-7} \text{ Henries per meter}$

Highly nonlinear, with hysteresis and saturation
Piecewise-linear modeling of core material characteristics

**No hysteresis or saturation**

\[ B = \mu H \]
\[ \mu = \mu_r \mu_0 \]

Typical \( \mu_r \) = 10³ to 10⁵

**Saturation, no hysteresis**

\[ B = B_{sat} \]

Typical \( B_{sat} \) = 0.3 to 0.5T, ferrite
0.5 to 1T, powdered iron
1 to 2T, iron laminations
# Units

## Table 12.1. Units for magnetic quantities

<table>
<thead>
<tr>
<th>quantity</th>
<th>MKS</th>
<th>unreationalized cgs</th>
<th>conversions</th>
</tr>
</thead>
<tbody>
<tr>
<td>core material</td>
<td>( B = \mu_0 \mu_r H )</td>
<td>( B = \mu_r H )</td>
<td></td>
</tr>
<tr>
<td>magnetic field</td>
<td>Tesla</td>
<td>Gauss</td>
<td>( 1 \text{T} = 10^4 \text{G} )</td>
</tr>
<tr>
<td>current density</td>
<td>Ampere / meter</td>
<td>Oersted</td>
<td>( 1 \text{A/m} = 4\pi \cdot 10^{-3} \text{Oe} )</td>
</tr>
<tr>
<td>flux</td>
<td>Weber</td>
<td>Maxwell</td>
<td>( 1 \text{Wb} = 10^8 \text{Mx} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 1 \text{T} = 1 \text{Wb} / \text{m}^2 )</td>
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</tbody>
</table>
Example: a simple inductor

**Faraday’s law:**
For each turn of wire, we can write

\[ v_{\text{turn}}(t) = \frac{d\Phi(t)}{dt} \]

Total winding voltage is

\[ v(t) = n v_{\text{turn}}(t) = n \frac{d\Phi(t)}{dt} \]

Express in terms of the average flux density \( B(t) = \bar{\Phi}(t)/A_c \)

\[ v(t) = nA_c \frac{dB(t)}{dt} \]
Inductor example: Ampere’s law

Choose a closed path which follows the average magnetic field line around the interior of the core. Length of this path is called the mean magnetic path length $l_m$.

For uniform field strength $H(t)$, the core MMF around the path is $H l_m$.

Winding contains $n$ turns of wire, each carrying current $i(t)$. The net current passing through the path interior (i.e., through the core window) is $ni(t)$.

From Ampere’s law, we have

$$H(t) l_m = n i(t)$$
Inductor example: core material model

\[ B = \begin{cases} 
B_{\text{sat}} & \text{for } H \geq B_{\text{sat}}/\mu \\
\mu H & \text{for } |H| < B_{\text{sat}}/\mu \\
-B_{\text{sat}} & \text{for } H \leq -B_{\text{sat}}/\mu 
\end{cases} \]

Find winding current at onset of saturation: substitute \( i = I_{\text{sat}} \) and \( H = B_{\text{sat}}/\mu \) into equation previously derived via Ampere’s law. Result is

\[ I_{\text{sat}} = \frac{B_{\text{sat}} l_m}{\mu n} \]
Electrical terminal characteristics

We have:

\[ v(t) = nA_c \frac{dB(t)}{dt} \quad H(t) \ell_m = n i(t) \]

\[ B = \begin{cases} 
B_{sat} & \text{for } H \geq B_{sat}/\mu \\
\mu H & \text{for } |H| < B_{sat}/\mu \\
-B_{sat} & \text{for } H \leq -B_{sat}/\mu 
\end{cases} \]

Eliminate \( B \) and \( H \), and solve for relation between \( v \) and \( i \). For \( |i| < I_{sat} \),

\[ v(t) = \mu n A_c \frac{dH(t)}{dt} \quad \Rightarrow \quad v(t) = \frac{\mu n^2 A_c}{\ell_m} \frac{di(t)}{dt} \]

which is of the form

\[ v(t) = L \frac{di(t)}{dt} \quad \text{with} \quad L = \frac{\mu n^2 A_c}{\ell_m} \]

— an inductor

For \( |i| > I_{sat} \) the flux density is constant and equal to \( B_{sat} \). Faraday’s law then predicts

\[ v(t) = nA_c \frac{dB_{sat}}{dt} = 0 \quad \text{—saturation leads to short circuit} \]
13.1.2 Magnetic circuits

Uniform flux and magnetic field inside a rectangular element:

\[ \Phi = H \ell \]

MMF between ends of element is

\[ \mathcal{F} = H \ell \]

Since \( H = B / \mu \) and \( B = \Phi / A_c \), we can express \( \mathcal{F} \) as

\[ \mathcal{F} = \Phi \mathcal{R} \]

with

\[ \mathcal{R} = \frac{\ell}{\mu A_c} \]

A corresponding model:

\[ \Phi \quad \mathcal{R} \]

\[ \mathcal{R} = \text{reluctance of element} \]
Magnetic circuits: magnetic structures composed of multiple windings and heterogeneous elements

- Represent each element with reluctance
- Windings are sources of MMF
- MMF $\rightarrow$ voltage, flux $\rightarrow$ current
- Solve magnetic circuit using Kirchoff’s laws, etc.
Magnetic analog of Kirchoff’s current law

Divergence of $\mathbf{B} = 0$

Flux lines are continuous and cannot end

Total flux entering a node must be zero
Magnetic analog of Kirchoff’s voltage law

Follows from Ampere’s law:

$$\oint_{\text{closed path}} H \cdot dl = \text{total current passing through interior of path}$$

Left-hand side: sum of MMF’s across the reluctances around the closed path

Right-hand side: currents in windings are sources of MMF’s. An \(n\)-turn winding carrying current \(i(t)\) is modeled as an MMF (voltage) source, of value \(ni(t)\).

Total MMF’s around the closed path add up to zero.
Example: inductor with air gap

Core permeability $\mu$

Cross-sectional area $A_c$

Air gap $\ell_g$

Magnetic path length $\ell_m$

$i(t)$

$+ n$ turns

$- v(t)$

$\Phi$
Magnetic circuit model

\[ \Phi_c + \Phi_g = ni \]

\[ ni = \Phi \left( R_c + R_g \right) \]

\[ R_c = \frac{\ell_c}{\mu A_c} \]

\[ R_g = \frac{\ell_g}{\mu_0 A_c} \]
Solution of model

Faraday’s law:

\[ v(t) = n \frac{d\Phi(t)}{dt} \]

Substitute for \( \Phi \):

\[ v(t) = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g} \frac{di(t)}{dt} \]

Hence inductance is

\[ L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g} \]
Effect of air gap

\[ ni = \Phi \left( R_c + R_g \right) \]

\[ L = \frac{n^2}{R_c + R_g} \]

\[ \Phi_{sat} = B_{sat} A_c \]

\[ I_{sat} = \frac{B_{sat} A_c}{n} \left( R_c + R_g \right) \]

Effect of air gap:
- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability
13.2 Transformer modeling

Two windings, no air gap:

\[ R = \frac{\ell_m}{\mu A_c} \]

\[ \mathcal{J}_c = n_1 i_1 + n_2 i_2 \]

\[ \Phi R = n_1 i_1 + n_2 i_2 \]

Magnetic circuit model:
13.2.1 The ideal transformer

In the ideal transformer, the core reluctance $R$ approaches zero.

MMF $\mathcal{F}_c = \Phi R$ also approaches zero. We then obtain

$$0 = n_1 i_1 + n_2 i_2$$

Also, by Faraday’s law,

$$v_1 = n_1 \frac{d\Phi}{dt}$$
$$v_2 = n_2 \frac{d\Phi}{dt}$$

Eliminate $\Phi$:

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Ideal transformer equations:

$$\frac{v_1}{n_1} = \frac{v_2}{n_2}$$
and
$$n_1 i_1 + n_2 i_2 = 0$$
13.2.2 The magnetizing inductance

For nonzero core reluctance, we obtain

\[ \Phi R = n_1 i_1 + n_2 i_2 \]

with \( v_1 = n_1 \frac{d\Phi}{dt} \)

Eliminate \( \Phi \):

\[ v_1 = \frac{n_2^2}{R} \frac{d}{dt} \left[ i_1 + \frac{n_2}{n_1} i_2 \right] \]

This equation is of the form

\[ v_1 = L_M \frac{di_M}{dt} \]

with

\[ L_M = \frac{n_1^2}{R} \]

\[ i_M = i_1 + \frac{n_2}{n_1} i_2 \]
Magnetizing inductance: discussion

- Models magnetization of core material
- A real, physical inductor, that exhibits saturation and hysteresis
- If the secondary winding is disconnected:
  - we are left with the primary winding on the core
  - primary winding then behaves as an inductor
  - the resulting inductor is the magnetizing inductance, referred to the primary winding
- Magnetizing current causes the ratio of winding currents to differ from the turns ratio
Transformer saturation

- Saturation occurs when core flux density $B(t)$ exceeds saturation flux density $B_{sat}$.
- When core saturates, the magnetizing current becomes large, the impedance of the magnetizing inductance becomes small, and the windings are effectively shorted out.
- Large winding currents $i_1(t)$ and $i_2(t)$ do not necessarily lead to saturation. If
  \[ 0 = n_1i_1 + n_2i_2 \]
  then the magnetizing current is zero, and there is no net magnetization of the core.
- Saturation is caused by excessive applied volt-seconds
Saturation vs. applied volt-seconds

Magnetizing current depends on the integral of the applied winding voltage:

\[ i_M(t) = \frac{1}{L_M} \int v_1(t) \, dt \]

Flux density is proportional:

\[ B(t) = \frac{1}{n_1 A_c} \int v_1(t) \, dt \]

Flux density becomes large, and core saturates, when the applied volt-seconds \( \lambda_1 \) are too large, where

\[ \lambda_1 = \int_{t_1}^{t_2} v_1(t) \, dt \]

limits of integration chosen to coincide with positive portion of applied voltage waveform
13.2.3 Leakage inductances
Transformer model, including leakage inductance

Terminal equations can be written in the form

\[
\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}
\]

Mutual inductance:

\[ L_{12} = \frac{n_1 n_2}{R} = \frac{n_2}{n_1} L_M \]

Primary and secondary self-inductances:

\[ L_{11} = L_{\ell 1} + \frac{n_1}{n_2} L_{12} \]
\[ L_{22} = L_{\ell 2} + \frac{n_2}{n_1} L_{12} \]

Effective turns ratio

\[ n_e = \sqrt{\frac{L_{22}}{L_{11}}} \]

Coupling coefficient

\[ k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}} \]