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13.1 Review of Basic Magnetics

13.1.1 Basic relationships

**Faraday’s law**

\[ v(t) \longleftrightarrow B(t), \Phi(t) \]

**Ampere’s law**

\[ i(t) \longleftrightarrow H(t), \mathcal{F}(t) \]

**Terminal characteristics**

**Core characteristics**

Fundamentals of Power Electronics
Basic quantities

*Magnetic quantities*

- Length $\ell$
- Magnetic field $H$
- MMF $\mathcal{F} = H\ell$
- Total flux $\Phi$
- Flux density $B$

*Electrical quantities*

- Length $\ell$
- Electric field $E$
- Voltage $V = E\ell$
- Total current $I$
- Current density $J$

Fundamentals of Power Electronics

Chapter 13: Basic Magnetics Theory
Magnetic field $H$ and magnetomotive force $\mathcal{F}$

Magnetomotive force (MMF) $\mathcal{F}$ between points $x_1$ and $x_2$ is related to the magnetic field $H$ according to

$$\mathcal{F} = \int_{x_1}^{x_2} H \cdot dl$$

**Example: uniform magnetic field of magnitude $H$**

- Length $\ell$
- Magnetic field $H$
- MMF $\mathcal{F} = H\ell$

**Analogous to electric field of strength $E$, which induces voltage (EMF) $V$:**

- Length $\ell$
- Electric field $E$
- Voltage $V = E\ell$
Flux density $B$ and total flux $\Phi$

The total magnetic flux $\Phi$ passing through a surface of area $A_c$ is related to the flux density $B$ according to

$$\Phi = \int_{\text{surface } S} B \cdot dA$$

**Example: uniform flux density of magnitude $B$**

Analogous to electrical conductor current density of magnitude $J$, which leads to total conductor current $I$:
Faraday’s law

Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to

$$v(t) = \frac{d\Phi(t)}{dt}$$

For uniform flux distribution, $\Phi(t) = B(t)A_c$ and hence

$$v(t) = A_c \frac{dB(t)}{dt}$$
Lenz’s law

The voltage $v(t)$ induced by the changing flux $\Phi(t)$ is of the polarity that tends to drive a current through the loop to counteract the flux change.

**Example: a shorted loop of wire**

- Changing flux $\Phi(t)$ induces a voltage $v(t)$ around the loop.
- This voltage, divided by the impedance of the loop conductor, leads to current $i(t)$.
- This current induces a flux $\Phi'(t)$, which tends to oppose changes in $\Phi(t)$.

![Diagram](image)
Ampere’s law

The net MMF around a closed path is equal to the total current passing through the interior of the path:

$$\int_{\text{closed path}} H \cdot dl = \text{total current passing through interior of path}$$

**Example: magnetic core. Wire carrying current } i(t) \text{ passes through core window.**

- Illustrated path follows magnetic flux lines around interior of core
- For uniform magnetic field strength } H(t), \text{ the integral (MMF) is } H(t)l_m. \text{ So}

$$F(t) = H(t)l_m = i(t)$$
Ampere’s law: discussion

• Relates magnetic field strength $H(t)$ to winding current $i(t)$
• We can view winding currents as sources of MMF
• Previous example: total MMF around core, $\mathcal{F}(t) = H(t)\ell_m$, is equal to the winding current MMF $i(t)$
• The total MMF around a closed loop, accounting for winding current MMF’s, is zero
Core material characteristics: the relation between $B$ and $H$

**Free space**

$$B = \mu_0 H$$

$\mu_0 =$ permeability of free space

$= 4\pi \cdot 10^{-7}$ Henries per meter

**A magnetic core material**

Highly nonlinear, with hysteresis and saturation
Piecewise-linear modeling of core material characteristics

No hysteresis or saturation

\[ B = \mu H \]
\[ \mu = \mu_r \mu_0 \]

Typical \( \mu_r = 10^3 \) to \( 10^5 \)

Saturation, no hysteresis

Typical \( B_{sat} = 0.3 \) to \( 0.5T \), ferrite

0.5 to 1T, powdered iron

1 to 2T, iron laminations
## Units

**Table 12.1. Units for magnetic quantities**

<table>
<thead>
<tr>
<th>quantity</th>
<th>MKS equation</th>
<th>unrationlized cgs equation</th>
<th>conversions</th>
</tr>
</thead>
<tbody>
<tr>
<td>core material</td>
<td>$B = \mu_0 \mu_r H$</td>
<td>$B = \mu_r H$</td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>Tesla</td>
<td>Gauss</td>
<td>$1 \text{T} = 10^4 \text{G}$</td>
</tr>
<tr>
<td>$H$</td>
<td>Ampere / meter</td>
<td>Oersted</td>
<td>$1 \text{A/m} = 4\pi \cdot 10^{-3} \text{Oe}$</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Weber</td>
<td>Maxwell</td>
<td>$1 \text{Wb} = 10^8 \text{Mx}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$1 \text{T} = 1 \text{Wb} / \text{m}^2$</td>
</tr>
</tbody>
</table>
Example: a simple inductor

Faraday’s law:
For each turn of wire, we can write
\[ v_{\text{turn}}(t) = \frac{d\Phi(t)}{dt} \]

Total winding voltage is
\[ v(t) = n v_{\text{turn}}(t) = n \frac{d\Phi(t)}{dt} \]

Express in terms of the average flux density \( B(t) = \Phi(t)/A_c \)

\[ v(t) = nA_c \frac{dB(t)}{dt} \]
Inductor example: Ampere’s law

Choose a closed path which follows the average magnetic field line around the interior of the core. Length of this path is called the *mean magnetic path length* $\ell_m$.

For uniform field strength $H(t)$, the core MMF around the path is $H \ell_m$.

Winding contains $n$ turns of wire, each carrying current $i(t)$. The net current passing through the path interior (i.e., through the core window) is $ni(t)$.

From Ampere’s law, we have

$$H(t) \ell_m = n i(t)$$
Inductor example: core material model

$$B = \begin{cases} 
  B_{sat} & \text{for } H \geq B_{sat}/\mu \\
  \mu H & \text{for } |H| < B_{sat}/\mu \\
  -B_{sat} & \text{for } H \leq -B_{sat}/\mu 
\end{cases}$$

Find winding current at onset of saturation: substitute $i = I_{sat}$ and $H = B_{sat}/\mu$ into equation previously derived via Ampere’s law. Result is

$$I_{sat} = \frac{B_{sat} l_m}{\mu n}$$
Electrical terminal characteristics

We have:

\[ v(t) = nA_c \frac{dB(t)}{dt} \quad H(t) \ell_m = n \, i(t) \]

\[ B = \begin{cases} 
B_{sat} & \text{for } H \geq B_{sat}/\mu \\
\mu H & \text{for } |H| < B_{sat}/\mu \\
-B_{sat} & \text{for } H \leq -B_{sat}/\mu 
\end{cases} \]

Eliminate \( B \) and \( H \), and solve for relation between \( v \) and \( i \). For \( |i| < I_{sat} \),

\[ v(t) = \mu nA_c \frac{dH(t)}{dt} \quad \rightarrow \quad v(t) = \frac{\mu n^2 A_c}{\ell_m} \frac{di(t)}{dt} \]

which is of the form

\[ v(t) = L \frac{di(t)}{dt} \quad \text{with} \quad L = \frac{\mu n^2 A_c}{\ell_m} \]

—an inductor

For \( |i| > I_{sat} \) the flux density is constant and equal to \( B_{sat} \). Faraday’s law then predicts

\[ v(t) = nA_c \frac{dB_{sat}}{dt} = 0 \quad \text{—saturation leads to short circuit} \]
13.1.2 Magnetic circuits

Uniform flux and magnetic field inside a rectangular element:

MMF between ends of element is

\[ \mathcal{F} = H \ell \]

Since \( H = B / \mu \) and \( B = \Phi / A_c \), we can express \( \mathcal{F} \) as

\[ \mathcal{F} = \Phi \mathcal{R} \]

with

\[ \mathcal{R} = \frac{\ell}{\mu A_c} \]

A corresponding model:

\[ \Phi \rightarrow \mathcal{R} \rightarrow - \]

\( \mathcal{R} \) = reluctance of element
Magnetic circuits: magnetic structures composed of multiple windings and heterogeneous elements

- Represent each element with reluctance
- Windings are sources of MMF
- MMF $\rightarrow$ voltage, flux $\rightarrow$ current
- Solve magnetic circuit using Kirchoff’s laws, etc.
Magnetic analog of Kirchoff’s current law

Divergence of $B = 0$
Flux lines are continuous and cannot end
Total flux entering a node must be zero

\[ \Phi_1 = \Phi_2 + \Phi_3 \]
Magnetic analog of Kirchoff’s voltage law

Follows from Ampere’s law:

\[ \oint_{\text{closed path}} \mathbf{H} \cdot d\mathbf{l} = \text{total current passing through interior of path} \]

Left-hand side: sum of MMF’s across the reluctances around the closed path

Right-hand side: currents in windings are sources of MMF’s. An \( n \)-turn winding carrying current \( i(t) \) is modeled as an MMF (voltage) source, of value \( ni(t) \).

Total MMF’s around the closed path add up to zero.
Example: inductor with air gap

\[ v(t) \]

\[ i(t) \]

\[ + \]

\[ - \]

\[ n \text{ turns} \]

\[ \Phi \]

Core permeability \( \mu \)

Cross-sectional area \( A_c \)

Air gap \( \ell_g \)

Magnetic path length \( \ell_m \)
Magnetic circuit model

\[ \Phi_c + \Phi_g = ni \]

\[ ni = \Phi \left( R_c + R_g \right) \]

\[ R_c = \frac{\ell_c}{\mu A_c} \]

\[ R_g = \frac{\ell_g}{\mu_0 A_c} \]
Solution of model

Faraday’s law:

\[ v(t) = n \frac{d\Phi(t)}{dt} \]

Substitute for \( \Phi \):

\[ v(t) = \frac{n^2}{R_c + R_g} \frac{di(t)}{dt} \]

Hence inductance is

\[ L = \frac{n^2}{R_c + R_g} \]
Effect of air gap

\[ ni = \Phi \left( \frac{1}{R_c + R_g} \right) \]

\[ L = \frac{n^2}{R_c + R_g} \]

\[ \Phi_{sat} = B_{sat} A_c \]

\[ I_{sat} = \frac{B_{sat} A_c}{n} \left( \frac{1}{R_c + R_g} \right) \]

Effect of air gap:
- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability
13.2 Transformer modeling

Two windings, no air gap:

\[ \mathcal{R} = \frac{\ell_m}{\mu A_c} \]

\[ \mathcal{F}_c = n_1 i_1 + n_2 i_2 \]

\[ \Phi \mathcal{R} = n_1 i_1 + n_2 i_2 \]

Magnetic circuit model:
13.2.1 The ideal transformer

In the ideal transformer, the core reluctance $\mathcal{R}$ approaches zero.

$\text{MMF } \mathcal{F}_c = \Phi \mathcal{R}$ also approaches zero. We then obtain

$$0 = n_1 i_1 + n_2 i_2$$

Also, by Faraday’s law,

$$v_1 = n_1 \frac{d\Phi}{dt}$$
$$v_2 = n_2 \frac{d\Phi}{dt}$$

Eliminate $\Phi$:

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Ideal transformer equations:

$$\frac{v_1}{n_1} = \frac{v_2}{n_2} \quad \text{and} \quad n_1 i_1 + n_2 i_2 = 0$$
13.2.2 The magnetizing inductance

For nonzero core reluctance, we obtain

$$\Phi R = n_1 i_1 + n_2 i_2 \quad \text{with} \quad v_1 = n_1 \frac{d\Phi}{dt}$$

Eliminate $\Phi$:

$$v_1 = \frac{n_2}{n_1} \frac{1}{R} \frac{d}{dt} \left[ i_1 + \frac{n_2}{n_1} i_2 \right]$$

This equation is of the form

$$v_1 = L_M \frac{di_M}{dt}$$

with

$$L_M = \frac{n_2^2}{R}$$

$$i_M = i_1 + \frac{n_2}{n_1} i_2$$
Magnetizing inductance: discussion

- Models magnetization of core material
- A real, physical inductor, that exhibits saturation and hysteresis
- If the secondary winding is disconnected:
  - we are left with the primary winding on the core
  - primary winding then behaves as an inductor
  - the resulting inductor is the magnetizing inductance, referred to the primary winding
- Magnetizing current causes the ratio of winding currents to differ from the turns ratio
Transformer saturation

- Saturation occurs when core flux density $B(t)$ exceeds saturation flux density $B_{\text{sat}}$.
- When core saturates, the magnetizing current becomes large, the impedance of the magnetizing inductance becomes small, and the windings are effectively shorted out.
- Large winding currents $i_1(t)$ and $i_2(t)$ do not necessarily lead to saturation. If
  \[ 0 = n_1 i_1 + n_2 i_2 \]
  then the magnetizing current is zero, and there is no net magnetization of the core.
- Saturation is caused by excessive applied volt-seconds
Saturation vs. applied volt-seconds

Magnetizing current depends on the integral of the applied winding voltage:

\[ i_M(t) = \frac{1}{L_M} \int v_1(t) \, dt \]

Flux density is proportional:

\[ B(t) = \frac{1}{n_1 A_c} \int v_1(t) \, dt \]

Flux density becomes large, and core saturates, when the applied volt-seconds \( \lambda_1 \) are too large, where

\[ \lambda_1 = \int_{t_1}^{t_2} v_1(t) \, dt \]

limits of integration chosen to coincide with positive portion of applied voltage waveform.
13.2.3 Leakage inductances

\[ \Phi_M + v_1(t) - i_1(t) + v_2(t) - i_2(t) \]

\[ \Phi_{l1} \]

\[ \Phi_{l2} \]

\[ i_1(t) \quad v_1(t) \quad i_2(t) \quad v_2(t) \]
Transformer model, including leakage inductance

Terminal equations can be written in the form

\[
\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}
\]

mutual inductance:

\[ L_{12} = \frac{n_1 n_2}{R} = \frac{n_2}{n_1} L_M \]

primary and secondary self-inductances:

\[ L_{11} = L_{\ell 1} + \frac{n_1}{n_2} L_{12} \]
\[ L_{22} = L_{\ell 2} + \frac{n_2}{n_1} L_{12} \]

effective turns ratio \[ n_e = \sqrt{\frac{L_{22}}{L_{11}}} \]

coupling coefficient \[ k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}} \]