Introduction to Power Electronics
ECEN 4797/5797

Lecture 36
November 26, 2018

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Mutual flux $\Phi_M$ is large and is mostly confined to the core.

Leakage flux is present, which does not completely link both windings.

Because of symmetry of winding geometry, leakage flux runs approximately vertically through the windings.
Analysis of leakage flux using Ampere’s law

Ampere’s law, for the closed path taken by the leakage flux line illustrated:

\[ \text{Enclosed current} = \mathcal{J}(x) = H(x)l_w \]

(note that MMF around core is small compared to MMF through the air inside the winding, because of high permeability of core)
Ampere’s law for the transformer example

For the innermost leakage path, enclosing the first layer of the primary:

This path encloses four turns, so the total enclosed current is $4i(t)$.

For the next leakage path, enclosing both layers of the primary:

This path encloses eight turns, so the total enclosed current is $8i(t)$.

The next leakage path encloses the primary plus four turns of the secondary. The total enclosed current is $8i(t) - 4i(t) = 4i(t)$. 
MMF diagram, transformer example

Enclosed current = $\mathcal{F}(x) = H(x)l_w$

Leakage path

Primary winding

Secondary winding

Layer 1

Layer 2

Layer 1

Layer 2
Two-winding transformer example

**Winding layout**

**MMF diagram**

Use Ampere’s law around a closed path taken by a leakage flux line:

\[
\left( m_p - m_s \right) i = \mathcal{F}(x)
\]

- \( m_p \) = number of primary layers enclosed by path
- \( m_s \) = number of secondary layers enclosed by path

\[ \mathcal{F}(x) \]

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Two-winding transformer example with proximity effect

Flux does not penetrate conductors.

Surface currents cause net current enclosed by leakage path to be zero when path runs down interior of a conductor.

Magnetic field strength $H(x)$ within the winding is given by

$$H(x) = \frac{\mathcal{F}(x)}{\ell_w}$$
Interleaving the windings: MMF diagram

Greatly reduces the peak MMF, leakage flux, and proximity losses
A partially-interleaved transformer

For this example, there are three primary layers and four secondary layers. The MMF diagram contains fractional values.
Definition of the layer number $m$

$$m = \frac{\mathcal{F}(h)}{\mathcal{F}(h) - \mathcal{F}(0)}$$
13.4.3 Foil windings and layers

Approximating a layer of round conductors as an effective single foil conductor:

Square conductors (b) have same cross-sectional area as round conductors (a) if

\[ h = \sqrt{\frac{\pi}{4}} \cdot d \]

Eliminate space between square conductors: push together into a single foil turn (c)

(d) Stretch foil so its width is \( \ell_w \). The adjust conductivity so its dc resistance is unchanged.
Winding porosity $\eta$

Stretching the conductor increases its area. Compensate by increasing the effective resistivity $\rho$, to maintain the same dc resistance. Define winding porosity $\eta$ as the ratio of cross-sectional areas. If layer of width $l_w$ contains $n_t$ turns of round wire having diameter $d$, then the porosity is

$$\eta = \frac{\sqrt{\frac{\pi}{4}} d n_t}{l_w}$$

Typical $\eta$ for full-width round conductors is $\eta = 0.8$.

The increased effective resistivity increases the effective skin depth:

$$\delta' = \frac{\delta}{\sqrt{\eta}}$$

Define $\varphi = h/d$. The effective value for a layer of round conductors is

$$\varphi = \frac{h}{\delta'} = \sqrt{\eta} \sqrt{\frac{\pi}{4} \frac{d}{\delta}}$$
13.4.4 Power loss in a layer

Approximate computation of copper loss in one layer

Assume uniform magnetic fields at surfaces of layer, of strengths $H(0)$ and $H(h)$. Assume that these fields are parallel to layer surface (i.e., neglect fringing and assume field normal component is zero).

The magnetic fields $H(0)$ and $H(h)$ are driven by the MMFs $\mathcal{F}(0)$ and $\mathcal{F}(h)$.

Sinusoidal waveforms are assumed, and rms values are used. It is assumed that $H(0)$ and $H(h)$ are in phase.
Solution for layer copper loss $P$

Solve Maxwell's equations to find current density distribution within layer. Then integrate to find total copper loss $P$ in layer. Result is

$$P = R_{dc} \frac{q}{n_l} \left[ \left( E^2(h) + E^2(0) \right) G_1(\varphi) - 4 E(h)E(0) G_2(\varphi) \right]$$

where

$$R_{dc} = \rho \frac{\ell_b}{A_w} = \rho \frac{(MLT)n_l^3}{\eta \ell_w}$$

$n_l$ = number of turns in layer, $R_{dc}$ = dc resistance of layer, $\rho$ = mean-length-per-turn, or circumference, of layer.

$$G_1(\varphi) = \frac{\sinh (2\varphi) + \sin (2\varphi)}{\cosh (2\varphi) - \cos (2\varphi)}$$

$$G_2(\varphi) = \frac{\sinh (\varphi) \cos (\varphi) + \cosh (\varphi) \sin (\varphi)}{\cosh (2\varphi) - \cos (2\varphi)}$$

$$\varphi = \frac{h}{\delta} = \sqrt{\eta} \sqrt{\frac{\pi}{4} \frac{d}{\delta}}$$

$$\eta = \sqrt{\frac{\pi}{4} d n_l \ell_w}$$
Winding carrying current $I$, with $n_l$ turns per layer

If winding carries current of rms magnitude $I$, then

$$\mathcal{F}(h) - \mathcal{F}(0) = n_l I$$

Express $\mathcal{F}(h)$ in terms of the winding current $I$, as

$$\mathcal{F}(h) = mn_l I$$

The quantity $m$ is the ratio of the MMF $\mathcal{F}(h)$ to the layer ampere-turns $n_l I$. Then,

$$\frac{\mathcal{F}(0)}{\mathcal{F}(h)} = \frac{m - 1}{m}$$

Power dissipated in the layer can now be written

$$P = I^2 R_{dc} \varphi Q'(\varphi, m)$$

$$Q'(\varphi, m) = \left(2m^2 - 2m + 1\right) G_1(\varphi) - 4m(m - 1) G_2(\varphi)$$
Increased copper loss in layer

\[ P = I^2 R_{dc} \varphi Q'(\varphi, m) \]
Layer copper loss vs. layer thickness

\[ \frac{P}{P_{dc, \varphi = 1}} = Q'(\varphi, m) \]

Relative to copper loss when \( h = \delta \)

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13.4.5 Example: Power loss in a transformer winding

Two winding transformer

Each winding consists of $M$ layers

Proximity effect increases copper loss in layer $m$ by the factor $\varphi Q'(\varphi, m)$

Sum losses over all primary layers:

$$F_R = \frac{P_{pri}}{P_{pri,dc}} = \frac{1}{M} \sum_{m=1}^{M} \varphi Q'(\varphi, m)$$
Increased total winding loss

Express summation in closed form:

\[ F_R = \varphi \left[ G_1(\varphi) + \frac{2}{3} \left( M^2 - 1 \right) \left( G_1(\varphi) - 2G_2(\varphi) \right) \right] \]
Total winding loss

\[
\frac{P_{pri}}{P_{pri,dc}} \bigg|_{\psi = 1} = G_1(\psi) + \frac{2}{3} (M^2 - 1) \left( G_1(\psi) - 2G_2(\psi) \right)
\]
13.4.6 Interleaving the windings

Same transformer example, but with primary and secondary layers alternated

Each layer operates with $F = 0$ on one side, and $F = i$ on the other side

Proximity loss of entire winding follows $M = 1$ curve

For $M = 1$: minimum loss occurs with $\varphi = \pi/2$, although copper loss is nearly constant for any $\varphi \geq 1$, and is approximately equal to the dc copper loss obtained when $\varphi = 1$. 
Partial interleaving

Partially-interleaved example with 3 primary and 4 secondary layers

Each primary layer carries current $i$ while each secondary layer carries $0.75i$. Total winding currents add to zero. Peak MMF occurs in space between windings, but has value $1.5i$.

We can apply the previous solution for the copper loss in each layer, and add the results to find the total winding losses. To determine the value of $m$ to use for a given layer, evaluate

$$m = \frac{\mathcal{F}(h)}{\mathcal{F}(h) - \mathcal{F}(0)}$$
Determination of $m$

Leftmost secondary layer:

$$m = \frac{\mathcal{F}(h)}{\mathcal{F}(h) - \mathcal{F}(0)} = \frac{-0.75i}{-0.75i - 0} = 1$$

Next secondary layer:

$$m = \frac{\mathcal{F}(h)}{\mathcal{F}(h) - \mathcal{F}(0)} = \frac{-1.5i}{-1.5i - (-0.75i)} = 2$$

Next layer (primary):

$$m = \frac{\mathcal{F}(0)}{\mathcal{F}(0) - \mathcal{F}(h)} = \frac{-1.5i}{-1.5i - (-0.5i)} = 1.5$$

Center layer (primary):

$$m = \frac{\mathcal{F}(h)}{\mathcal{F}(h) - \mathcal{F}(0)} = \frac{0.5i}{0.5i - (-0.5i)} = 0.5$$

Use the plot for layer loss (repeated on next slide) to find loss for each layer, according to its value of $m$. Add results to find total loss.
Layer copper loss vs. layer thickness

\[ \frac{P}{P_{dc}} = Q'(\varphi, m) \]

Relative to copper loss when \( h = \delta \)
Discussion: design of winding geometry to minimize proximity loss

- Interleaving windings can significantly reduce the proximity loss when the winding currents are in phase, such as in the transformers of buck-derived converters or other converters.
- In some converters (such as flyback or SEPIC) the winding currents are out of phase. Interleaving then does little to reduce the peak MMF and proximity loss. See *Vandelac and Ziogas* [10].
- For sinusoidal winding currents, there is an optimal conductor thickness near $\varphi = 1$ that minimizes copper loss.
- Minimize the number of layers. Use a core geometry that maximizes the width $l_w$ of windings.
- Minimize the amount of copper in vicinity of high MMF portions of the windings.
Litz wire

- A way to increase conductor area while maintaining low proximity losses
- Many strands of small-gauge wire are bundled together and are externally connected in parallel
- Strands are twisted, or transposed, so that each strand passes equally through each position on inside and outside of bundle. This prevents circulation of currents between strands.
- Strand diameter should be sufficiently smaller than skin depth
- The Litz wire bundle itself is composed of multiple layers
- Advantage: when properly sized, can significantly reduce proximity loss
- Disadvantage: increased cost and decreased amount of copper within core window
13.4.7 PWM waveform harmonics

**Fourier series:**

\[ i(t) = I_0 + \sum_{j=1}^{\infty} \sqrt{2} I_j \cos(j \omega t) \]

with

\[ I_j = \frac{\sqrt{2} I_{pk}}{j \pi} \sin(j \pi D) \quad I_0 = DI_{pk} \]

**Copper loss:**

\[ P_{dc} = I_0^2 R_{dc} \]

\[ P_j = I_j^2 R_{dc} \sqrt{j} \varphi_1 \left[ G_1(\sqrt{j} \varphi_1) + \frac{2}{3} \left( M^2 - 1 \right) \left( G_1(\sqrt{j} \varphi_1) - 2G_2(\sqrt{j} \varphi_1) \right) \right] \]

Total, relative to value predicted by low-frequency analysis:

\[ \frac{P_{cu}}{DI_{pk}^2 R_{dc}} = D + \frac{2 \varphi_1}{D \pi^2} \sum_{j=1}^{\infty} \frac{\sin^2(j \pi D)}{j \sqrt{j}} \left[ G_1(\sqrt{j} \varphi_1) + \frac{2}{3} \left( M^2 - 1 \right) \left( G_1(\sqrt{j} \varphi_1) - 2G_2(\sqrt{j} \varphi_1) \right) \right] \]
Harmonic loss factor $F_H$

Effect of harmonics: $F_H = \text{ratio of total ac copper loss to fundamental copper loss}$

$$F_H = \frac{\sum_{j=1}^{\infty} P_j}{P_1}$$

The total winding copper loss can then be written

$$P_{cu} = I_0^2 R_{dc} + F_H F_R I_1^2 R_{dc}$$
Increased proximity losses induced by PWM waveform harmonics: $D = 0.5$
Increased proximity losses induced by PWM waveform harmonics: $D = 0.3$
Increased proximity losses induced by PWM waveform harmonics: $D = 0.1$
Discussion: waveform harmonics

- Harmonic factor $F_H$ accounts for effects of harmonics.
- Harmonics are most significant for $\varphi_1$ in the vicinity of 1.
- Harmonics can radically alter the conclusion regarding optimal wire gauge.
- A substantial dc component can drive the design towards larger wire gauge.
- Harmonics can increase proximity losses by orders of magnitude, when there are many layers and when $\varphi_1$ lies in the vicinity of 1.
- For sufficiently small $\varphi_1$, $F_H$ tends to the value $1 + (\text{THD})^2$, where the total harmonic distortion of the current is:

$$\text{THD} = \sqrt{\sum_{j=2}^{\infty} \frac{I_j^2}{I_1}}$$