Introduction to Power Electronics
ECEN 4797/5797

Lecture 6
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3.4. How to obtain the input port of the model

Buck converter example — use procedure of previous section to derive equivalent circuit

Average inductor voltage and capacitor current:

\[
\langle v_L \rangle = 0 = D V_g - I_L R_L - V_C \\
\langle i_C \rangle = 0 = I_L - V_C / R
\]
Construct equivalent circuit as usual

Construct equivalent circuit as usual

\[
\langle v_L \rangle = 0 = DV_g - I_L R_L - V_C \quad \langle i_C \rangle = 0 = I_L - V_C/R
\]

What happened to the transformer?
• Need another equation
Modeling the converter input port

Input current waveform $i_g(t)$:

Dc component (average value) of $i_g(t)$ is

$$I_g = \frac{1}{T_s} \int_0^{T_s} i_g(t) \, dt = DI_L$$
Input port equivalent circuit

\[ I_s = \frac{1}{T_s} \int_0^{T_s} i_g(t) \, dt = DI_L \]
Complete equivalent circuit, buck converter

Input and output port equivalent circuits, drawn together:

Replace dependent sources with equivalent dc transformer:
3.5. Example: inclusion of semiconductor conduction losses in the boost converter model

Models of on-state semiconductor devices:
- MOSFET: on-resistance $R_{on}$
- Diode: constant forward voltage $V_D$ plus on-resistance $R_D$

Insert these models into subinterval circuits
Boost converter example: circuits during subintervals 1 and 2

\[ V_S \]

\[ L \]

\[ \text{switch in position 1} \]

\[ \text{switch in position 2} \]
Average inductor voltage and capacitor current

\[ \langle v_L \rangle = D(V_g - IR_L - IR_{on}) + D'(V_g - IR_L - V_D - IR_D - V) = 0 \]

\[ \langle i_C \rangle = D(-V/R) + D'(I - V/R) = 0 \]
Construction of equivalent circuits

\[ V_g - IR_L - IDR_{on} - D'V_D - ID'R_D - D'V = 0 \]

\[ D'I - V/R = 0 \]
Complete equivalent circuit
Solution for output voltage

\[ V_s = \left( \frac{1}{D'} \right) \left( V_s - D' V_D \right) \left( \frac{D'^2 R}{D'^2 R + R_L + D'R_{on} + D'R_D} \right) \]

\[ \frac{V}{V_s} = \left( \frac{1}{D'} \right) \left( 1 - \frac{D' V_D}{V_s} \right) \left( \frac{1}{1 + \frac{R_L + D'R_{on} + D'R_D}{D'^2 R}} \right) \]
Solution for converter efficiency

\[ P_{in} = (V_s)(I) \]

\[ P_{out} = (V)(D'I) \]

\[ \eta = D' \frac{V}{V_g} = \frac{\left( 1 - \frac{D'V_D}{V_g} \right)}{\left( 1 + \frac{R_L + DR_{on} + D'R_D}{D'^2R} \right)} \]

Conditions for high efficiency:

\[ V_g/D' \gg V_D \]

\[ D'^2R \gg R_L + DR_{on} + D'R_D \]
Accuracy of the averaged equivalent circuit in prediction of losses

• Model uses average currents and voltages

• To correctly predict power loss in a resistor, use rms values

• Result is the same, provided ripple is small

MOSFET current waveforms, for various ripple magnitudes:

\[
i(t) = \begin{cases} 
0 & 0 \leq t < DT_s \\
I & DT_s \leq t < T_s \\
2I & T_s \leq t
\end{cases}
\]

<table>
<thead>
<tr>
<th>Inductor current ripple</th>
<th>MOSFET rms current</th>
<th>Average power loss in ( R_{on} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( \Delta i = 0 )</td>
<td>( I \sqrt{D} )</td>
<td>( D \hat{f} R_{on} )</td>
</tr>
<tr>
<td>(b) ( \Delta i = 0.1 I )</td>
<td>( (1.00167) I \sqrt{D} )</td>
<td>( (1.0033) D \hat{f} R_{on} )</td>
</tr>
<tr>
<td>(c) ( \Delta i = I )</td>
<td>( (1.155) I \sqrt{D} )</td>
<td>( (1.3333) D \hat{f} R_{on} )</td>
</tr>
</tbody>
</table>
Summary of chapter 3

1. The dc transformer model represents the primary functions of any dc-dc converter: transformation of dc voltage and current levels, ideally with 100% efficiency, and control of the conversion ratio $M$ via the duty cycle $D$. This model can be easily manipulated and solved using familiar techniques of conventional circuit analysis.

2. The model can be refined to account for loss elements such as inductor winding resistance and semiconductor on-resistances and forward voltage drops. The refined model predicts the voltages, currents, and efficiency of practical nonideal converters.

3. In general, the dc equivalent circuit for a converter can be derived from the inductor volt-second balance and capacitor charge balance equations. Equivalent circuits are constructed whose loop and node equations coincide with the volt-second and charge balance equations. In converters having a pulsating input current, an additional equation is needed to model the converter input port; this equation may be obtained by averaging the converter input current.