ECEN 4797/5797

Introduction to Power Electronics

Lecture #12

Monday, September 21, 2009

Discontinuous Conduction Mode

Chapter 5

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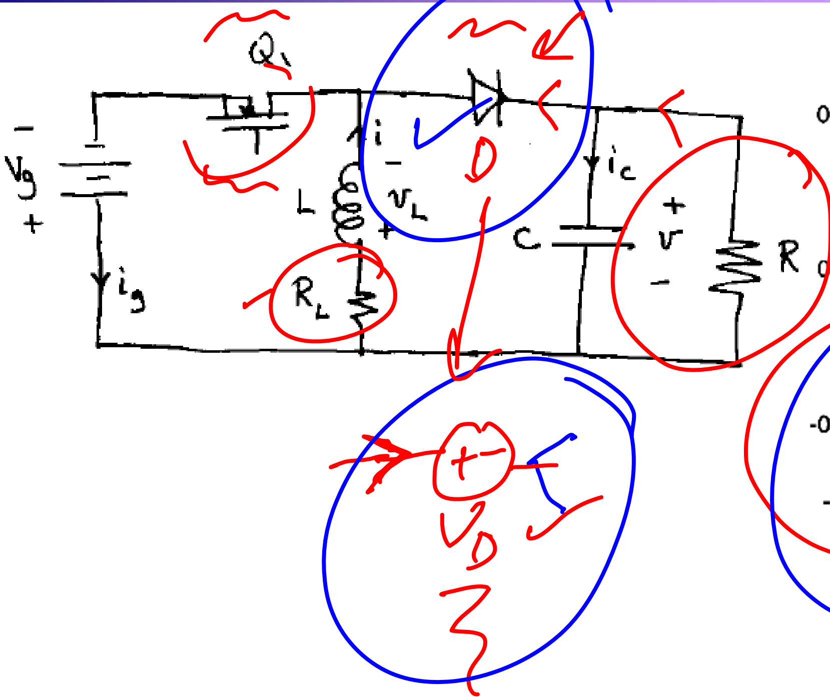
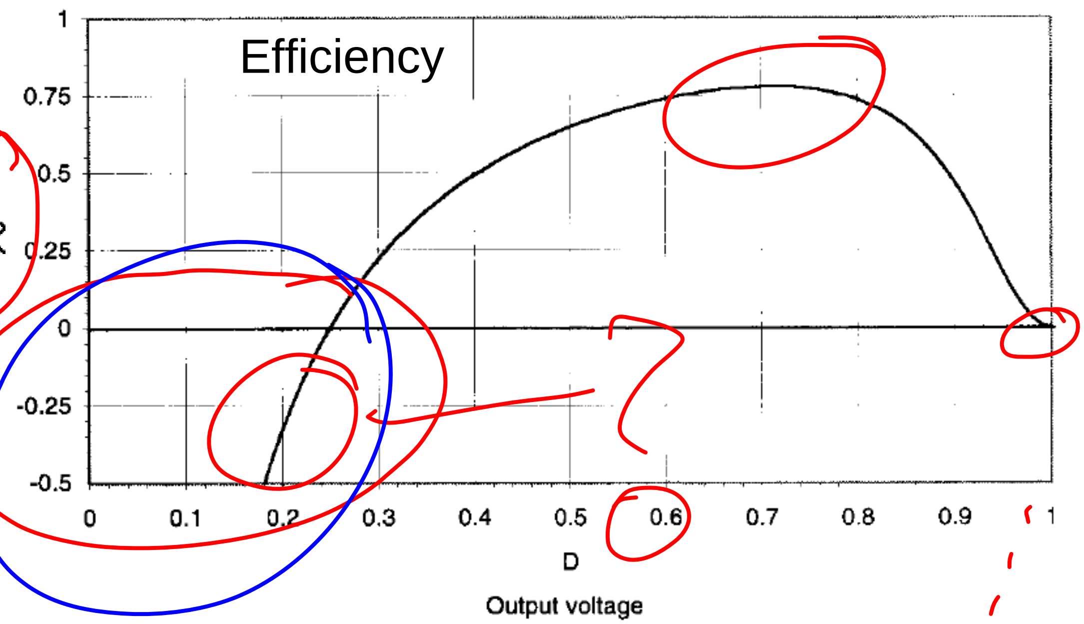
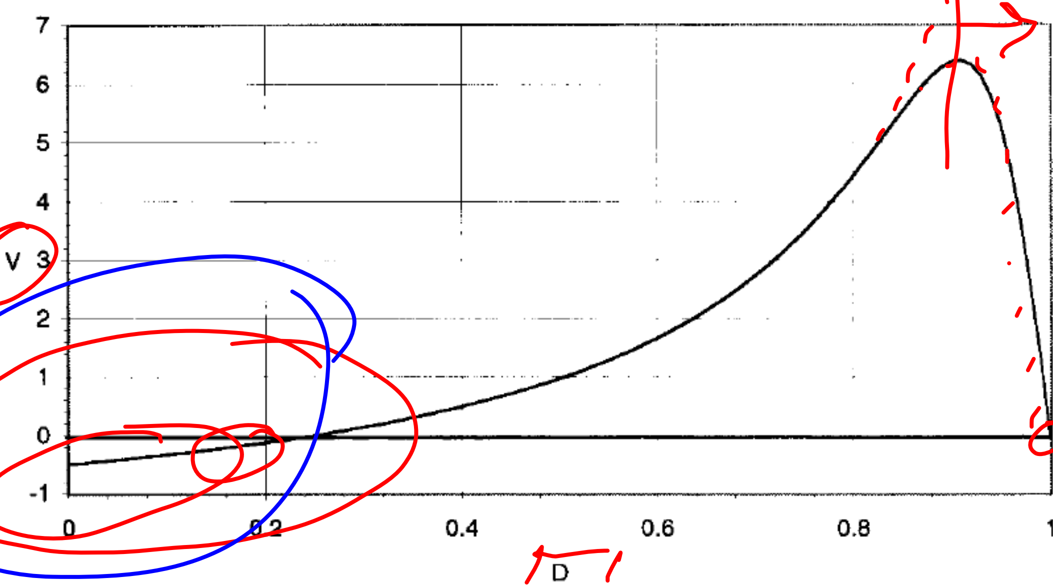
SPST

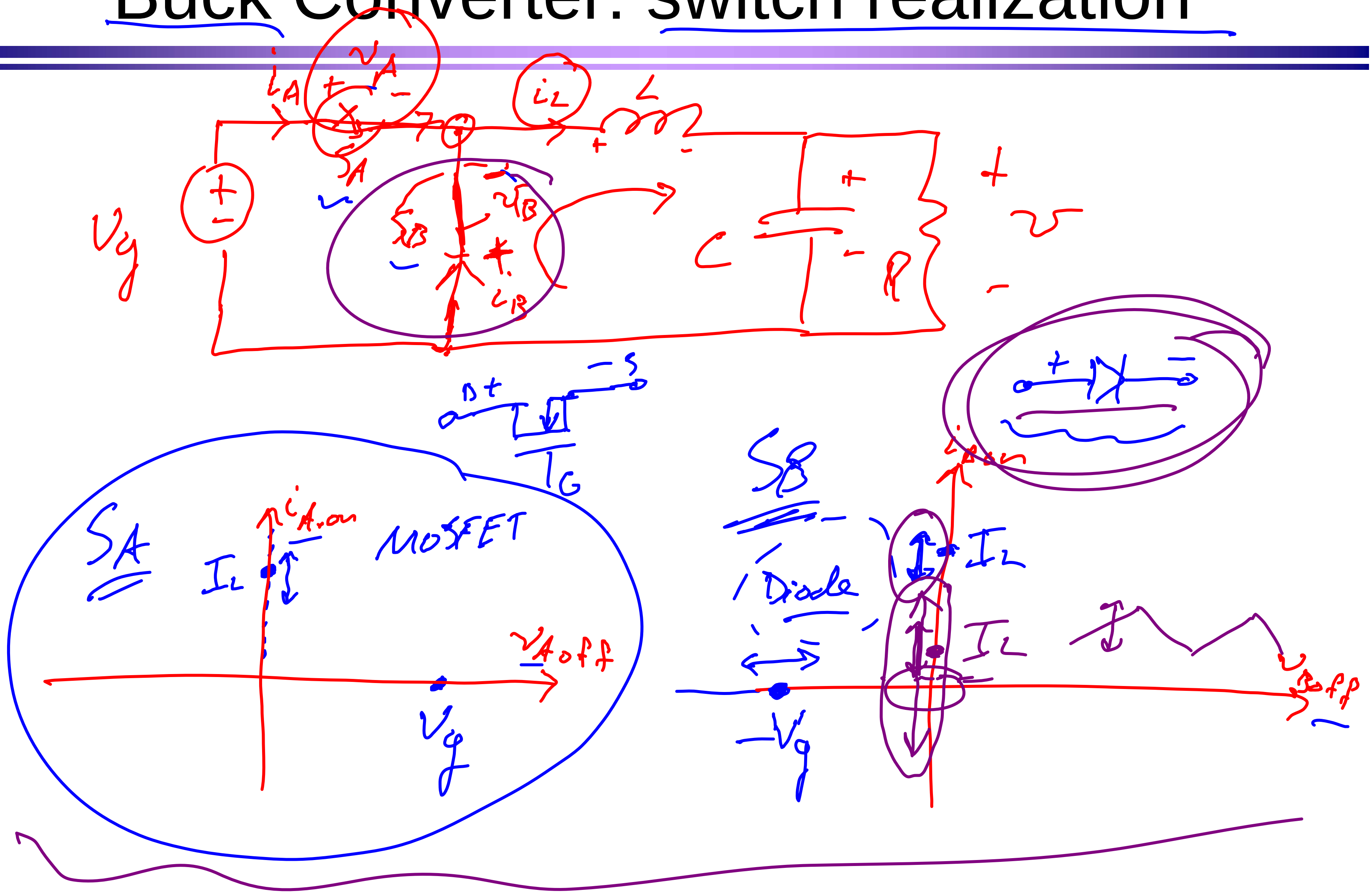
HW #2: Text Prob. 3.7





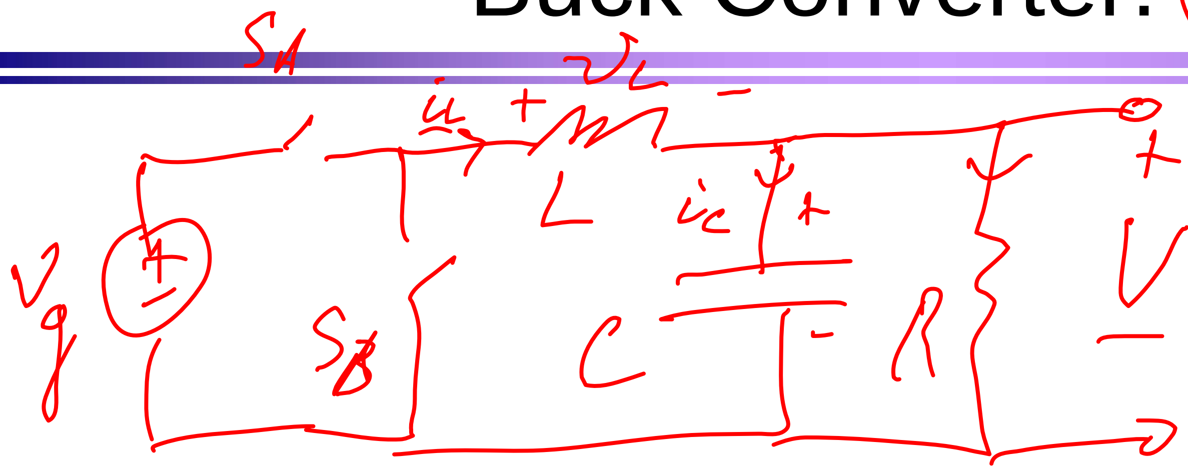


Buck Converter: switch realization



Buck Converter: CCM

"ideal"



steady state:

$V = D \cdot V_g$

$$\boxed{i_L \approx I_L = \frac{V}{R} = \frac{DV_g}{R}}$$

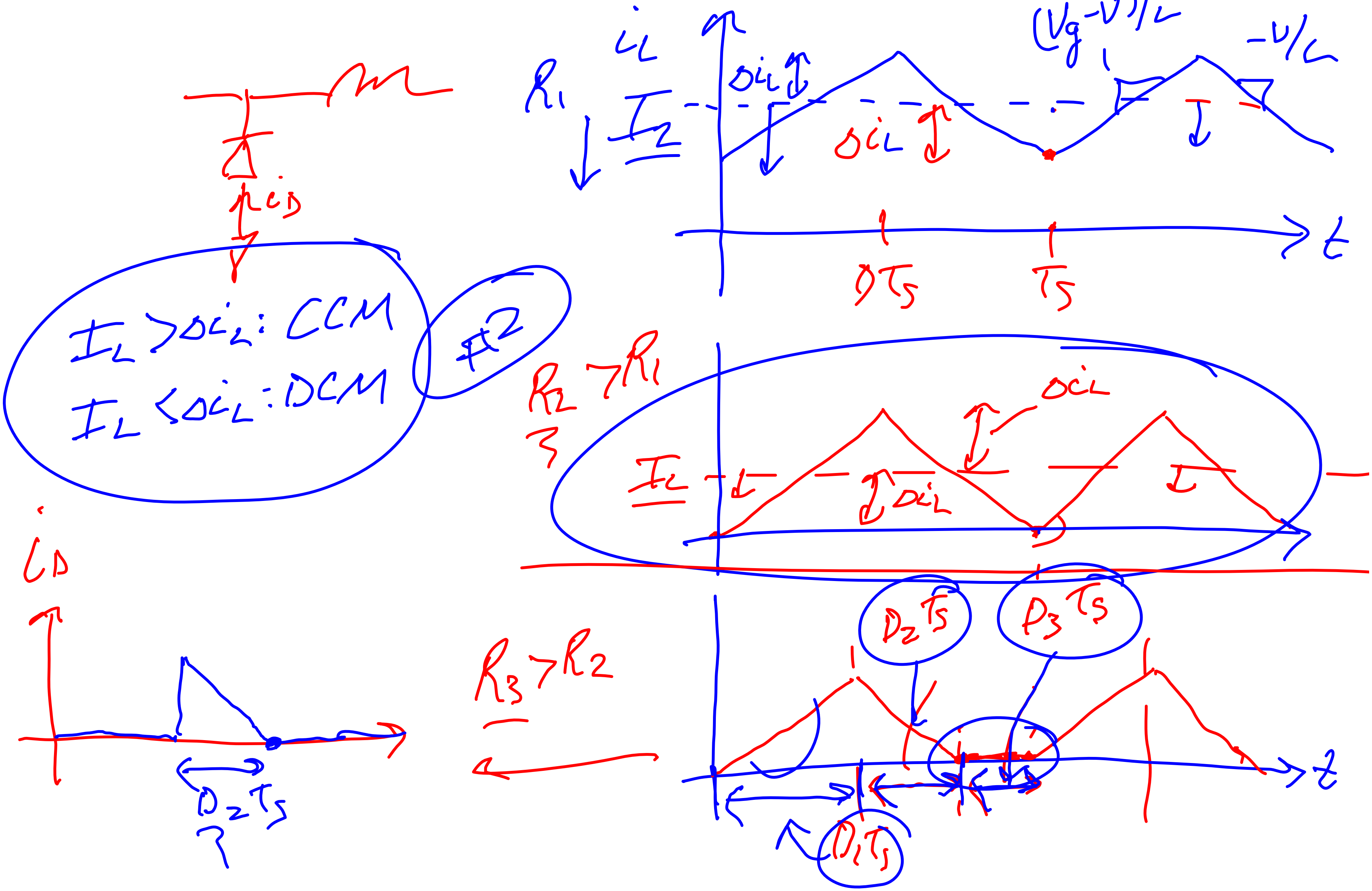
$D'T_s V = 2 \cdot 2\Delta i_L$

$$\boxed{\Delta i_L = \frac{D'T_s V}{2L} = \cancel{\frac{DD'T_s V_g}{2LR}}}$$

$$\boxed{\Delta i_L = \frac{D \cdot D' T_s V_g}{2L}}$$

not $f(R)$

Buck Converter: Inductor current



Mode boundary

$$I > \Delta i_L \quad \textit{for CCM}$$
$$I < \Delta i_L \quad \textit{for DCM}$$

Insert buck converter expressions for I and Δi_L :

I_L $$\frac{DV_g}{R} < \frac{DD'T_sV_g}{2L}$$ Δi_L

Simplify:

L, R, T_s $$\boxed{\frac{2L}{RT_s} < D'}$$ $\leftarrow f(D)$

$$K = \frac{2L}{RT_s}$$

?

This expression is of the form

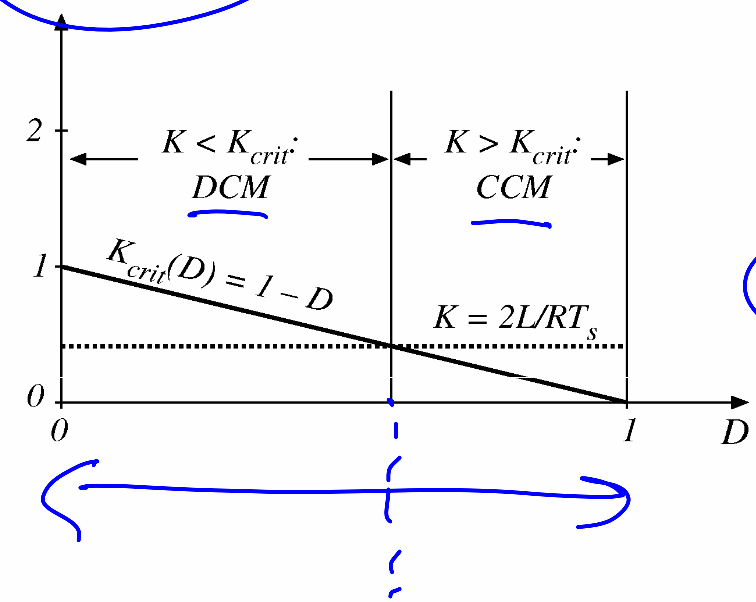
$$K < K_{crit}(D) \quad \textit{for DCM}$$

$$\text{where} \quad K = \frac{2L}{RT_s} \quad \textit{and} \quad K_{crit}(D) = D'$$

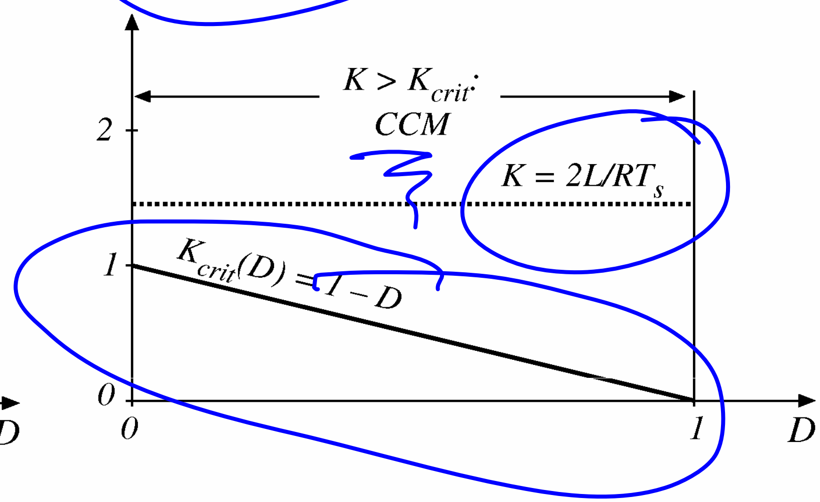
$K_{crit} = D' = 1 - D \rightarrow 0 \leq K_{crit} \leq 1$

K and K_{crit} vs. D

for $K < 1$:



for $K > 1$:



Critical load resistance R_{crit}

Solve K_{crit} equation for load resistance R:

$$R < R_{crit}(D) \qquad \textit{for CCM}$$
$$R > R_{crit}(D) \qquad \textit{for DCM}$$

where $$R_{crit}(D) = \frac{2L}{D'T_s}$$

Buck

Summary: mode boundary

$$K > K_{crit}(D) \quad or \quad R < R_{crit}(D) \quad for\ CCM$$
$$K < K_{crit}(D) \quad or \quad R > R_{crit}(D) \quad for\ DCM$$

Table 5.1. CCM-DCM mode boundaries for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	$\max_{0 \le D \le 1}(K_{crit})$	$R_{crit}(D)$	$\min_{0 \le D \le 1}(R_{crit})$
Buck	$(1 - D)$	1	$\frac{2L}{(1-D)T_s}$	$2\frac{L}{T_s}$
Boost	$D(1-D)^2$	$\frac{4}{27}$	$\frac{2L}{D(1-D)^2 T_s}$	$\frac{27}{2}\frac{L}{T_s}$
Buck-boost	$(1-D)^2$	1	$\frac{2L}{(1-D)^2 T_s}$	$2\frac{L}{T_s}$

$V/V_g = M(D, R_{on}, R_L, V_D, Q_R, t_{rr}, t_s, R)$

5.2. Analysis of the conversion ratio $M(D,K)$

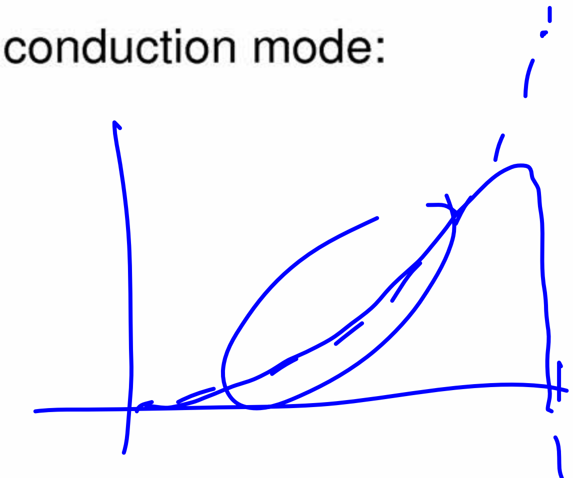
Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance

$$\langle v_L \rangle = \frac{1}{T_s} \int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\langle i_C \rangle = \frac{1}{T_s} \int_0^{T_s} i_C(t)\, dt = 0$$



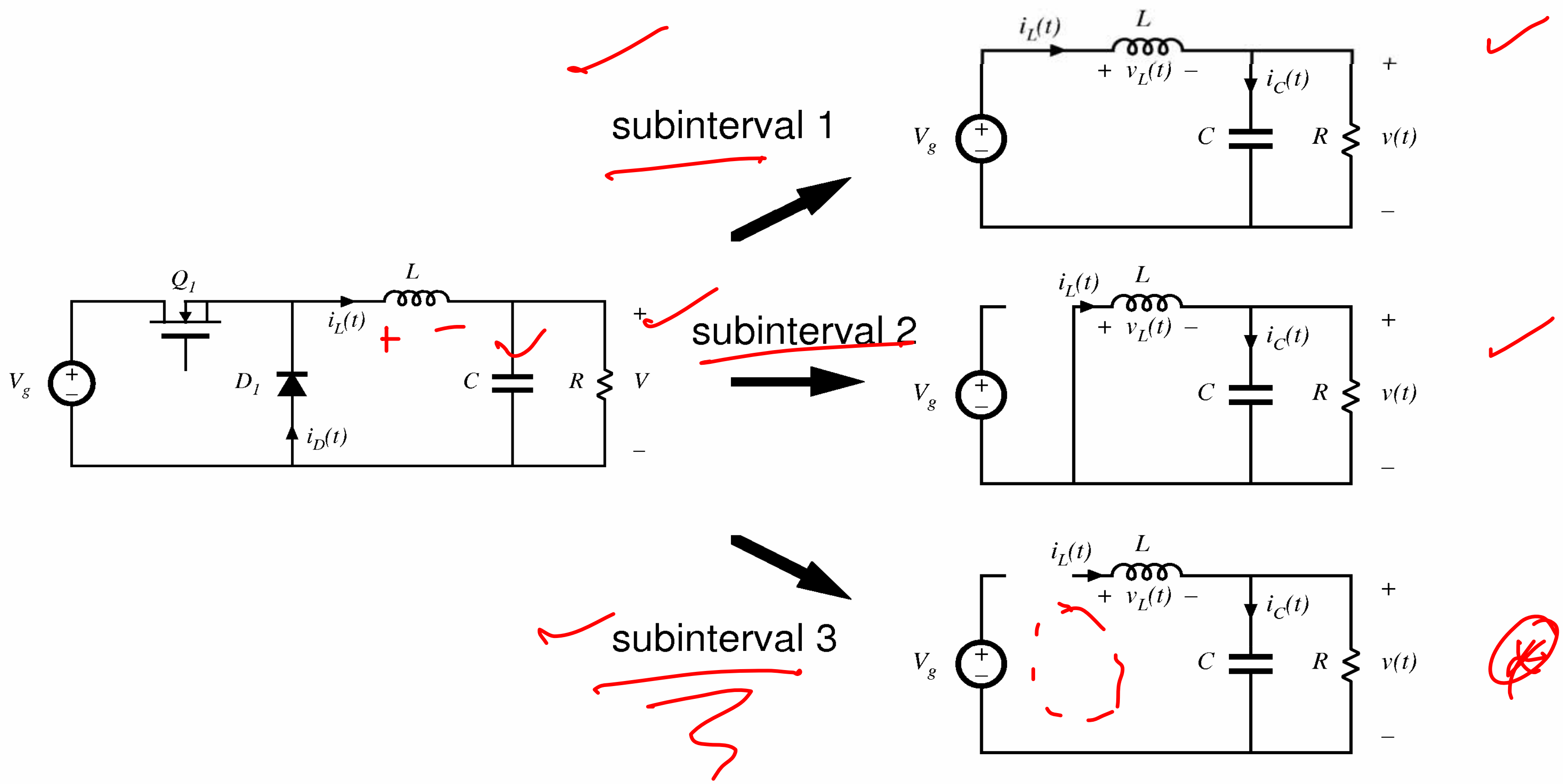
Small ripple approximation sometimes applies:

$v(t) \approx V$ because $\Delta v << V$

$i(t) \approx I$ is a poor approximation when $\Delta i > I$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Example: Analysis of DCM buck converter $M(D,K)$



DC $\approx$ DC

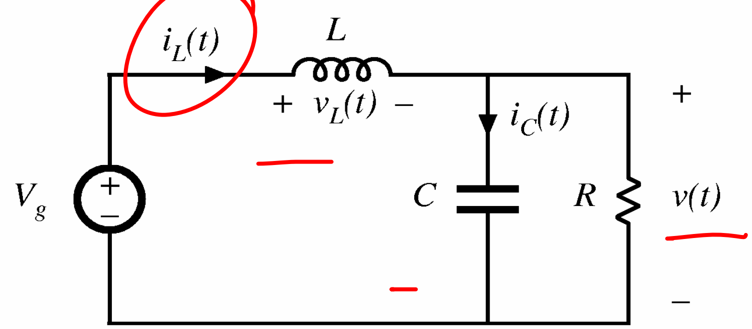
Subinterval 1

$$v_L = V_g - v(t)$$

$$\boxed{v_L \approx V_g - V}$$

$$i_C = i_L(t) - v/R$$

$$\boxed{i_C \approx i_L - V/R}$$



5V

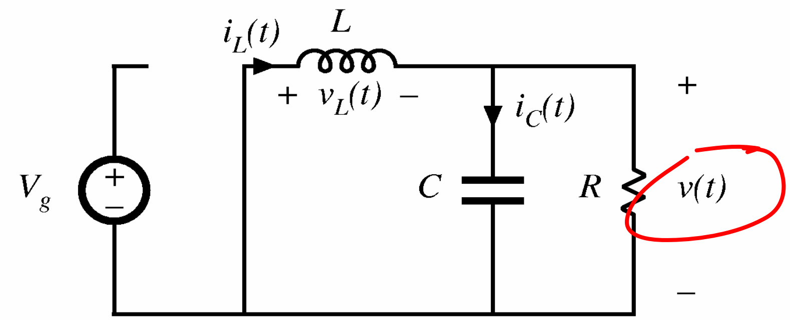
+/- 2%

Subinterval 2

$$\boxed{v_L = -v \approx -V}$$

$$i_C = i_L - v/R$$

$$\boxed{i_C \approx i_L - V/R}$$



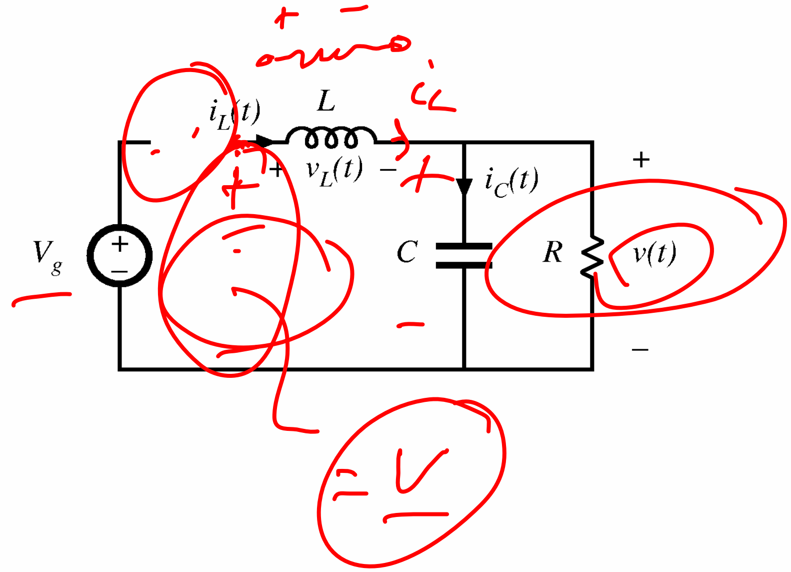
$$v_L = L\frac{di}{dt}$$

Subinterval 3

$$\boxed{v_L = \phi}$$

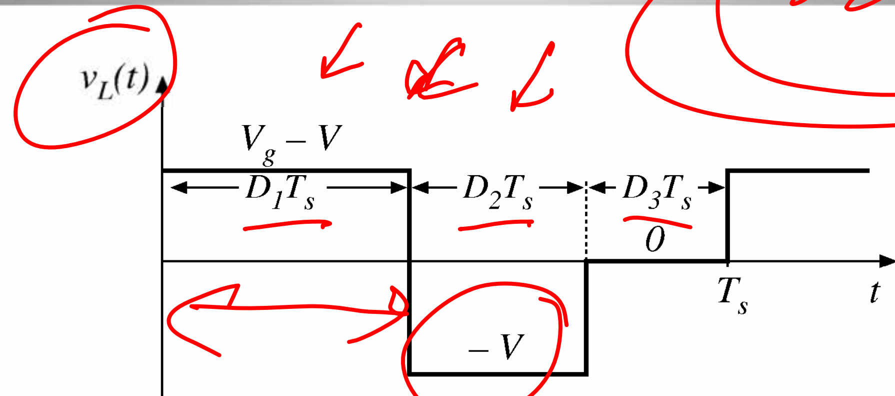
$$i_C = i_L - \frac{v}{R}$$

$$\boxed{i_C = -\frac{V}{R}}$$



Inductor volt-second balance

$D_2 = ?$



$$D_1(V_g - V) - D_2 V + D_3 \cdot (0) = 0$$

$$D_1 V_g - (D_1 + D_2) V = 0$$

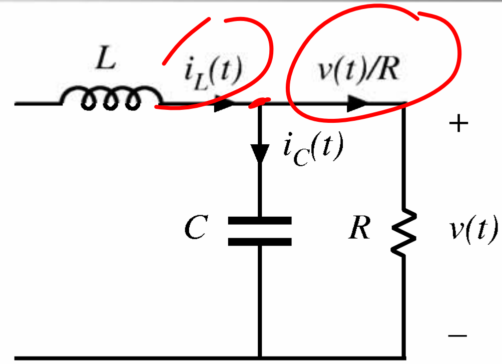
$$V = V_g \cdot \frac{D_1}{D_1 + D_2}$$

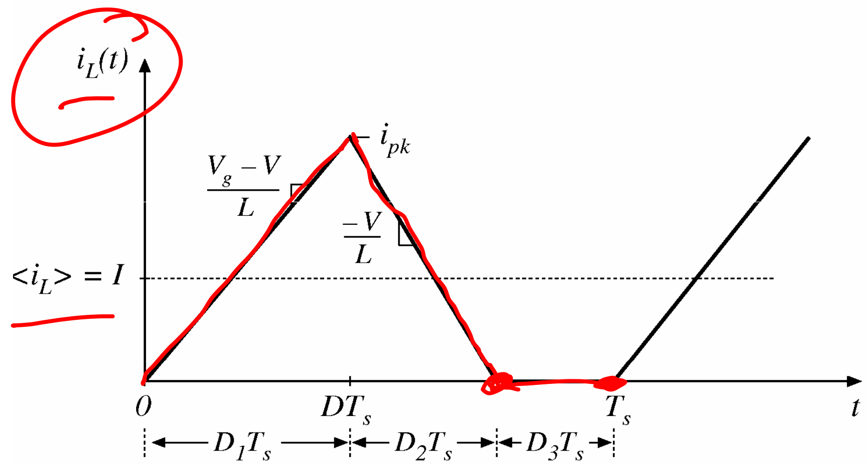
Capacitor charge balance

$\langle i_C \rangle = 0$

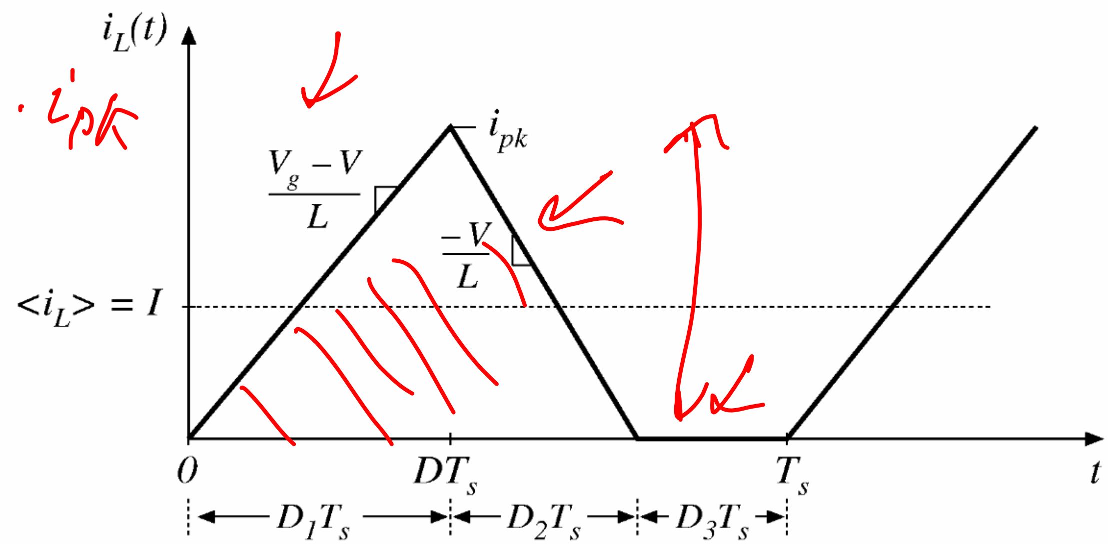
$\langle i_L \rangle = V/R$

I





Inductor Current Waveform



$$\langle i_L \rangle = \frac{1}{T_s}\cdot\frac{1}{2}\cdot(D_1+D_2)\,T_s\cdot i_{pk}$$

$$i_{pk} = \frac{(V_g - V)}{L}\cdot D_1 T_s$$

$D_1 T_s$

$$\langle i_L \rangle = \frac{T_s^{\cancel{2}}(D_1+D_2)\,(V_g-V)\,D_1}{2\cancel{T_s}\cdot L} = \frac{D_1 T_s}{2L}\cdot(D_1+D_2)(V_g-V)$$

$$= V/R$$

DCM

Solution for V

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2}$$ (from inductor volt-second balance)

$$\frac{V}{R} = \frac{D_1 T_s}{2L}(D_1 + D_2)(V_g - V)$$ (from capacitor charge balance)

Eliminate D_2 , solve for V :

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

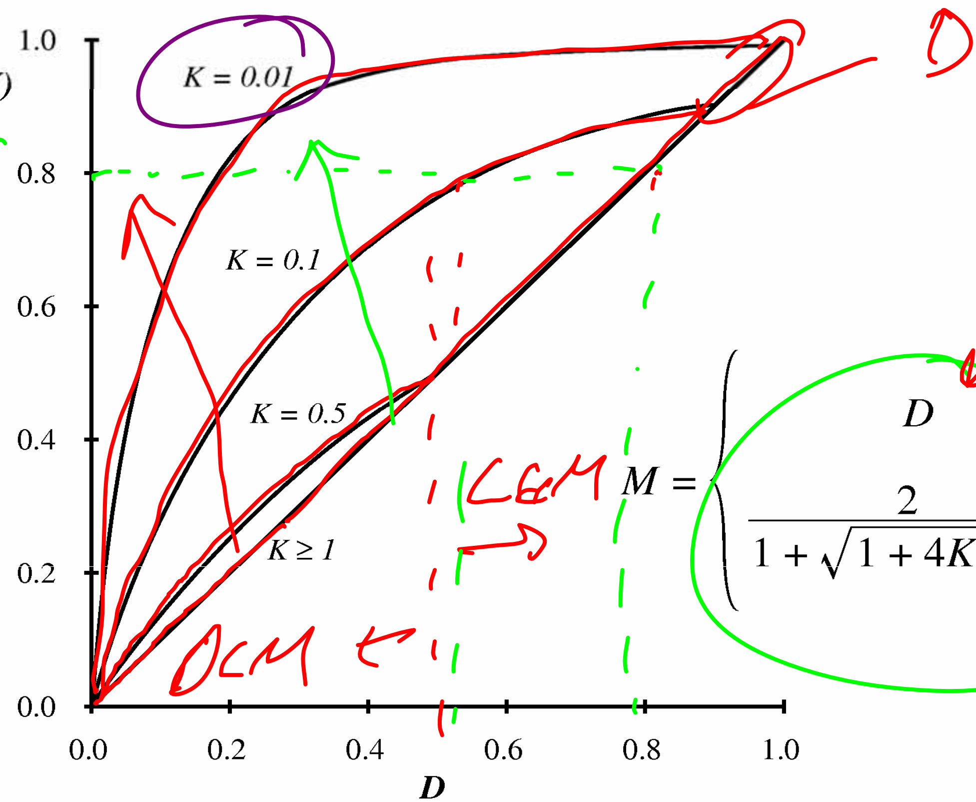
where $K = 2L / RT_s$

valid for $K < K_{crit}$

$$K = \frac{2L}{RT_s}$$

Load dep.!

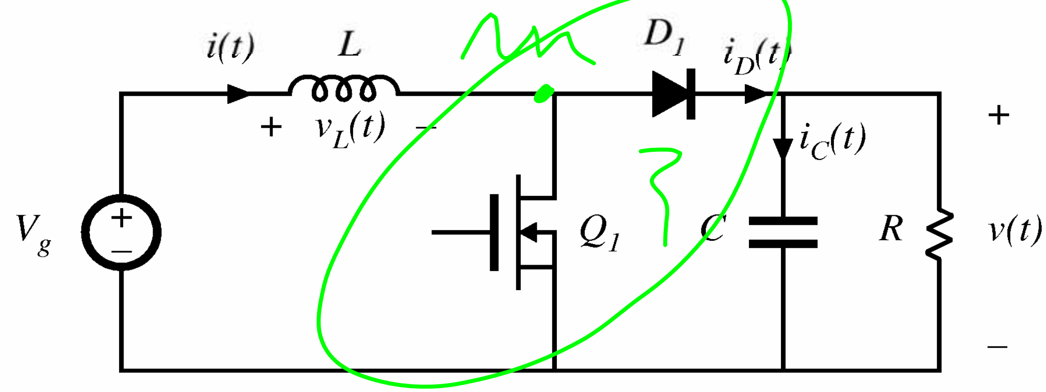
Buck converter $M(D,K)$



$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1+\sqrt{1+4K/D^2}} & \text{for } K < K_{crit} \end{cases}$$

$\frac{V}{V_g} = M$

5.3. Boost converter example



Mode boundary:

$$I > \Delta i_L \quad for\ CCM$$
$$I < \Delta i_L \quad for\ DCM$$

Previous CCM soln:

$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} DT_s$$