

# ECEN 4797/5797

Introduction to Power Electronics

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Lecture #20

Friday, October 9, 2009

Part II: Converter Dynamics & Control

Chapter 7: Intro ac equivalent circuit modeling

Sections 7.1 to 7.2

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# 7.1. Introduction

✓ nonlinear  
 ✓ time-variant  
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 sampled

Objective: maintain  $v(t)$  equal to an accurate, constant value  $V$ .

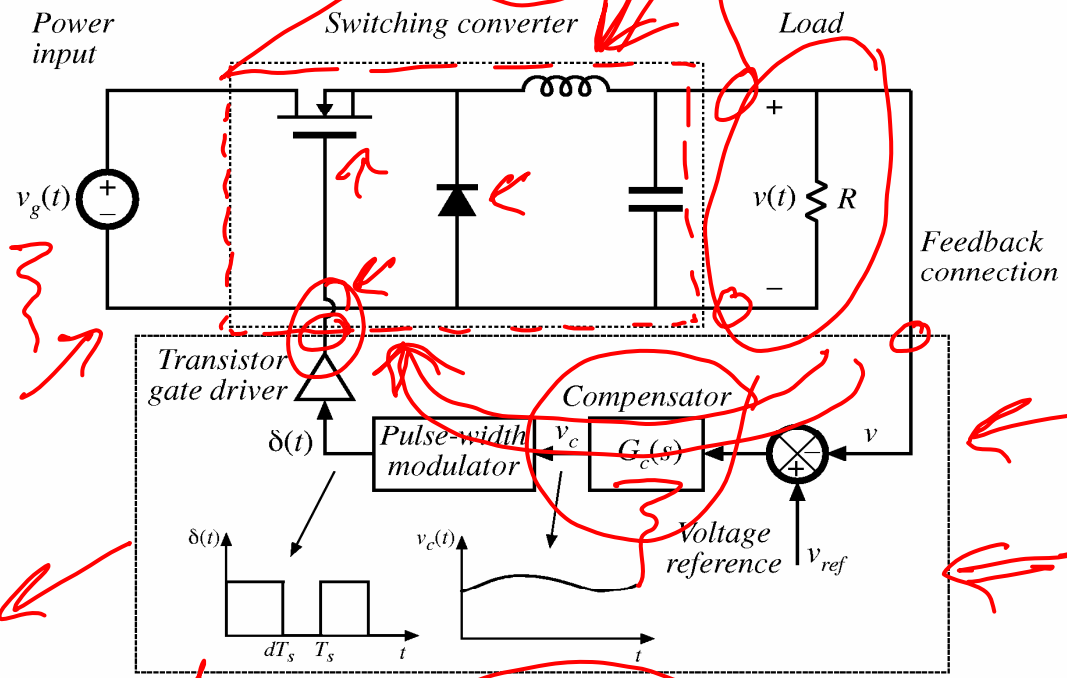
There are disturbances:

- in  $v_g(t)$
- in  $R$

There are uncertainties:

- in element values
- in  $V_g$
- in  $R$

A simple dc-dc regulator system, employing a buck converter



Linear  
 time-invariant  
 LTI

Controller

# Applications of control in power electronics

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## ✓ *DC-DC converters*

Regulate dc output voltage.

Control the duty cycle  $d(t)$  such that  $v(t)$  accurately follows a reference signal  $v_{ref}$ .

## ✓ *DC-AC inverters*

Regulate an ac output voltage.

Control the duty cycle  $d(t)$  such that  $v(t)$  accurately follows a reference signal  $v_{ref}(t)$ .

## ✓ *AC-DC rectifiers*

Regulate the dc output voltage.

Regulate the ac input current waveform.

Control the duty cycle  $d(t)$  such that  $i_g(t)$  accurately follows a reference signal  $i_{ref}(t)$ , and  $v(t)$  accurately follows a reference signal  $v_{ref}$ .

# Converter Modeling

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## Applications

Aerospace worst-case analysis

Commercial high-volume production: design for reliability and yield

## High quality design

Ensure that the converter works well under worst-case conditions

- Steady state (losses, efficiency, voltage regulation)
- Small-signal ac (controller stability and transient response)

## Engineering methodology

Simulate model during preliminary design (design verification)

Construct laboratory prototype converter system and make it work under nominal conditions

Develop a converter model. Refine model until it predicts behavior of nominal laboratory prototype

Use model to predict behavior under worst-case conditions

Improve design until worst-case behavior meets specifications (or until reliability and production yield are acceptable)

# Modeling

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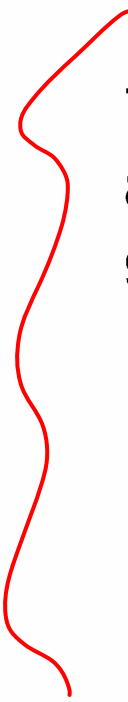
- Representation of physical behavior by mathematical means
- Model dominant behavior of system, ignore other insignificant phenomena
- Simplified model yields physical insight, allowing engineer to design system to operate in specified manner
- Approximations neglect small but complicating phenomena
- After basic insight has been gained, model can be refined (if it is judged worthwhile to expend the engineering effort to do so), to account for some of the previously neglected phenomena

# Part II

## Converter Dynamics and Control

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7. AC equivalent circuit modeling
  8. Converter transfer functions
  9. Controller design
  10. Input filter design
  11. AC and DC equivalent circuit modeling of the discontinuous conduction mode
  12. Current programmed control

# Objective of Part II

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Develop tools for modeling, analysis, and design of converter control systems

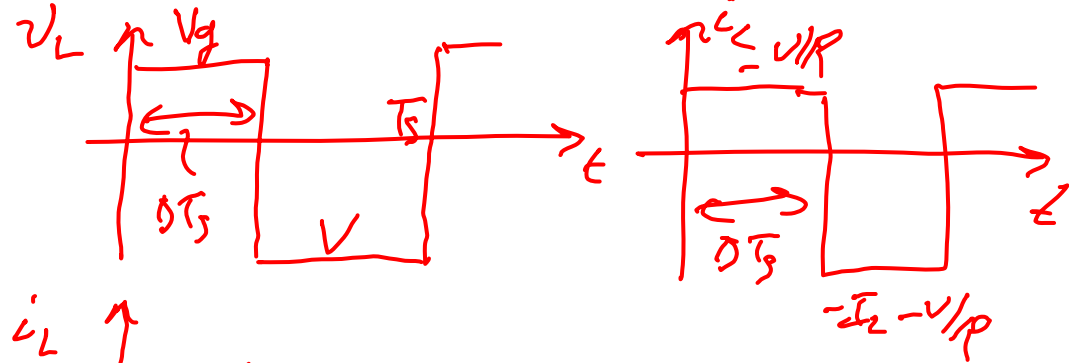
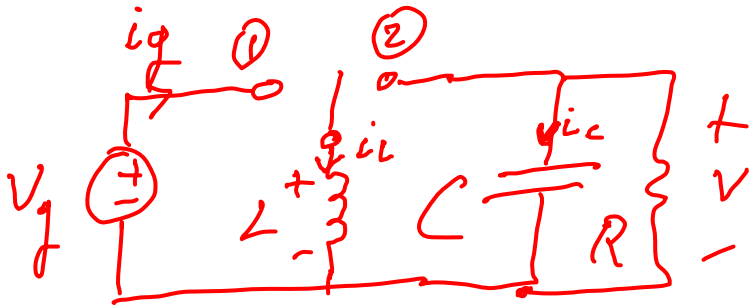
Need dynamic models of converters:

How do ac variations in  $v_g(t)$ ,  $R$ , or  $d(t)$  affect the output voltage  $v(t)$ ?

What are the small-signal transfer functions of the converter?

- Extend the steady-state converter models of Chapters 2 and 3, to include CCM converter dynamics (Chapter 7)
- Construct converter small-signal transfer functions (Chapter 8)
- Design converter control systems (Chapter 9)
- Design input EMI filters that do not disrupt control system operation (Chapter 10)
- Model converters operating in DCM (Chapter 11)
- Current-programmed control of converters (Chapter 12)

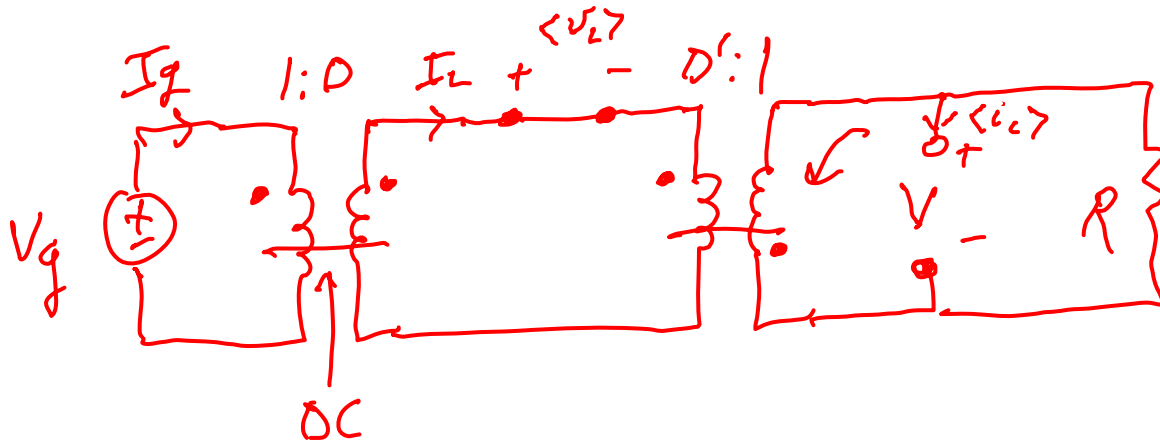
# Buck-boost converter modeling



$$\langle v_L \rangle = DV_g + D'V = 0 \quad \text{Steady state}$$

$$\langle i_C \rangle = -D'I_L - V/R = 0$$

$$\langle i_g \rangle = DI_L$$



# Buck-boost: add slow variations

$$V = -\frac{DV_g}{1-D} = \frac{-D^{\leftarrow} V_G^{\leftarrow}}{\frac{D^{\leftarrow}}{2}}$$

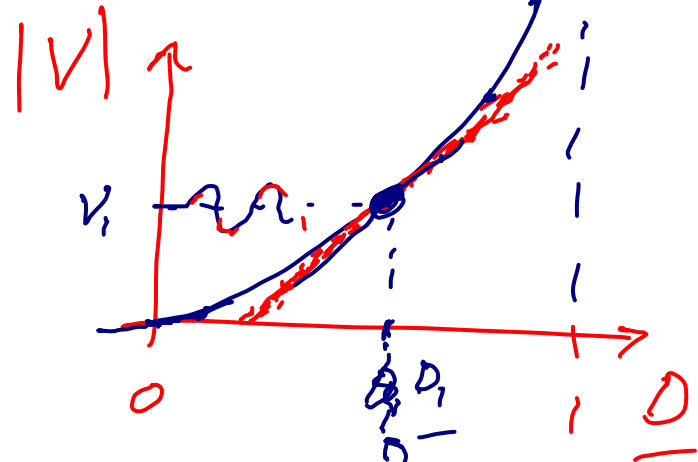
Nonlinear

Linearize

Linear

$$V = f(D, V_g) = V_1 + \hat{v} \quad \text{DC: know}$$

$$\hat{v} = \left. \frac{\partial f}{\partial D} \right|_{D_1} \hat{d} + \left. \frac{\partial f}{\partial V_g} \right|_{V_{g1}} \hat{v}_g$$



$$D = D_1 + \hat{d}$$

$$V = V_1 + \hat{v} + \text{"higher terms"}$$

1st order term

# The small-signal assumption

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If the ac variations are much smaller in magnitude than the respective quiescent values,

$$|\hat{v}_g(t)| \ll |V_g|$$

$$|\hat{d}(t)| \ll |D|$$

$$|\hat{i}(t)| \ll |I|$$

$$|\hat{v}(t)| \ll |V|$$

$$|\hat{i}_g(t)| \ll |I_g|$$

then the nonlinear converter equations can be linearized.

# Linearization: Taylor series expansion

$$\begin{aligned} \langle v_L \rangle &= \underbrace{DV_g}_{\uparrow \uparrow} + \underbrace{D'V}_{\uparrow} \approx 0 \end{aligned}$$

$$\langle i_C \rangle = -\underline{D'} I_L - \underline{V}/R \approx 0$$

$$\langle i_g \rangle = \underline{D} I_L$$

non linear, avg.

$$\underline{DC}: \quad \underline{V_L = 0 = D_i V_g + D'_i V_i = 0}$$

$$\underline{\hat{v}_L} = \underbrace{(D_i)}_{\uparrow \frac{\partial \langle v_L \rangle}{\partial v_g}} \cdot \hat{v}_g + \underbrace{(D'_i)}_{\uparrow \frac{\partial \langle v_L \rangle}{\partial v}} \hat{v} + \underbrace{(V_{g_i} - V_i)}_{\uparrow \frac{\partial \langle v_L \rangle}{\partial d}} \hat{d} \approx 0$$

$$\underline{\hat{i}_C} = -D'_i \hat{i}_L - \frac{\hat{v}}{R} + I_L \hat{d} \approx 0$$

$$\underline{\hat{i}_g} = D_i \hat{i}_L + I_L \hat{d}$$

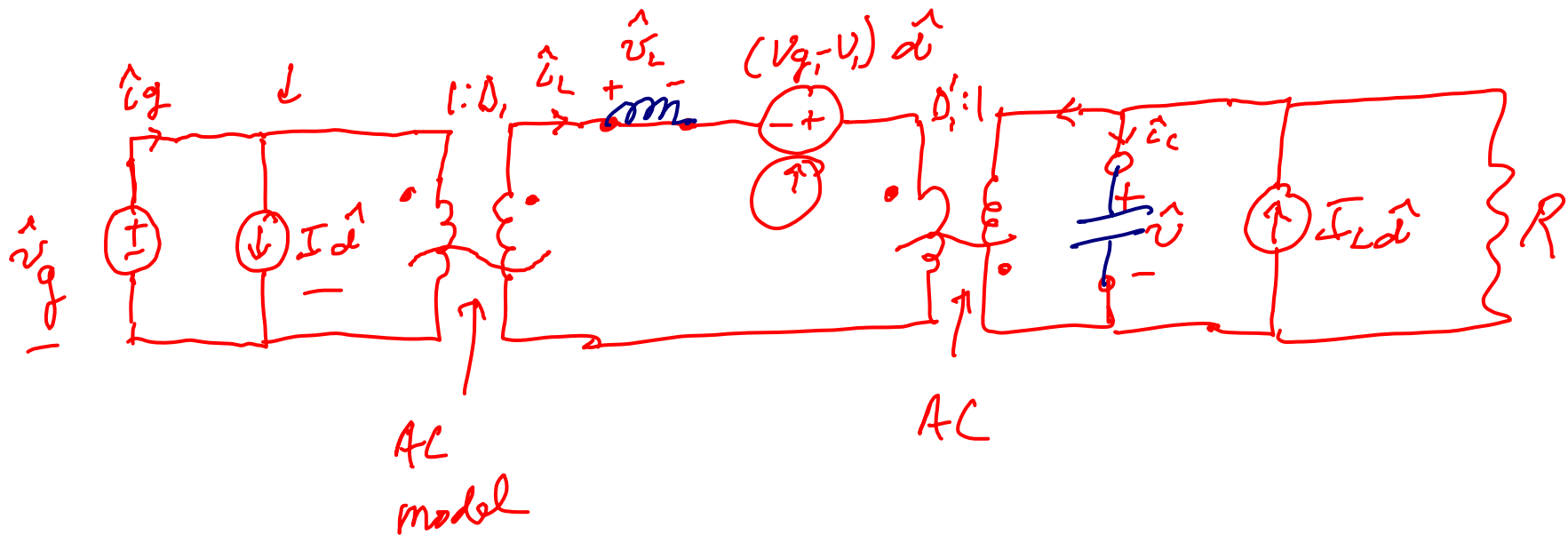
# Averaged, linear, small-signal (but ~steady-state [slow])

$$\hat{v}_L = \underline{D\hat{v}_g(t)} + \underline{D'\hat{v}(t)} + \underline{(V_g - V)\hat{d}(t)} \approx 0$$

$$\hat{i}_c = -\underline{D'\hat{i}(t)} - \underline{\frac{\hat{v}(t)}{R}} + \underline{I\hat{d}(t)} \approx 0$$

$$\hat{i}_g = \underline{D\hat{i}(t)} + \underline{I\hat{d}(t)}$$

$$\hat{v} = \frac{V_g - V}{D'} \hat{d}$$



# Step change

