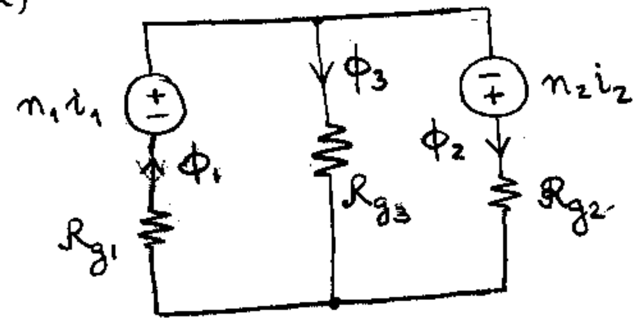
Problem 13.4

(a)



magnetic circuit model

$$\mathcal{R}_{g1} = \frac{g_1}{\mu_0 A_c}$$

$$\mathcal{R}_{g2} = \frac{g_2}{\mu_0 A_c}$$

$$\mathcal{R}_{g3} = \frac{g_3}{\mu_0 A_c}$$

$\mu >> \mu_0$, neglect core reluctances.

(b) Solve the model in (a):

$$\phi_1 = \frac{n_1 i_1}{\mathcal{R}_{g1} + \mathcal{R}_{g2} \| \mathcal{R}_{g3}} + \frac{n_2 i_2}{\mathcal{R}_{g2} + \mathcal{R}_{g1} \| \mathcal{R}_{g3}} \frac{\mathcal{R}_{g3}}{\mathcal{R}_{g1} + \mathcal{R}_{g3}}$$

$$\phi_2 = \frac{n_1 i_1}{\mathcal{R}_{g1} + \mathcal{R}_{g2} \| \mathcal{R}_{g3}} \frac{\mathcal{R}_{g3}}{\mathcal{R}_{g2} + \mathcal{R}_{g3}} + \frac{n_2 i_2}{\mathcal{R}_{g2} + \mathcal{R}_{g1} \| \mathcal{R}_{g3}}$$

$$v_1 = n_1 \frac{d\phi_1}{dt} = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

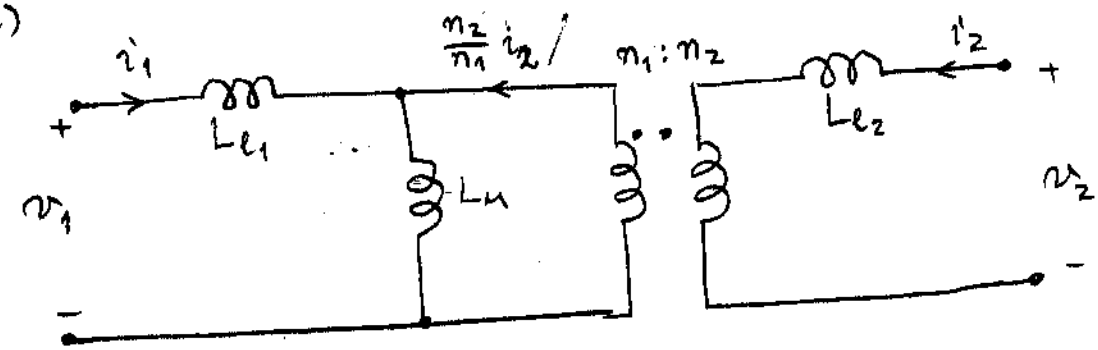
$$v_2 = n_2 \frac{d\phi_2}{dt} = L_{12} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt}$$

$$L_{11} = \frac{n_1^2}{\mathcal{R}_{g1} + \mathcal{R}_{g2} \| \mathcal{R}_{g3}} = \mu_0 A_c n_1^2 \frac{g_2 + g_3}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

$$L_{12} = \frac{n_1 n_2}{\mathcal{R}_{g2} + \mathcal{R}_{g1} \| \mathcal{R}_{g3}} \frac{\mathcal{R}_{g3}}{\mathcal{R}_{g1} + \mathcal{R}_{g3}} = \mu_0 A_c n_1 n_2 \frac{g_3}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

$$L_{22} = \frac{n_2^2}{\mathcal{R}_{g2} + \mathcal{R}_{g1} \| \mathcal{R}_{g3}} \frac{\mathcal{R}_{g3}}{\mathcal{R}_{g2} + \mathcal{R}_{g3}} = \mu_0 A_c n_2^2 \frac{g_1 + g_3}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

(c)



electrical circuit model

$$v_1 = L_{e1} \frac{di_1}{dt} + L_M \frac{d}{dt}\left(i_1 + \frac{n_2}{n_1} i_2\right)$$

$$v_1 = \underbrace{\left(L_{e1} + L_M\right)}_{L_{11}} \frac{di_1}{dt} + \underbrace{L_M \frac{n_2}{n_1}}_{L_{12}} \frac{di_2}{dt}$$

$$v_2 = L_{e2} \frac{di_2}{dt} + \frac{n_2}{n_1} L_M \frac{d}{dt}\left(i_1 + \frac{n_2}{n_1} i_2\right)$$

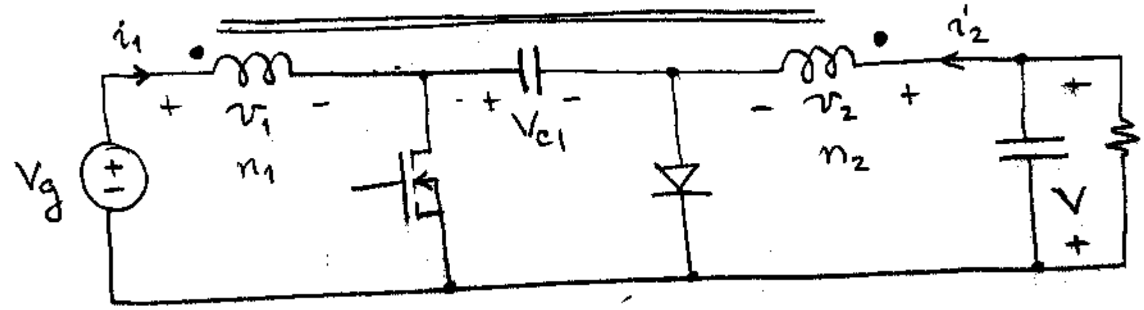
$$v_2 = \underbrace{L_M \frac{n_2}{n_1}}_{L_{12}} \frac{di_1}{dt} + \underbrace{\left(L_{e2} + \left(\frac{n_2}{n_1}\right)^2 L_M\right)}_{L_{22}} \frac{di_2}{dt}$$

$$L_M = \frac{n_1}{n_2} L_{12} = \mu_0 A_c n_1^2 \frac{g_3}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

$$L_{e1} = L_{11} - L_M = \mu_0 A_c n_1^2 \frac{g_2}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

$$L_{e2} = L_{22} - \left(\frac{n_2}{n_1}\right)^2 L_M = \mu_0 A_c n_2^2 \frac{g_1}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

(d)

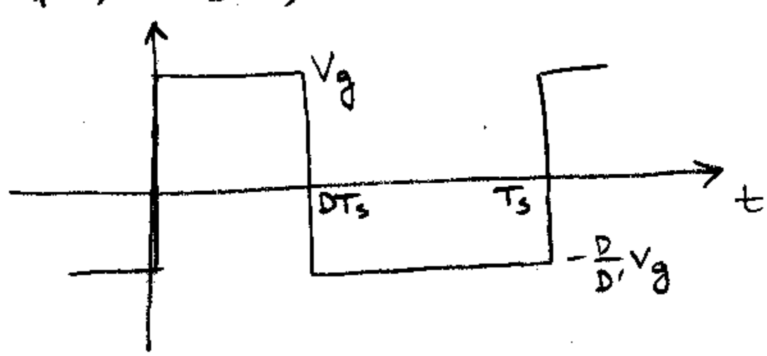


$$V = \frac{D}{D'} V_g \quad , \quad V_{c1} = V_g + V = \frac{1}{D'} V_g$$

$$v_1 = \begin{cases} V_g \ , & DT_s \\ V_g - V_{c1} = -\frac{D}{D'} V_g \ , & D'T_s \end{cases}$$

$$v_2 = \begin{cases} V_{c1} - V = V_g \ , & DT_s \\ -V = -\frac{D}{D'} V_g \ , & D'T_s \end{cases}$$

$v_1(t) = v_2(t)$



The voltage waveforms $v_1(t)$ and $v_2(t)$ <u>are the same</u>!

(e)
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{L_{11}L_{22} - L_{12}^2} \begin{bmatrix} L_{22} & -L_{12} \\ -L_{12} & L_{11} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

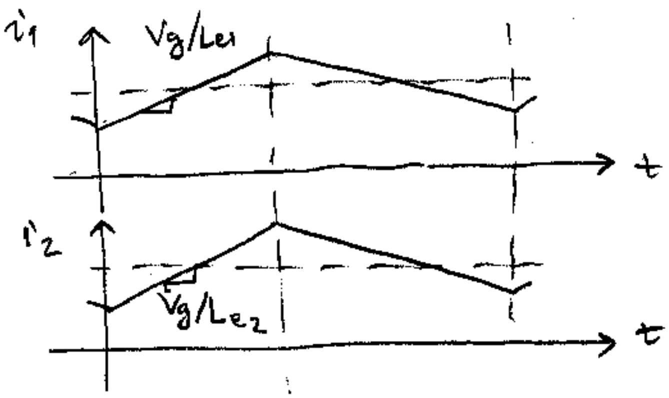
use $v_1 = v_2$ from part (d)

$$\frac{di_1}{dt} = \frac{L_{22} - L_{12}}{L_{11}L_{22} - L_{12}^2} v_1 = \frac{v_1}{L_{e1}}$$

$$\frac{di_2}{dt} = \frac{L_{11} - L_{12}}{L_{11}L_{22} - L_{12}^2} v_1 = \frac{v_1}{L_{e2}}$$

$$L_{e1} = \frac{L_{11}L_{22} - L_{12}^2}{L_{22} - L_{12}} = \mu_0 A_c n_1^2 \frac{1}{g_1 + g_3\left(1 - \frac{n_1}{n_2}\right)}$$

$$L_{e2} = \frac{L_{11}L_{22} - L_{12}^2}{L_{11} - L_{12}} = \mu_0 A_c n_2^2 \frac{1}{g_2 + g_3\left(1 - \frac{n_2}{n_1}\right)}$$



(f) To make $\Delta i_2 = 0$, we need $\frac{di_2}{dt} = 0$,

or, equivalently $L_{e2} \to \infty$

This can be accomplished by choosing

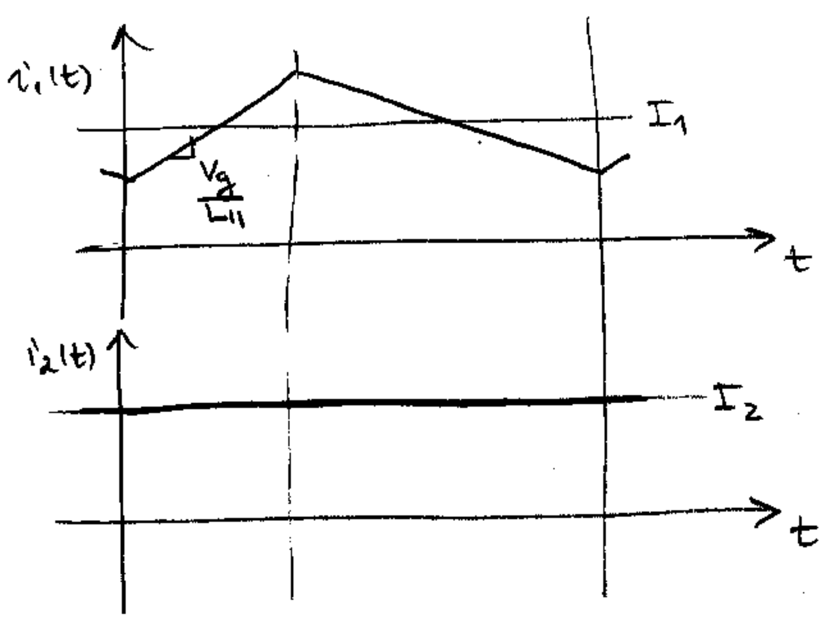
$$g_2 + g_3\left(1 - \frac{n_2}{n_1}\right) = 0$$

$$\boxed{\frac{n_2}{n_1} = \frac{g_2}{g_3} + 1 = \frac{g_2 + g_3}{g_3}.}$$

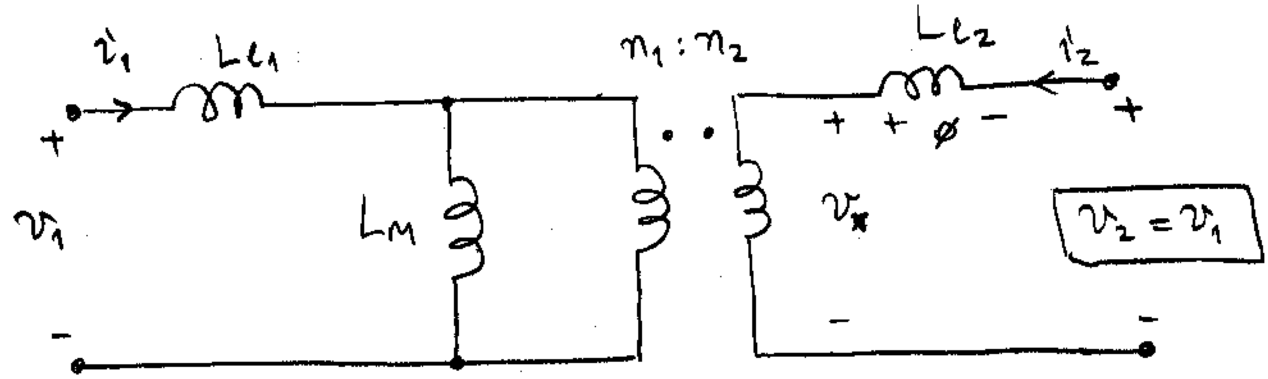
In this case,

$$L_{e1} = \mu_0 A_c n_1^2 \frac{g_2 + g_3}{g_1 g_2 + g_1 g_3 + g_2 g_3}$$

$$L_{e1} = L_{11}$$



A physical interpretation of the "zero-ripple" condition can be made in terms of the electrical circuit model of part (c):



The objective of zero ripple in i_2 can be achieved by having ϕ volts across $L_{\ell 2}$:

$$v_x = \underbrace{\frac{L_M}{L_M + L_{\ell 1}} \cdot \frac{n_2}{n_1}}_{1} v_1 = v_1$$

Zero-ripple in i_2 : $\boxed{\dfrac{L_M}{L_M + L_{\ell 1}} \dfrac{n_2}{n_1} = 1}$

From (c), we get: $\dfrac{g_3}{g_2 + g_3} \dfrac{n_2}{n_1} = 1$

or $\boxed{\dfrac{n_2}{n_1} = \dfrac{g_2 + g_3}{g_3}}$

which is the same as we already found algebraically from the inductance-matrix model.